BLACK HOLES

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INTRODUCTION:

Save for its limited astronomical applications, the General Theory of Relativity did not seem to have a bright future till the late fifties for the simple reason of lack of any extensive experimental or observational phenomena. However, this ended in the early sixties with numerous achievements on the experimental, observational and theoretical fronts, viz., the development of powerful experimental techniques making it feasible to conduct new and classical tests of the theory with unprecedented accuracy, the discoveries on the observational front of the explosions in galactic nuclei, the quasars, the pulsars and the X-ray sources, the 3° K cosmic black body radiation and developments on the theoretical fronts such as the maximal extension of the Schwarzschild metric, the Kerr metric, gravitational collapse, relativistic steller structure black holes, the singularity theorems and cosmology. The occurrence of singularities (infinite density, zero radius) in the relativistic cosmologies and the relativistic gravitational collapse has been a great difficulty to the relativists. There has seemed no plausible way out to remove the singularities from the solutions and there had been this feeling that a mass cannot collapse to the state of infinite density and zero radius. That is why the prediction by Oppenheimer and his students of the inevitable gravitational collapse of a certain amount of matter to a singularity in the late thirties didn't draw much notice till the sixties when the discovery of one of the most puzzling objects of our time, the Quasar, and the possibility of the explanation of the enormous energy output in terms of the gravitational collapse of supermassive stars gave a great boost to studies in this field. In fact, it was the pioneering work of Oppenheimer and Snyder (1939) which led to the prediction of the formation of black holes in the supernova explosion of stars with masses exceeding the Oppenheimer-Volkoff mass limit M_{OV} ($\sim 2.5\ M_{\odot}$) when the core of the star has collapsed to an extremely small size of a few kilometers and to densities beyond even that of the atomic nucleus. Black holes are interesting objects not just for their small size but for the exotic properties dictated by the surrounding spac.time, which is so greatly warped near the singularity that the laws of physics seem to break down. A great many advances have been made of late in black-hole theory and there are indications of the possible existence of black holes in systems like Cyg X-1, which offers the testing ground for the predictions of general relativity and for studying physics in strong gravitational fields. The present article attempts to highlight some of the interesting features of black hole physics. For greater detail, one may see Misner et al. (1973) and DeWitt and DeWitt (1973).

GRAVITATIONAL COLLAPSE, THE KERR-NEWMAN BLACK HOLE:

A black hole will form in the supernova explosion of a massive star provided the collapsing core shrinks to the point where its gravitational potential energy equals its rest mass energy, i.e., when all the matter has collapsed inside the so-called Schwarzschild radius (R_s) of the mass. But not every star can produce a black hole. It has to be massive enough. The pivotal role of mass would become clear if we trace in brief the end products of the evolution of stars of different masses.

It is widely held that a one solar mass star finally evolves to the white dwarf stage after it has consumed most of its nuclear fuel and thereafter derives its energy from gravitational contraction. Stars beginning with a mass exceeding the Chandrasekhar limit $M_{Ch} = 1.2\ M_{\odot}$ cannot become stable white dwarfs unless they have undergone a steady mass loss through a stellar wind or formed a planetary nebula because the balance between the forces of pressure and gravity fails to develop at white dwarf densities. The core is doomed to collapse to become either a neutron star or a black hole depending on whether its mass lies in the range $M_{OV} > M > M_{Ch}$ or $M > M_{OV}$. An exact value of M_{OV} cannot be given; the oft quoted values lie within $1.5 - 3M_{\odot}$ and the value is sensitive to the equation of state employed for the description of the superdense matter.

Consider first the smaller of these two mass ranges. The collapse is slow in the beginning but picks up soon and a substantial portion of the star mass implodes faster than the surrounding envelope. Actually, the stages subsequent to the onset of the collapse have a critical dependence on the generation and propagation of neutrinos, the shock wave generation and its propagation. The core is imploding in nearly free fall to higher and higher densities. However, the electrons cannot be squeezed to high values of Fermi energy. They tunnel into the nuclei and interact with the protons to form neutrons in the inverse β—decay process. The electron pressure thins, the internal pressure drops and the core collapses further under its own weight. Around $\rho \backsim 2 \times 10^{14}$ gm cm^{-3}, the neutron rich nuclei start disintegrating into free neutrons because the neutron binding energy becomes negative. The material is mostly neutronic, with a small admixture of electrons and protons. At slightly higher densities, the force of repulsion among neutrons becomes large enough (because of the exclusion principle) to balance the gravitational force of the falling layers of the core and halts the collapse at $\rho \backsim 10^{15}$ gm cm^{-3} and $R \backsim 10$Km. This happens so fast that a sudden conversion of the kinetic energy of the collapse into heat produces large pressure to blow off the outer envelope, still falling in at high speeds, and to accelerate the particles to very high energies (the supernova explosion). What is left, if anything, is a neutron star.

