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Activity dependence of solar supergranular fractal dimension

U. Paniveni,[1,2] V. Krishan,[1,3] Jagdev Singh[1] and R. Srikanth[3,4]★

[1] *Indian Institute of Astrophysics, Koramangala, Bangalore-560034, Karnataka, India*
[2] *NIE Institute of Technology, Hootagalli Industrial Area, Mysore-570018, Karnataka, India*
[3] *Raman Research Institute, Bangalore-560080, Karnataka, India*
[4] *Poornaprajna Institute of Scientific Research, Devanahalli, Bangalore-562110, Karnataka, India*

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ABSTRACT

We study the complexity of supergranular cells using the intensity patterns obtained at the Kodaikanal Solar Observatory during the solar maximum. Our data consist of visually identified supergranular cells, from which a fractal dimension D for supergranulation is obtained according to the relation $P \propto A^{D/2}$, where A is the area and P the perimeter of the supergranular cells. We find a fractal dimension of about 1.12 for active region cells and about 1.25 for quiet region cells, a difference that could be attributed to the inhibiting effect of the magnetic field.

Key words: methods: data analysis – methods: statistical – techniques: image processing – Sun: activity – Sun: granulation – Sun: photosphere.

1 INTRODUCTION

Heat flux transport is chiefly by convection rather than by photon diffusion in the convection zone of cool stars such as the Sun, the thickness of convection zone being 30 per cent of the solar radius below the photosphere. Convection is revealed predominantly on two scales – on the typical scale of 1–2 arcsec it is granulation, and on the typical scale of 30–40 arcsec it is supergranulation. The typical lifetime of a supergranular cell is 24 h. Horizontal motion in the supergranular cells transports the magnetic flux tubes from the centre to the edge of the cells where they are deposited. The excess heat resulting from these magnetic fields at the chromospheric level traces out a network of supergranulation structure. Supergranules are characterized by typical horizontal speeds of 0.3–0.4 km s^{-1}. The vertical downward motion at the cell boundary is typically in the range of 0.1–0.2 km s^{-1}. The speed of the central upwelling is believed to be about 0.01 km s^{-1}, though there is some uncertainty concerning this. Worden & Simon (1976) report a tentative value of 50 m s^{-1}, suggesting the need for further accurate studies. Giovanelli (1980) reports an upper bound of 10 m s^{-1} in the absence of measurement fluctuations, whereas a speed as high as 100 m s^{-1} has also been reported (Küveler 1983). Berrilli et al. (1999) report an upper limit of 2 per cent anisotropy for the chromospheric network cell orientation and a 30 per cent size reduction towards the poles.

Sykora (1970) finds a cell size dependence on solar latitude, as also confirmed by Raju, Srikanth & Singh (1998). A dependence of the network size on solar cycle with a smaller size at solar maxima has been reported (Singh & Bappu 1981; Meunier, Roudier & Rieutord 2008), which is arguably consistent with the findings of Meunier, Roudier & Tkaczuk (2007b), who suggest that supergranular sizes are anticorrelated with magnetic activity. Cells of a given size associated with a remnant magnetic field live longer than those in the field-free regions (Singh et al. 1994). Srikanth, Raju & Singh (1999a) have also found a positive correlation between cell sizes and cell lifetimes. Convective motion and magnetic inhibition of motion are both stronger in active regions thereby leading to similar speeds in all regimes (Srikanth, Singh & Raju 1999b). The interrelationships amongst the parameters of length L, lifetime T and the horizontal flow velocity $v_{\rm h}$ of supergranular structures throw light on the underlying dynamics. A relationship between the horizontal flow velocity and the size of the supergranular cell has been established by Krishan et al. (2002), which they find to be compatible with the Kolmogorov energy spectrum, in contrast to Meunier et al. (2007a). This difference can possibly be attributed to the differing ways by which cells are defined in these two works (cf. below). A relationship between the horizontal flow velocity and cell lifetime has been established by Paniveni et al. (2004).

