The magic of 0.177!

An Arthur Clarke fan? Then, you'll know that 0.177 holds special significance in orbital paths. If the earth were to "fall into" the sun if it were to stop in its orbit, the time taken would be 0.177 times a year, which is 70 days, writes **C Sivaram**
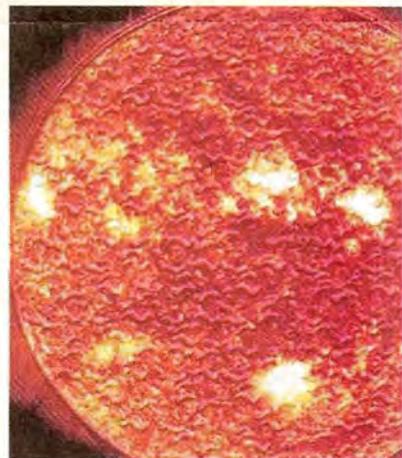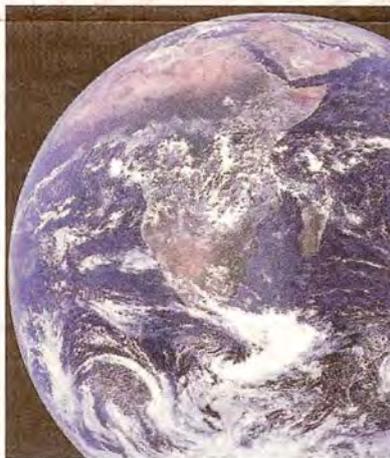

The Moon Impact Probe (MIP) was released for the orbiting Chandrayaan spacecraft in mid-November. Chandrayaan was then in its circular orbit, hundred kilometres above the lunar surface, with an orbital period of two hours. The probe weighing about thirty kilograms took just over twenty minutes to hit the lunar surface, after being released from the orbiting spacecraft.

In this connection, it is of interest to note that the above two time periods, i.e, the orbital period and the time taken to impact (after release from the orbit) are related by a well-known result in celestial mechanics. The latter time is square root of 2 divided by 8, of the orbital period of two hours.

This is also 0.177 of the orbital period, which works out to be just over twenty minutes.

Arthur Clarke apparently knew this result when in his science fiction story *Jupiter V*, he stated that by a well-known theorem, a probe dropped from a spacecraft orbiting Jupiter, time taken is 0.177 times the orbital period! Rarely do we have such knowledgeable science fiction writers. (This theorem may not be known to most present day students of astronomy or physics!)

A related 'textbook' problem is to estimate the time it would take for the earth to "fall into" the sun if it were to be suddenly stopped in its orbit. Again the answer is 0.177 times a year which gives about seventy days. So in case of such a contingency, humanity would have about two months to gear itself for the challenge (to devise means of escape)!

An explanation for the above result relating the two time periods involves the basic physics of orbits in a gravitational field.

An object in orbit has both kinetic and potential energy, but as it is still in a bound state while in orbit, the potential energy is dominant. However, when the object is stopped in circular orbit and it goes plunging into the central mass (around which it is orbiting), its total energy doubles (the energy is now all potential; the path becomes an elongated ellipse with eccentricity approaching unity).

The orbital period squared is related to the orbital radius cubed, but as gravitational potential energy scales inversely as the radius, the period squared is now proportional to the inverse cube of the energy. So when the energy doubles, the period squared decreases by a factor of 8, and the time period decreases by a factor of square root of 8, and as the time taken to hit the surface is half of this (only half the orbit is completed), we get the required result, that is one divided by four times square root of two or 0.177 as it can easily be verified.

This equation between the orbital period of any spacecraft and the time taken for a released object to hit the planet surface is universal and has the status of a theorem, similar to the well-known result that the escape velocity (at any orbit) is square root of two times the orbital velocity.

The time taken for the Chandrayaan spacecraft to touch the lunar surface was 20 minutes, and is related by a wellknown result in celestial mechanics, where 0.177 comes into play.