GM/R is now large enough to show its effect appreciably. One of these is the effect of pressure regeneration. In general relativity, pressure acts as a source of gravity. Thus, if the neutron star so formed or the initial core were made a bit more massive, then since pressure contributes to the effective mass of the collapsar, the latter would collapse still further, making the pressure still larger, and so on. If the collapsar has a mass $M > M_{OV}$ or if the neutron star (mass M) accretes at least a mass $\triangle M \geqslant M_{OV} - M$, the contribution of pressure to the gravitational attraction is so high that the balance between the forces of gravity and pressure fails to be established at neutron star densities: the material is doomed to be crushed further till it all falls inside the Schwarzschild radius R_S ($= 2GM/c^2$) and disappears from sight. The core has become what we call a black hole and warped so highly the surrounding spacetime around that it folds in over itself.

This is a brief sketch of the possible outcome of the late stages of stellar evolution . In fact, mass is not the only deciding parameter: rotation is also important, as is steady mass loss and mixing of chemical composition of the star among its different layers. These have an effect on whether the star must undergo a supernova explosion and how much mass it would lose in the explosion. Obviously, a star rapidly rotating initially might yield a neutron star. It is generally believed that medium mass stars ($\backsim$ 4-10 $M_\odot$) and most heavy stars ($M \geqslant 30\, M_\odot$) undergo supernova explosion to produce neutron stars and black holes, respectively, if they haven't lost a large amount of mass at relatively early phases of their evolution (e.g., see Taylor 1974). Granted that the object collapses to a radius $r < R_S$, but why call it a black hole? To find the answer, suppose the collapse is being watched by two observers, one attached to the infalling surface layer and the other staying away ($r \gg R_S$). To the remote observer, the collapse to the radius R_S appears to take an infinite co-ordinate time (which is his proper time). When $r \rightarrow R_S$, the collapsar appears to the remote observer to become fainter and redder because of the Doppler effect from the relativistically receding layers, the diminishing size, the aberration and the gravitational redshift. The signals emitted by the comoving observer (moving in at relativistic speeds) take an increasingly long time to reach his associate far away and once r crosses R_S, the communication is broken. Because of infinite time dilation at $r = R_S$ resulting in infinite redshift, the last photons escaping spend all their energy in their struggle to climb to the observer. The object is therefore invisible or "black". Also, since any signal sent down by the remote observer to find out the fate of the comoving observer may take an infinite co-ordinate time to reach him, the name "black hole" is justified.

The object is a black hole by virtue of the existence of the surface with $r=R_S$, also known as the "event horizon". It forms the boundary of all the events that can be connected to future infinity by means of tardyons or photons. Signals emitted from within the horizon have to move faster than light, i.e., travel along spacelike geodesics, in order to escape. The horizon is therefore a one-way membrane, and the relativist calls, any region of spacetime unable to communicate with the rest of the universe by means of photons or tardyons a black hole.

What happens if the geometry of the collapsing object departs from spherical symmetry? Can such a collapse also lead to the formation of a horizon, i.e. to a black hole? Non-sphericity might result from rotation, magnetic fields etc. Although more likely to happen, the non-spherical collapse of astronomical objects is a difficult problem to handle. The situation has been improved lately with some studies of small departures from sphericity made by a number of workers.

The small perturbations away from spherically symmetric collapse can be treated as test fields which are superimposed on the unperturbed , spherically symmetric background geometry. This assists one in seeing whether, under these small perturbations, the formation of the event horizon is stable. If so, a black hole can be expected to form in a nearly spherical collapse, too. Using perturbation theory, it has become clear that the collapse of a nearly spherical, non-rotating body also leads to the formation of a Schwarzschild black hole, after radiating away all the gravitational deformations in the form of gravitational waves. R. Price has shown that this happens for perturbing fields of any integral spin s that might be coupled to the collapsar. Price's theorem states that "in relativistic gravitational collapse with small non-spherical perturbations, anything that can be radiated will be radiated away completely". Capable of being radiated away are the multipoles which are not conserved: those of order $l \geqslant s$, i.e., l-pole radiation of a spin-s field. Therefore, all the multipoles with $l \geqslant s$ get radiated away in the collapse as vector (electromagnetic, $s = 1$) and tensor (gravitational, $s=2$) radiation; the final field is characterized completely by the conserved quantities (the multipole moments with $l < s$). The end-state is a black hole with no scalar field (since all scalar multipoles are radiatable), a monopole Coulomb field (non-existence of magnetic monopoles), a monopole gravitational field (plus a stationary dipole field corresponding to rotation, if the final black hole has spin), monopole, dipole and quadrupole moments of a spin-3 field, etc.