No unanimous agreement exists concerning the origin of supergranulation. Rieutord et al. (2000) suggest that supergranular flow is generated directly by the granular flow through a large-scale instability which fixes the scale in space and time of supergranulation. It is therefore conjectured that non-linear interaction between flows at the granular scale, in other words Reynold stresses, is sufficient to drive flows at the supergranular scale and that the energy released by the recombination of ionized helium plays no part. In the light of lack of conclusive proof required of a convective origin, other speculations have arisen to account for supergranulation: gravity wave modulation of the convective motions (Lindzen & Tung 1976; Rast 2003). Lisle, Rast & Toomre (2004) have noted a north–south alignment of supergranulation, consistent with an underlying dynamical cause at a larger scale identified with giant cells.

In an earlier work, we suggested (Paniveni et al. 2005) a possible turbulent origin of supergranulation based on a fractal analysis. However, theoretical approaches are undecided between scenarios

★E-mail: srik@rri.res.in

that either invoke a large-scale instability of the surface turbulent convection or a direct forcing by buoyancy. We suggest that this discrepancy concerning the role of turbulence in supergranulation is probably due to the ambivalence in our understanding of the relationship between the area and magnetic flux content of network cells. For example, Rieutord et al. (2007) have shown, through the kinetic energy spectrum derived from high-resolution data obtained from the Pic-du-Midi observatory, that supergranulation peaks at 36 Mm and spans on scales ranging between 20 Mm and 75 Mm. The decrease in supergranular flows in the small scales is close to the k^{-2} power law, steeper than the Kolmogorov one. Based on the probability distribution function of the divergence field, they find a signature of intermittency of supergranulation and therefore that the supergranulation field has turbulent nature. On the other hand, Meunier et al. (2007a) find that the velocity–scale relationship in supergranules is not compatible with the Kolmogorov turbulence and the 1/3 exponent. Part of this discrepancy is perhaps due to the way cells are defined and scales extracted. For example, it has been noted that the network may be more complete than it seems to visual methods like the one we have employed (Hagenaar, Schrijver & Title 1997). The set identified by visual inspection could thus be biased towards cells with well-demarcated boundaries. Clearly, observational constraints are needed to guide theoretical approaches.

Fractal analysis is a valuable mathematical tool to quantify the complexity of geometric structures (Mandelbrot 1977) and thereby gain insight into the underlying dynamics. For example, statistical analyses such as studies of the size distribution of active regions or of the fractal dimension of solar surface magnetic structures are useful for comparing observations and models. In the context of solar physics, fractal analysis was first adopted by Roudier & Muller (1986), who measured the fractal dimension of granular perimeters. Complex phenomena such as distribution of the flux tubes and their interaction with the convective pattern can be understood by their fractal analysis which also helps to test models. The structures are not strictly self-similar and therefore one should consider the fractal dimension computed over small ranges of size (Meunier 2004). Fractal analysis can shed light on the turbulence of magnetoconvective processes that generate the magnetic structures (Stenflo & Holzreuter 2003a; Lawrence, Ruzmaikin & Cadavid 1993), and has been applied to a study of the formation of solar active regions (Meunier 1999).

A number of authors have studied the temporal evolution and geometric properties of supergranular cells and the complexity of their network pattern. Berrilli, Florio & Ermolli (1998) have employed fractal analysis to characterize the complexity of supergranular flows using Ca K images of the chromospheric network. They use an intensity threshold scheme to produce binary image versions of the filtergram. The skeleton representing a medial axis transform is then used to extract geometrical information such as the area and the perimeter of the cells, from which we obtain the fractal dimension. Their method of data analysis permits a statistically large number of cells to be analysed. By contrast, visual inspection restricts the number of cells we have studied. On the other hand, what is novel to our work is that our method allows us to identify reasonably well the cells in both quiescent and active regions and thus to compare cells at different activity levels. It is not clear that a threshold scheme like that of Berrilli et al. (1998) can be directly applied to quiet regions (where cells are known to be less well defined) and thence to the study of activity dependence. Perhaps a future study would be to extend the threshold scheme to cover active regions, which may not be straightforward, but require careful inspection.

