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COMMENT ON "SOUTHWARD MAGNETIC FIELD IN THE NEUTRAL SHEET PRODUCED BY WAVY MOTIONS PROPAGATING IN THE DAWN-DUSK DIRECTION"

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Abstract. In a recent paper Nakagawa and Nishida [1989] have suggested that wavy motions of the neutral sheet can be generated by the Kelvin-Helmholtz instability if the dawn-dusk flow of only several tens of km/s is present. However, their mathematical analysis is based on the choice of particular magnetic field directions in the three regions consisting of north, south lobes and the neutral sheet. In an earlier paper Uberoi [1986] discussed the Kelvin-Helmholtz instability of a similar structured plasma layer without any assumptions either on velocity field directions or on the magnetic field directions, thus pointing out the angle effect due to variation in magnetic field directions on the instability criterion. The relevance of these results to the problem of wavy motions of the neutral sheet are pointed out. In particular it is found that when the y-component of the magnetic field in each lobe is taken into consideration the Kelvin-Helmholtz instability can be exicted only when the dawn-dusk flow is of several hundreds of km/s a order of ten higher than that arrived in the analysis by Nakagawa and Nishida [1989].

Magnetic field observations by IMP 6 at about $20R_E$ tailward from the earth show that magnetic field in the neutral sheet sometimes takes north and south polarities alternately at successive crossings. In a recent paper Nakagawa and Nishida [1989] (therein after NN89) have shown that this can be understood by wavy motion of the neutral sheet that propagates in the dawn-dusk direction in the presence of the dawn-dusk component of the magnetic field. They further suggest that a possible cause of the wavy motion of the neutral sheet is the Kelvin-Helmholtz (K-H) instability caused by the duskward flow in the neutral sheet. To illustrate this point NN89 study the K-H instability of a three layered plasma system: the three regions being the neutral sheet (region 2), the northern and the southern lobes (regions 1 and 3). For mathematical simplicity they assume plasma parameters to be same in the regions 1 and 3, the flow to be present in the neutral sheet and finally they choose only particular directions for the flow velocity and the magnetic fields in the regions 1 and 2 though as they point out their observations show variations from the assumed directions of the magnetic field in the three regions. Moreover, while discussing the instability criterion they find the critical speed of the flow, above which the instability sets in as a function of the wavenumber. Therefore, it is not necessary that this may be the minimum critical speed above which the system will be unstable for all wavenumbers. For finite systems it becomes necessary to find the minimum critical speed above which the instability will set in for a range of wavenumbers. These aspects of the K-H instability mechanism of a three layer plasma system have been considered in detail by Uberoi [1986] [therein after U86]. In fact this work discusses exactly the same dispersion relation as eqn.(3) in NN89 without making any assumptions regarding the presence of flow velocity in the three regions or on the flow field and the magnetic field directions. Therefore, the general discussion of the instability criterion in this work has a relevance to the problem of wavy motions of the neutral sheet as discussed by NN89.

Considering the magnetic field and the plasma flow velocity in the x-y plane without any restrictions on direction such that

$$\mathbf{B}_i = (B_i \sin\beta_i B_i \cos\beta_i, 0) \text{ and}$$

$$\mathbf{V}_i = (V_i \sin\phi_i, V_i \cos\phi_i, 0), i = 1, 2$$

and the wave vector $\mathbf{k} = (\mathrm{k} \sin\theta, \mathrm{k} \cos\theta, 0)$ such that the relative flow speed of the plsma on the two sides of the interface separating the inner and the surrounding layer $\mathrm{U} = [V_1 \cos(\theta - \phi) - V_2 \cos(\theta - \phi_2)]$ it is found [U86] that there are two critical speeds $U_c^2 = [V_{A1}^2 \cos^2(\theta - \beta_2) + V_{A2}^2 \cos(\theta - \beta_2)]$ and $U^{*2} = \frac{\rho_1 + \rho_2}{\rho_1 \rho_2} [\rho_1 V_{A1}^2 \cos^2(\theta - \beta_1) + \rho_2 V_{A2}^2 \cos^2(\theta - \beta_1)]$. When $U < U_c$, the system is stable for symmetric (sausage mode) and asymmetric (kink mode) perturbations. However, U_c is no longer the minimum critical speed above which the system will be unstable for all wavenumbers. When $U_c < U < U^*$, the instability can set in through kink or sausage modes depending on the ratio $Q \equiv \rho_1 V_{A1} \cos(\theta - \beta_1) / \rho_2 V_{A2} \cos(\theta - \beta_2)$. When $Q > 1$, the instability sets in through the sausage mode for wavenumbers in the range $k_1^* < k < k_2^*$. When $Q < 1$, kink mode is unstable for $k_1 < k < k_2$. For exact expressions giving the values of the wavenumbers $k_{1,2}$ and $k_{1,2}^*$ the reader is refered to the Equations. (13) and (14) in the paper by [U86].