Highly non-spherical collapse is very poorly understood and it is not known for sure whether horizons form in this case also. The situation is reviewed by Thorne (1972) who conjectures that horizons form when and only when a mass M gets squeezed into a region whose proper circumference in every direction is $\gtrsim 4\pi m$. The horizon forms the boundary of all the events which can be connected to future infinity by means of light, or slower-than-light, signals. Therefore any particle that falls into the horizon is lost for ever from the outside. Assuming that matter, in the form, e.g., of an accretion disk or a satellite, is absent from the exterior, the only conserved integrals left to govern the final horizon and the exterior spacetime are mass, angular momentum and charge that went down into the hole. The exterior field is the Kerr-Newman solution of the Einstein-Maxwell equations

$$ds^2 = \left(\frac{\Delta - a^2 \operatorname{Sin}^2\theta}{\Sigma}\right)dt^2 + \frac{2a \operatorname{Sin}^2\theta \,(r^2 + a^2 - \Delta)dt\, d\varphi}{\Sigma} \tag{1}$$

$$-\frac{\Sigma}{\Delta} dr^2 - \Sigma\, d\theta^2 - \frac{[(r^2+a^2)^2 - \Delta a^2 \operatorname{Sin}^2\theta]}{\Sigma} \operatorname{Sin}^2\theta d\varphi^2$$

where * $\Sigma = r^2 + a^2 \operatorname{Cos}^2\theta$; $\Delta = r^2 + a^2 + e^2 - 2mr$

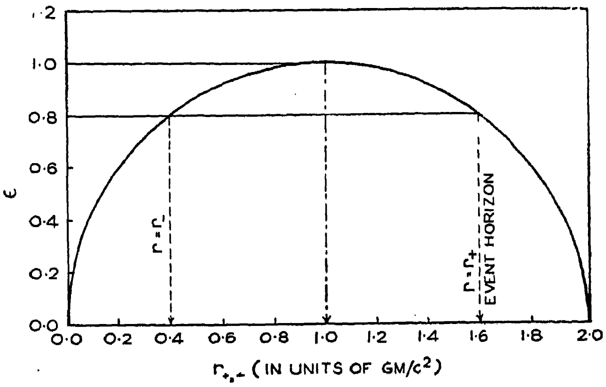
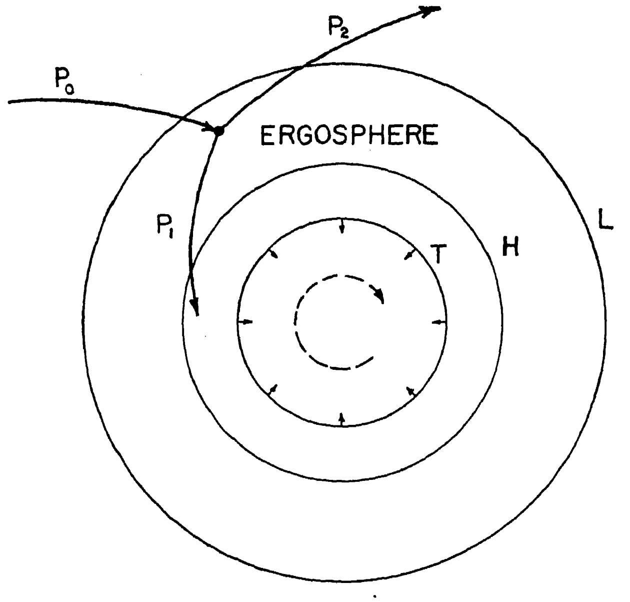


Fig. 1. Location of outer and inner event horizons $r_{+,-}$ in a Kerr-Newman spacetime from the origin, denoted by the points of intersection of lines of ϵ = const. with the solid curve, $r_{+,-} = m\ [1 \pm (1-e^2)^{\frac{1}{2}}]$ vs ϵ. There is one horizon $r_+ = r_- = m$ for extreme configurations ($\epsilon = 1$) and none for those with $\epsilon > 1$, which have consequently, naked singularities. (Here $\epsilon = (a^2 + e^2)^{\frac{1}{2}}/m$, e is charge of the black hole, m its gravitational mass and J = ma its spin angular momentum, all in units with G = c = 1.)

Provided that the charge and angular momentum are not too large—$a^2 + e^2 \leqslant m^2$—an event horizon exists; and this solution then describes the so-called Kerr-Newman black hole. The above metric specializes to the Schwarzschild solution if $m \neq 0$; a, e = 0 (static, neutral) Kerr if m, $a \neq 0$; e = 0 (stationary, neutral), and Reissner-Nordström if m, $e \neq 0$; a = 0 (charged, static).