The variation of fractal dimension with the solar cycle is an important observation, which theoretical models should also be able to reproduce. Thus, many issues could be resolved by studying the maximum of the solar cycle, such as the variation of the fractal dimension with the activity level. It will also be interesting to establish some comparison with other types of data (Paniveni et al. 2005). Magnetohydrodynamics (MHD) models due to Rincon & Rieutord (2003) and Benson, Stein & Nordlund (2006) are of relevance here, similar to that of Crouch, Charbonneau & Thibault (2007), who, employing an *n*-body diffusion-limited aggregation model to simulate the dispersal and interaction of small-scale magnetic elements at the solar surface, obtain a spatial distribution of clusters of magnetic concentrations, comparable to the supergranule cell pattern. A dependence of the fractal dimension of active region magnetic structures on the activity level (spots, flares) and solar cycle phase (Meunier 2004) has been observed.

In this work, we study supergranular fractal dimension by the method of visual inspection on the intensitygrams of Kodaikanal Solar Observatory during the solar maximum of 2001 May–August and the activity level dependence of the fractal dimension.

2 DATA ANALYSIS

We analysed intensity data, consisting of Ca II K filtergrams ($\lambda = 3934$ Å) of the Sun, obtained between 2001 May 16 and 2002 November 26, during the solar maximum phase of the 23rd solar cycle, at the Solar Observatory, Kodaikanal. Light from a 46 cm siderostat is diverted to a 15 cm Zeiss achromat objective which provides an $f/15$ beam and a 2 cm image. A pre-filter and a Daystar Ca K narrow-band filter are used together with a Photometrix 1k × 1k CCD to record the K filtergram. The images have a resolution of about 2 arcsec, which is twice the granular scale. Only cells lying within 60° angular distance from the disc centre were selected in order to minimize projection effects. Regions that were clearly identifiable as either quiescent (quiet) or active were noted, and visually identifiable cells were selected from these regions. Regions that were not unequivocally quiescent or active were avoided, for simplicity. Depending on the region in which a cell is found, it is called quiescent or active.

An example of region and cell selection is depicted in Fig. 1 in a zoomed-in view. In all, a set of 239 cells were analysed, comprising 87 quiescent and 152 active cells. Clearly, the cell perimeter that is detected, and hence the fractal dimension that is derived, depends on the smoothing level, with the greater degree of smoothing reducing the fractal dimension. It thus seems that the fractal dimension attributed to a feature must be qualified by the resolution at which it is derived. It cannot be discounted that our method of visual inspection probably rejects or fails to select many cells with gaps in the network, and thus with less well-defined contours, compared to the whole cell boundary, thereby favouring active area cells, where such gaps are less likely. The greater clarity of contours in the active region cells could also mean that a larger perimeter is found in this case relative to quiet cells, and thus that the fractal dimension for this region is an over-estimate.

The profile of an identified cell was scanned as follows. We chose a fiducial *y*-direction on the cell and performed intensity profile scans along the *x*-direction for all the pixel positions on the *y*-axis (Krishan et al. 2002; Paniveni et al. 2004). In each scan, the cell extent is taken to be marked by two juxtaposed ‘crest’ (separated by a ‘trough’) as expected in the intensitygrams. This set of data points was used to determine the area and perimeter of a given cell and