When $U > U^*$, the system is unstable for all the wavenumbers $k > k_1$.

In order to understand the effect of variation of angle β on the wavy motion of the neutral sheet generated

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due to onset of K-H instability we first consider the case considered by NN89. In their case $\beta_1 = \frac{\pi}{2}, \beta_2 = 0, \theta = V_1 = \phi_2 = 0$ and as $Q < 1$, the instability sets in through the kink mode. When $\rho_2/\rho_1 = 10, V_{A2} = 44$ km/sec, the range of critical flow speed is $44 < U < 145$ km/s. For any value of U in this range, the mode will be unstable for all the wavenumbers $0 < kL < 0.24$. The critical speed U = 65 km/s as derived by NN89 lies in this range and the corresponding wavenumber kL = 0.24 calculated from the observational values is the maximum value of this range. We find that the neutral sheet can become unstable not just for U = 65 km/s pointed out by NN89 but for flow speeds as low as 44 km/s, though kL being very small for these low flow speeds shows that this can happen only for thin neutral sheet. When the flow speed is increased to 100 km/s the value of kL lies in the range $0 < kL < 0.9$. The instability sets in for all the wavenumbers in this range and not just for the single value as calculated by NN89.

It is important to point out that when $U > 145$ km/sec the K-H instabiliity can excite both the kink and the sausage modes. Since the kink mode is expected to explain the neutral sheet crossings with alternating polarity [NN89] it appears that the instability criterion for the kink mode when flow speed is in the range $U_c < U < U^*$, is more relevant to the wavy motions of the neutral sheet.

The effect of considering the y-component of the magnetic field in the lobes on the value of critical speed has been mentioned by NN89 but no attempt was made to give an estimate of the modified speed. In order to get the quantitative measure of this modification we take a simple case by considering the y-component of the magnetic field in the lobes in the same direction as in the neutral sheet. In this case $\beta_1 = \beta_2 = 0 = V_1 = \phi_2 = 0$. Taking $\rho_2/\rho_1 = 10, V_{A1}/V_{A2} = 9$ such that when $V_{A2} = 44$ km/s, $V_{A1} = 396$ km/s, we find $\rho_1 V_{A1}/\rho_2 V_{A2} = 0.9 < 1$, so the instability will set in by exciting the kink mode for the flow speed in the range 399 km/s$< U <$ 441 km/s. The range of wavenumbers is given by $k_1 < k < k_2$. The minimum value [U86] of U for which instability will set in is U = 440 km/s and the minimum wavenumber is kL = 2 tan $h^{-1}0.9 \sim 3$. Now consider the case $Q > 1$. Choosing $V_{A1}/V_{A2} = 20$ we have $\rho_1 V_{A1}/\rho_2 V_{A2} = 2, V_{A2} = 44$ km/s, $V_{A1} = 880$ km/s. In this case the sausage mode will be unstable for the flow speed $882 < U < 933$ km/s. The minimum flow speed is U = 924 km/s with corresponding wavenumber kL = 1.1. These values of the critical flow speeds and the wavenumbers differ very much from the earlier case when $\beta_1 = \pi/2, \beta_2 = 0$ thus showing the importance of the angle effect on the K-H instability when layered plasma systems are considered. In particular we note that when y-component of the lobe magnetic field is taken into account the dawn-dusk flow velocity required for the onset of the K-H instability which generates the wavy motions of the neutral sheet, is of the order of several hundred of km/s a order of ten times higher than that arrived in the analysis by NN89. The minimum value of the wavenumber for which instability sets in is also higher in this case.

Finally, we like to mention that when the y-component of the lobe fields are taken into account, either of the modes, kink or sausage can become unstable depending on the value of Q. In fact for $Q > 1$, it is the sausage mode which is unstable and not the kink mode as suggested by NN89 from their analysis.

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