* Here m = mass, a = angular momentum/mass and c = charge. The units are such that G = c = 1 and all physical quantities are given dimensions of length to some power. Thus the conventional quantities M(gm), Q(e.s.u.) and J/M ($cm^2\ sec^{-1}$) are all "geometrized" to lengths: $m = \frac{GM}{c^2}$, $e = \frac{\sqrt{G}}{c^2} Q$ and $a = \frac{G}{c^3}\left(\frac{J}{M}\right)$.



THE PENROSE PROCESS

Fig. 2. A rotating black hole, viewed along the axis of rotation. Dotted arrow indicates sense of spin. L is the surface of infinite redshift. H is the event horizon or "one-way membrane". T is one of countless trapped surfaces inside the horizon. Between L and H lies the ergosphere. All particles in the ergosphere are dragged in the sense of rotation of the black hole. The Penrose process is illustrated. Particle P_0 breaks up in the ergosphere into P_1 and P_2. P_1, with negative energy and angular mumentum, falls into the horizon decreasing m and J of the hole; P_2 escapes to infinity carrying the surplus. (Although P_1 is dragged in sense of rotation of the hole, we figuratively represent its negative angular momentum by a backward-pointing arrow.)

The Kerr-Newman spacetime has co-ordinate singularities for two values of r viz., $r_{+,-} = m\,(1 \pm \sqrt{1-e^2})$ where $\epsilon = \sqrt{a^2 + e^2}/_r\, m$. The outer horizon is located at $r = r_+$ (Fig. 1). Fo a Schwarzschild black hole $r_+ = R_S$, $r_- = 0$. Pathological violations as casuality ("time machines") occur unless $0 \leqslant \epsilon \leqslant 1$. Objects with $\epsilon = 1$ are "extreme" Kerr-Newman black holes and the spacetime has only one co-ordinate singularity; objects with $\epsilon > 1$ are not black holes, there is no horizon and the physical singularity (infinite curvature) at r = 0 (and $\theta = \pi / 2$ if $a \neq 0$) is visible from future timelike infinity and is, therefore, a "naked" singularity. Whether such objects exist in the Universe remains still a bone of contention.

Surrounding the (outer) horizon H is an infinite redshift surface L, called also the stationary—or ergosurface (Fig. 2) located at radius

$$r_L = m + (m^2 - e^2 - a^2 \operatorname{Cos}^2\theta)^{\frac{1}{2}}$$

This surface coincides with the horizon only in the non-rotating case, a = 0. Otherwise, it touches the horizon only at the points $\theta = 0$, π (not shown in the Fig. 2, where the hole is seen from above the pole). The region bound by L and H is called the ergosphere. It is peculiar in the sense that the "time-lines" r, θ, φ = const. are space-like here. Here there can be no observers at rest with respect to stationary observers at infinity: anything which stays at fixed r and θ would move in the φ -direction (the dragging of inertial frames). The dynamical importance of the ergosphere will become clear a little further on.

That the parameters m, a and e alone determine uniquely the external gravitational and the electromagnetic fields of the hole, leaving no other degrees of freedom, is suggested by the various theorems of Israel, Carter and Hawking (the Israel-Carter conjecture).

Once a black hole has settled down to the state of equilibrium, having radiated away all multipole moments with $l \geqslant s$, the only information you can obtain about it is its mass, charge, and angular momentum, by means of, say, a study of the orbits of particles and the precession of the gyroscopes. In the language of quantum field theory, measurability of m, a and e stems from the fact that a black hole can interact with its surroundings via virtual photons and gravitons because they do not decouple from the source at the horizon. This does not happen to other field quanta. That's why it is not possible to define the baryon number or lepton number of a black hole. In order to understand this, let us drop a baryon or a lepton into the hole. At the horizon the baryon or lepton decouples from the meson or neutrino field and it is hard to say where the particle has gone or whether the baryon or lepton number of the hole has increased by one. Now this leaves us with the question: do the laws of conservation of baryons and leptons lose their meaning in black holes? Actually, although the local conservation of a quantity such as lepton or baryon number or strangeness remains valid, it is irrelevant, decause these quantities cannot govern the exterior spacetime. The exterior is influenced only by the dynamically conserved surface integrals, viz., m, a, and e. Not that the global conservation laws, known to be so well established, are violated; they are transcended, in the sense of impossibility of their verification. Hence Wheeler's statement, "A black hole has no hair". No way has been found to destroy the horizon so far. Whatever falls down into a black hole adds to its mass, angular momentum, and charge, and nothing else.

It has been shown by Bekenstein (1972) and Teitelboim (1972) that no exterior, massive Klein-Gordon (scalar), Proca (Vector) or Fierz-Pauli (spin-2) meson fields can be associated with a black hole. At the horizon, the gravitational field effectively sets the coupling constant equal to zero. Therefore, a black hole, be it static ($a = 0$) or stationary ($a \neq 0$), is incapable of interacting with the rest of the universe by means of strong interactions that are mediated by π (scalar), ρ (vector) or f (spin-2) virtual meason fields. The conclusion that black holes have no meson hair holds good in Brans-Dicke theory too, confirming a generalized Israel-Carter conjecture.