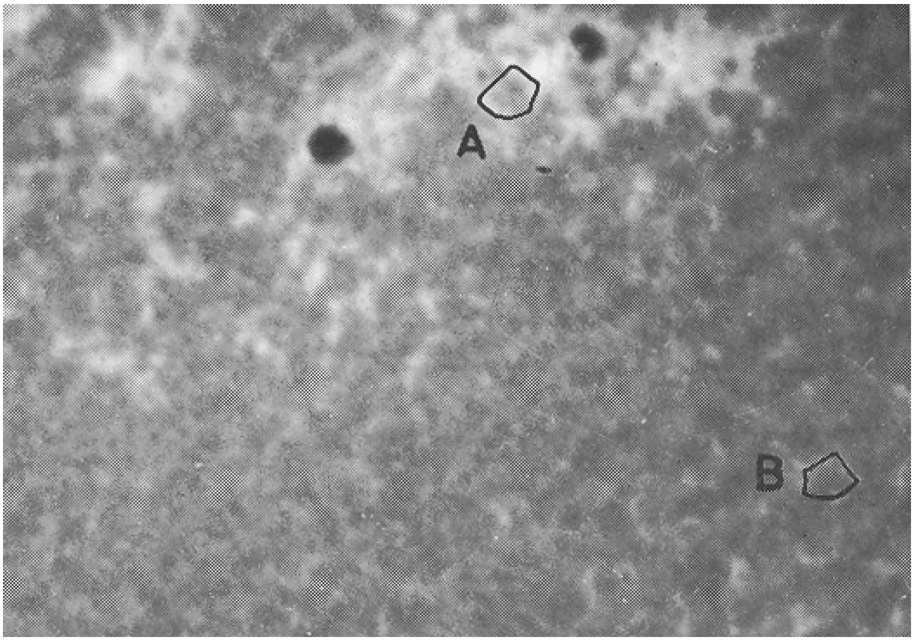

Figure 1. Zoomed-in view of the solar chromosphere. Cell A is present in an active region, while cell B in a quiet region.

of the spectrum of all selected supergranules. The area–perimeter relation is used to evaluate the fractal dimension.

The use of Ca II K images here has the advantage with respect to Dopplergrams (Paniveni et al. 2005) that the latter are much more sensitive to projection effects. Furthermore, cell identification and activity level recognition are easier with intensitygrams. The use of intensitygrams also usefully complements other forms of image data we have earlier used.

For our purpose, fractal dimension D is characterized by the area–perimeter relation of the structures. Self-similarity, meaning the same degree of complexity regardless of the scale at which the structures are observed, is expressed by a linear relationship between $\log P$ and $\log A$ over some range of scales.

3 RESULTS AND DISCUSSION

The main results pertaining to fractional dimension are derived from Figs 2 and 3, which are discussed below. A fractal analysis is relevant to such irregularly shaped features because we can quantify the supergranular irregularity and shed light on the nature of solar turbulence. Along the lines of various works reported in the introduction on granules, we analysed planar shapes by analysing the area–perimeter relation $P \propto A^{D/2}$. For smooth shapes such as circles and squares, $P \propto A^{1/2}$ and thus $D = 1$, the dimension of a line. As the perimeter becomes more and more contorted and tends to double back on itself, filling the plane so that $P \propto A$ and D approaches the value 2.

3.1 Main results

Our data, in which 152 active region cells and 87 quiet region cells were analysed separately, demonstrate an anticorrelation between the activity level and fractal dimension.

Active regions. The $\log A$ versus $\log P$ relation is linear as shown in the lower frame of Fig. 2 for the active region. The linear relation in this (as well as in Fig. 3 for the quiescent region) suggests that

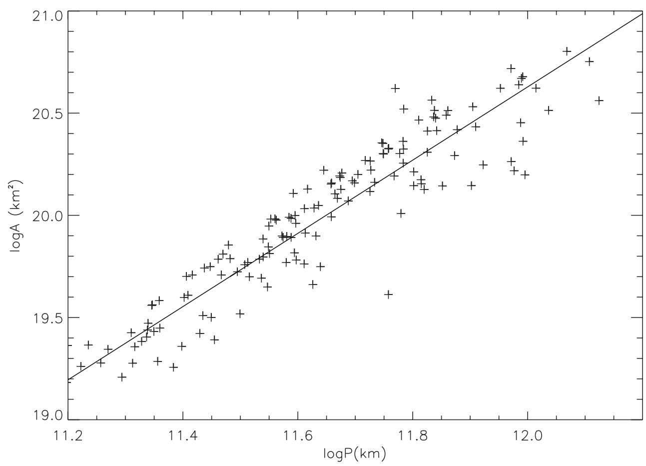

Figure 2. Active region: plot of the natural logarithm of the supergranular area (in km^2) against the natural logarithm of perimeter (km).

supergranules are self-similar and can be regarded as fractal objects over the observed range of scale. A correlation coefficient of 0.94 indicates strong correlation. Fractal dimension D, calculated as a 2/slope, is found to be about 1.12, the exact value derived being 1.12 ± 0.07. The average area of the cells analysed in this region is 247 Mm^2.

Quiet region. The $\log A$ versus $\log P$ relation is linear as shown in the lower frame of Fig. 3. A correlation coefficient of 0.88 indicates strong correlation. Fractal dimension D, calculated as a 2/slope, is found to be about 1.25, the exact value derived from the 87 cells being 1.25 ± 0.14. The average area of the analysed cells of this region is 272 Mm^2.