That a black hole also has no neutrino hair, i.e., that it cannot exert long-range neutrino forces on leptons in its exterior follows from the fact that no long-range weak interaction potential can exist in its exterior (Hartle 1972; Teitelboim 1972). Actually, the neutrinos do not totally decouple from the source (such as an infalling electron) at the horizon but the remaining interaction is given by an unobservable phase factor.

It is for these reasons, given two black holes with identical charge, mass and angular momentum, you cannot tell whether baryons went into the formation of one black hole and antimatter or radiation into that of the other.

THE END OF THE COLLAPSE:

Once the collapse (of an uncharged, spherical body) has become relativistic ($r \gtrsim R_S$), it is difficult to reverse, and it is impossible to reverse after the mass M has been squeezed to a dimension smaller than its event horizon. What then is the end of the collapse? Inside the horizon the co-ordinates r and t interchange their role dramatically: the future light cone is tipped inward towards decreasing r and since flow of time is not stopped the observer comoving with the collapsing mass, like the mass itself, is doomed to be crushed at the singularity $r = 0$ (infinite density and infinite curvature) in a proper time of $\backsim 10^{-5}$ $M/M_\odot$ sec once he passes through the horizon.

Now the existence of a singularity heralds the breakdown of all known physical formalism. The burning question is then: "Is the singularity real?" Can a body undergoing general non-symmetric collapse manage to avoid the occurrence of a spacetime singularity? The singularity theorems of Penrose and Hawking (1970) and recent studies of non-spherical collapse offer some insight into the problem which indicates that in non-spherical gravitational collapse, the collapsing matter may reach a critical stage after which it cannot communicate with the rest of the universe. Thereafter, if a spacetime-geometric structure called "trapped surface" (Fig. 2) forms, and our universe obeys certain conditions, then either a singularity may evolve in the geometry of spacetime, or a universe with which we had no previous connection may get suddenly joined to ours. The trapped surface forming in the region surrounding the matter of the collapsar, is a closed 2-surface, like the surface of a sphere, and exhibits the following property to mark the vicinity of the singularity: null rays emitted normally from a trapped surface converge as they propagate (wave front of shrinking area) regardless of whether they are emitted from the inner or outer surface. Finally the rays intersect either in a point or in what is called a caustic surface—a surface like the peak of a tent. The occurrence of a trapped surface signifies a point of no return for collapse since the theorems then suggest the existence of a singularity. However, there is an important disparity between the singularity that occurs in a spherical collapse and the one that evolves in a nonspherical collapse. In the case of spherical collapse, the trapped surfaces occur throughout inside the Schwarzschild radius. The convergence of photon paths is then by virtue of the greatly warped spacetime. The fate of the collapsing matter is sealed: it would be crushed to the state of infinite density and zero radius at the singularity. In the case of nonspherical collapse, the Penrose-Hawking theorem indicates that only some of the photons emitted from a trapped surface hit the singularity although it says nothing about the behavior of matter in the collapsar. It may be that the singularity has a non-zero, though small size and all or most of the matter that passes through the horizon is saved from being crushed to infinite density as it may get squeezed to a maximum density and then explode into another, possibly distant, region of spacetime in this or some other universe. When this matter emerges through the horizon, the event is witnessed as an explosion releasing tremendous amounts of energy. That is what a "white hole" is like. The bridge connecting the black hole with a white hole is known as the Einstein-Rosen bridge or a "worm hole". Also possible is the situation where matter is shot out of the $r=0$ singularity but lacks sufficient energy to emerge from the horizon into the external universe — a "grey hole" (Novikov and Thorne 1973).

MASS-ENERGY OF A BLACK HOLE ; THE LAWS OF BLACK HOLE MECHANICS:

Whatever be the black hole process, be it extraction of energy or simply accretion of matter or a collision with another black hole, it has to proceed according to the standard laws of physics. The four laws of black hole Physics bears similarity to the thermodynamics.

These laws become manifest when we study the interaction of a black hole with its surroundings. The interactions change the mass-energy of the black hole. But we must make one point clear : the very fact that everything which falls down a hole adds to its mass, charge and angular momentum and is then lost for ever beyond all recall is in no way an indication that energy in black holes is trapped once and for all and cannot be extracted. There are a number of ways to do so, eg. the Penrose process, black hole collisions and the super-radiance effect.

It was the remarkable idea of Penrose (1969) that particles falling into Kerr-Newman black holes might act as devices to extract their rotational energy. Suppose a particle with energy E_0 enters the ergosphere and decays into (1) a particle with energy E_1 and (2) one with energy $E_2=E_0—E_1$ (Figure 2). Now one can arrange the break-up such that, provided the first particle has negative angular momentum, its energy E_1 is negative as measured by a remote observer (though positive in the local Lorentz frame); then $E_2 > E_0$ The particle with energy E_1 falls down the horizon decreasing both the mass and spin of the black hole whereas that with energy E_2 flies away, with an excess energy (E_1), provided by the hole. Christodoulou (1970) has shown that all of the rotational energy of the rotating black hole can be extracted by a series of suitably chosen Penrose processes.

The mass left behind after " reversible " extraction of all the charge and spin is known as the irreducible mass m_{ir} of the black hole. The total mass - energy of the black hole consists of three parts : the irreducible mass, the rotational, and the electrical energy :

$$m^2 = \left(m_{ir} + \frac{e^2}{4\, m_{ir}} \right)^2 + \frac{J^2}{4\, m_{ir}^2} \qquad (2)$$

The irreducible mass is related to the horizon surface area of the black hole by $A = 16\pi\, m_{ir}^2$. Whatever transformations a black hole undergoes, its irreducible mass always tends to increase (irreversible transformations) at most, it can stay constant (reversible tansformations) :

$$\frac{dm_{ir}}{dt} \geqslant O \qquad (3)$$

(Christodoulou 1970). The equation (3) is just a special case of Hawking's theorem (not to be confused with the Hawking - Penrose theorem on singularities) to which we shall return later. Most of the transformations that a black hole undergoes are irreversible. Less frequent are the reversible ones where one has the possibility of extracting 50 percent of the energy of a maximally charged hole ($e = m$, $a = 0$) and 29 percent that of an extreme Kerr hole ($a = m$, $e = 0$) by a repetition of the Penrose process (Christodoulou and Ruffini 1971). Thus, in a sense, m_{ir} represents the energy of a black hole which is inert and cannot be transformed to work. A Schwarzschild black hole has no surplus energy ($m = m_{ir}$); hence no such extraction is possible. Still, according to Hawking (1971a), one can get energy out of the merger of two such black holes. What needs be done is to let two black holes (masses m and α m with $\alpha \leqslant 1$) spiral round each other, eventually colliding to form a single hole of mass m_0 and there you are with an energy $\Delta m = m + \alpha m - m_0 \leqslant m\,[1 + \alpha - (1+\alpha^2)^{\frac{1}{2}}]$, in the form of gravitational radiation; it might be as high as $(2 - \sqrt{2})m$ when $\alpha = 1$. Energy is available in the collisions of Kerr - Newman black holes also. The above upper limit on the available energy results from the constraint that the horizon surface area A of the final black hole must exceed the sum of surface areas of the colliding holes :

$$A \geqslant \sum_i A_i \qquad (4a)$$

This is known as Hawking's therorem and says that the horizon surface area of a black hole,

$$A = 4\pi\,(r_+^2 + a^2) = 4\pi\,(2mr_+ - e^2) \qquad (4b)$$

(for an isolated, stationary hole) cannot decrease towards the future in any process whatsover, e.g., accretion, collisions, or the Penrose process. At most it can stay constant (reversible transformations) i.e., $dA / dt \geqslant 0$. This is known as the second law of black hole mechanics, discovered simultaneously and independently by Hawking (1971a) and Christodoulou (1970, see Eq. 3). The theorem presupposes that spacetime is future asymptotically predictable, i.e., naked singularties ($\epsilon > 1$) are non - existent. In other words, causality holds. When there is no horizon, communication between the physical sigularity and the rest of the universe is possible. But if naked singularities are possible, then the end-state of complete gravitational collapse can differ from the Kerr-Newman form, invalidating the Israel - Carter conjecture; moreover the spacetime is then no longer future asymptotically predictable. Every attempt of an outside agency to destroy the horizon of an extreme Kerr-Newman black hole, e.g., through shooting charged particles, in a suitably chosen way, into the hole in order to raise ϵ above 1, fails. Once formed, a black hole remains a " horizon clothed " singularity that cannot be stripped. One does not know whether a naked singularity might form during collapse. Some indications have been given, though, by Yodzis et al. (1973) and Steinhmuller et al. (1975). Or is it that there exists some "cosmic censor" (Penrose 1969, 1974) which forbids the appearance of naked singularities, concealing each one in an absolute event horizon?

In some respects a black hole resembles a closed thermodynamic system. For instance, does not the irreducibility of horizon surface area sound like that of entropy in the second law of thermodynamics? The latter is a statement of the fact that no transformation, be it reversible or irreversible, can result in the decrease in the entropy of a closed thermodynamic system. Whenever there is an increase in entropy of such a system, some energy has been degraded. In the case of a black hole the increase in m_{ir} means that there is some degradation of energy of the hole. The area of a black hole can be so expressed (by suitably choosing the units) that it can be regarded as the entropy of the hole (Bekenstein 1973). However, this is not to be confused with the thermal entropy of the matter which fell into the hole.