The above results are compatible with the observation that there is an anticorrelation between the activity level and cell size (Singh & Bappu 1981; Meunier et al. 2007b, 2008). The observed fractal nature of supergranulation is also in accordance with earlier

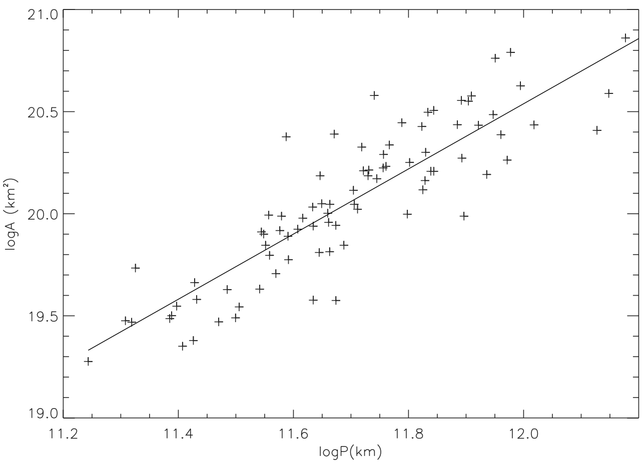

Figure 3. Quiet region: plot of the natural logarithm of the supergranular area (in km^2) against the natural logarithm of perimeter (km).

works where we presented some evidence for turbulent convection based on horizontal flow velocity, lifetime and length-scale data for supergranulation (Krishan et al. 2002; Paniveni et al. 2004). For this earlier analysis, we analysed 33 h of full disk Dopplergrams obtained in 1996 by the Michelson Doppler Interferometer (MDI) on board the Solar and Heliospheric Observatory (SOHO; Scherrer et al. 1995,).

3.2 Discussion

The spectral distribution of the temperature, a passive scalar, is related to the spectral distribution of kinetic energy. It can be easily shown that the Kolmogorov energy spectrum, $k^{-5/3}$, both in two- and in three-dimensional turbulence leads to a temperature spectrum of $k^{-5/3}$ (Krishan 1996; Zahn 1997).

Thus the temperature variance $\langle\theta^2\rangle$ varies as $r^{2/3}$, as a function of the distance r (Tennekes & Lumley 1970). According to Mandelbrot (1975), an isosurface for temperature has a fractal dimension given by D_I = (Euclid dimension) − 1/2 (exponent of the variance). Thus for two-dimensional supergranulation, $D_T = 2 - (1/2 \times 2/3) = 5/3 = 1.66$ for an isotherm. The pressure variance $\langle p^2\rangle$ on the other hand is proportional to the square of the velocity variance, i.e. $\langle p^2\rangle \propto r^{4/3}$ (Batchelor 1953). The fractal dimension of an isobar is therefore found to be $D_p = 2 - (1/2 \times 4/3) = 1.33$.

Our data furnish evidence that the fractal nature of the supergranular network is closer to being isobaric than isothermal. However our measurements are not compatible with the isobaric fractal dimension derived from a Kolmogorov turbulence, suggesting a more complex origin of supergranulation. We also observe a trend for quiet regions having a larger fractal dimension than active region supergranules.

It is interesting to note that Roudier & Muller (1986) obtained an almost similar dimension as we do, for smaller granules. However, in contrast to granules, we do not find any evidence for two different regimes of fractal dimension; both Figs 2 and 3 show that a single linear fit is suitable for the entire observed range of supergranules. But this could be an artefact due to our specific method of cell identification or the limited sample. Future works, for example by applying the threshold technique of Berrilli et al. (1998) to cells of different activity levels, can potentially improve on this result.

In Section 1, we noted theoretical or observational support for a relationship between the supergranular scale size and activity level (Singh & Bappu 1981; Meunier et al. 2007b). Nevertheless, the variation of cell size with its magnetic environment remains controversial. Part of this state of affairs probably stems from lack of a consistent definition of activity level in that they do not distinguish between intra-cellular activity and network activity (as indicated by Meunier et al. 2007b) and the magnetic sensitivity of the data. It may be hoped that an extension of theoretical models that can account for the relationship between the scale and absolute field could also shed light on how magnetic fields may influence fractal dimension. It is known that strong magnetic fields have an inhibiting effect on large-scale flows, but a causal connection linking restricted velocity flows in the presence of magnetic fields to smaller fractal dimension is not obvious. A quantitative description of this picture would be an interesting future exercise.