Black hole entropy S_{bh} is the horizon surface area multiplied by a constant $\mathbf{K} = \frac{1}{8\pi} \, l_n 2 \frac{kc^3}{G\hbar} = 1.46 \times 10^{48}$ erg K^{-1} cm^{-2}. For $M = 1M_{\odot}$, $S_{bh} = 10^{60}$ erg K^{-1} which is 10^{18} times the entropy of the Sun. Such an entropy implies complete irreversibility of black - hole formation. In fact, the black hole state represents the maximum entropy state of a certain amount of matter.

Now, isn't it true that the ordinary second law of thermodynamics also is transcended in black hole physics? For, when a package of entropy is dropped down a hole, the entropy of the exterior universe decreases. However, there is no way of getting information about the black hole interior and hence the exterior observer finds no way to exclude the possibility that the total entropy of the universe has decreased in this experiment. That is what one means by the "transcendance" of the second law of thermodynamics in black hole physics. The law needs to be redefined if it is to regain its usefulness. A careful investigation shows that the black hole area would always increase by an amount sufficient to compensate for the disappearance of any entropy carried by the package. Hence, if one generalizes the entropy to incorporate both the black hole entropy S_{bh} and the ordinary entropy S_o of everything outside the black hole, one recovers Bekenstein's (1974) generalized second law of thermodynamics in black hole physics.

$$\Delta S_g = \Delta (S_{bh} + S_o) \geqslant 0 \tag{5}$$

as the statement that the generalized entropy, S_g, cannot decrease in any process.

On the basis of Eq. 2, We can write the black hole analog of first law of thermodynamics (dE=TdS - PdV) as

$$dm = \frac{\Theta}{4\pi} dA + \Omega . dJ + \Phi \, de \tag{6}$$

$$\Omega = \frac{a}{r_+^2 + a^2}, \quad \Theta = \frac{2m\pi (1-\epsilon^2)^{\frac{1}{2}}}{A}, \quad \Phi = \frac{er_+}{r_+^2 + a^2}$$

where Ω and Φ are the angular frequency and electric potential of the hole; $\Omega . dJ$ and Φde can be thought of as the work done (-PdV) on the hole by an outside agency which enhances its angular momentum by dJ and charge by de. The first term on the right side of Eq. (6) indicates an analogy between A and S and between Θ and T. The area A has already been identified with black hole entropy; Θ is called the black hole temperature.

The quantity $K = 2\Theta$ is called the surface gravity of the hole (Bardeen et al. 1973). Just as the temperature in a system in thermodynamic equilibrium is the same at every point, so the surface gravity of a stationary hole is uniform over the horizon. This is called the zeroth law of black hole mechanics. And, there is no way to reduce K to zero by means of any finite series of operations. This statement, the third law of black hole mechanics, is the analog of the statement that absolute zero is attainable only asymptotically. One can reduce K by means of shooting particles down a black hole, but $K = 0$ would only be approached, never exactly attained. Once K had become zero, it could be made negative too, thereby producing a naked singularity. The assumed non-existence of naked singularities therefore implies that K can never be made equal to zero.

It was suggested by Misner (1972) that, in analogy with Penrose's particle process, it is also possible to deplete a Kerr black hole ($a \neq 0$, $e = 0$), of its rotational energy by scattering waves upon it. This is known as "superradiance". The outgoing wave has the same frequency but increased amplitude. If m_h is the harmonic number of the wave $\psi \sim e^{-i(\omega t - m_h \varphi)}$ of frequency ω, then the mass of the black hole decreases by an amount $dm = (\omega / m_h) \, dJ$. This makes the first law of black hole mechanics, Eq. (6), (with $e = 0$) take the form

$$\left(1 - \frac{\Omega \, m_h}{\omega}\right) dm = \frac{K}{8\pi} dA \tag{7}$$

Now the second law requires $dA \geqslant 0$. Hence $dm < 0$ if $0 < \omega < m_h \Omega$ and the scattered wave carries more energy than the incident wave. The reflection coefficient, which measures the enhancement of the power, is defined by (Power) out / (Power) in. It varies with the frequency ω and mode numbers (l, m_h) of the spherical wave considered, as well as with a / m of the hole. The reflection co-efficient can be as large as ~ 1.02 for electromagnetic waves and ~ 2.38 for gravitational waves. (Starobinsky 1973; Starobinsky and Churilov 1973; Press 1974.) As shown by Bekenstein (1973a), it is possible to extract the charge and Coulomb energy of a Kerr - Newman black hole also, by scattering a charged wave-field upon it. Far from the hole, the charged wave - field can be taken to be composed of many quanta (mesons, electrons, etc.), with $\hbar\omega$ and q as the energy per quantum and electric charge per quantum, respectively. In superradiant scattering, the outgoing wave removes energy from the black hole when