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REFERENCES

Batchelor G. K., 1953, The Theory of Homogeneous Turbulence. Cambridge Univ. Press, Cambridge
Benson D., Stein R., Nordlund A., 2006, in Uitenbroek H., Leibacher J., Stein R. F., eds, Solar MHD Theory and Observations: A High Spatial Resolution Perspective. p. 354
Berrilli F., Florio A., Ermolli I., 1998, Sol. Phys., 180, 29
Berrilli F., Ermolli I., Florio A., Pietropaolo E., 1999, A&A, 344, 965
Crouch A. D., Charbonneau P., Thibault K., 2007, ApJ, 662, 715
Giovanelli R. G., 1980, Sol. Phys., 67, 211
Hagenaar H. J., Schrijver C. J., Title A. M., 1997, 481, 988
Krishan V., 1996, Bull. Astron. Soc. India, 24, 285
Krishan V., Paniveni U., Singh J., Srikanth R., 2002, MNRAS, 334, 230
Küveler G., 1983, Sol. Phys., 88, 14
Lawrence J. K., Ruzmaikin A. A., Cadavid A. C., 1993, ApJ, 417, 805
Lindzen R. S., Tung K. K., 1976, Mon. Weather Rev., 104, 1602
Lisle J. P., Rast M. P., Toomre J., 2004, ApJ, 608, 1167
Mandelbrot B. B., 1975, J. Fluid Mech., 72, 401
Mandelbrot B., 1977, Fractals. Freeman, San Francisco
Meunier N., 1999, ApJ, 515, 801
Meunier N., 2004, A&A, 420, 333
Meunier N., Tkaczuk R., Roudier T., Rieutord M., 2007a, A&A, 461, 1141
Meunier N., Roudier T., Tkaczuk R., 2007b, A&A, 466, 1123
Meunier N., Roudier T., Rieutord M., 2008, A&A, 488, 1109
Paniveni U., Krishan V., Singh J., Srikanth R., 2004, MNRAS, 347, 1279
Paniveni U., Krishan V., Singh J., Srikanth R., 2005, Sol. Phys., 231, 1
Raju K. P., Srikanth R., Singh J., 1998, Sol. Phys., 180, 47
Rast M. P., 2003, ApJ, 597, 1201
Rieutord M., Roudier T., Malherbe J. M., Rincon F., 2000, A&A, 357, 1071
Rieutord M., Meunier N., Roudier T., Rondi S., Beigbeder F., Parès L., 2007, A&A, 479
Rincon F., Rieutord M., 2003, in Combes F., Barret D., Contini T., eds, Proc. SF2A-2003: Semaine de l'Astrophysique Francaise. EDP Sciences, Les Ulis, p. 103
Roudier Th., Muller R., 1986, Sol. Phys., 107, 11
Scherrer P. H. et al., 1995, Sol. Phys., 162, 129
Singh J., Bappu M. K. V., 1981, Sol. Phys., 71, 161
Singh J., Nagabhushana B. S., Babu G. S. D., Wahab Uddin, 1994, Sol. Phys., 153, 157
Srikanth R., Raju K. P., Singh J., 1999a, Sol. Phys., 184, 267
Srikanth R., Singh J., Raju K. P., 1999b, Sol. Phys., 187, 1

Stenflo J. O., Holzreuter R., 2003a, in Pevtsov A. A., Uitenbroek H., eds, ASP Conf. Ser. Vol. 286, Current Theoretical Models and Future High Resolution Solar Observations: Preparing for ATST. Astron. Soc. Pac., San Francisco, EDP Sciences, Les Ulis, p. 169
Sykora J., 1970, Sol. Phys., 71, 161
Tennekes H., Lumley J. L., 1970, A First Course in Turbulence. MIT Press, Boston
Worden S. P., Simon G. W., 1976, Sol. Phys., 46, 73
Zahn J. P., 1987, in Schröter E. H., Schussler M., eds, Solar and Stellar Physics. Springer-Verlag, Berlin, p. 55

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