$$\omega < m_h \Omega + \frac{q}{\hbar} \Phi .$$

In his celebrated investigation of 1917, Einstein used the principles of equilibrium statistical mechanics together with Planck's radiation law to prove that whenever a system can be "induced" to absorb or emit quanta by application of an external field, then that system must also display "spontaneous" emission: raised to an excited state, the system will de-excite of its own accord, even in the absence of any external field (Fermi 1966). Any charged, rotating black hole can be viewed as an excited state of the Schwarzschild ground state, according to Eq. (2). Superradiance, calculated in the framework of macroscopic, non-quantum physics is none other than the classical, correspondence limit of induced emission. Einstein's general argument then immediately implies that any charged, rotating black hole, even though left to itself, will spontaneously lose all its charge and angular momentum through emission of photons, gravitons, mesons, etc. This quantum process (vanishing as $\hbar \rightarrow 0$) is, however, negligible for stellar mass black holes for which the decay time is far longer than the age of the universe.

Distinct from the preceding is another quantum process of emission which works even for "dead" Schwarzschild black holes (Hawking 1974 a, b; De Witt 1974; Gibbons 1975). Inside the event horizon of a black hole there occur particle states of negative energy (with respect to an observer at infinity). A virtual pair of particles can therefore be spontaneously created, one with negative, the other with positive energy. If the pair is close enough to the horizon, the positive-energy particle may tunnel out quantum mechanically and escape to infinity, while the negative - energy particle remains behind. It is interesting to note that the particles of any given type, i.e., mass and spin, are emitted with the thermal spectrum appropriate to a body at temperature Θ (see Eq. (6)). The emitted flux carries off mass - energy and the hole "evaporates" in a time $\sim 10^{-28} M^3$ sec., where M is the mass of the hole in gms. Because of this process, Hawking's theorem, $\triangle A \geqslant 0$, is actually *violated* on the quantum mechanical level. However, the generalized 2nd Law of Thermodynamics, $\triangle (S_o + S_b) \geqslant 0$, remains valid: decrease of the black hole entropy $S_b \propto A$ is compensated by the increase in ordinary, external entropy S_o, due to the emitted particles.

The black hole temperature $\Theta \simeq 10^{26} M^{-1}\ ^\circ K$. For a black hole of stellar mass, $M = 1 M_\odot \sim 10^{33}$ gm., $\Theta \simeq 10^{-7}\ ^\circ K$, far less than the $3\ ^\circ K$ temperature of the microwave background. Such a black hole will therefore absorb more rapidly than it emits and it will grow larger.

There are reasons to believe that black holes with very small masses, i.e., $\sim 10^{-5}$ gm and upwards were created by the density fluctuations in the early period of the universe (Hawking 1971 b; Carr and Hawking 1974). The departures from homogeneity and isotropy in the early phases of the universe may have been large enough to overcome the pressure forces and kinetic energy of expansion so that small amounts of mass would have collapsed to form "micro black holes." A black hole formed at the threshold epoch in cosmology $t^* \sim \left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}}$ $\sim 10^{-43}$ sec should have a mass of $\sim 10^{-5}$ gm and a radius $\sim 10^{-33}$ cm. Being quite hot ($\Theta \gg 3\ ^\circ K$), the micro black holes would emit more than they absorb, and lose mass thereby. This makes Θ rise still further and enhances radiation even more. Once $K\Theta$ exceeds $m_e c^2$ and $M_\mu c^2$, respectively, electrons and muons are emitted. And when $\Theta > 10^{12}\ ^\circ K$, i.e., $M < 10^{14}$ gm., hadrons are emitted. The radiation rate rises cataclysmically towards the end : 10^{30} ergs are explosively released in the last 0.1 secs. of its existence. Only those primordial black holes could have survived which were born with masses $\geqslant 10^{-18} M_\odot \sim 10^{15}$ gm, i.e., with Schwarzschild radii of the order of the radius of a proton (since that of the sun is 3 km.). Although black hole area and entropy decreases in this way, still the generalized entropy, S_g, remains irreducible. Through the evaporation, the collapsar therefore converts baryons and leptons into entropy — an attractive explanation for excess entropy in the universe (10^8 photons per baryon).

OBSERVATIONAL PROSPECTS :

One does not know whether micro black holes exist. But with the discovery of quasars, pulsars and X-ray sources, the hopes of discovering the black holes have increased. Stellar black holes are expected to be present in binary systems, globular clusters, etc., while the best places to find supermassive black holes ($\geqslant 10^{4-5} M_\odot$) are the nuclei of galaxies and probably quasars.

But, by definition, a black hole is invisible. The only way out is its interaction with the surroundings and the subsequent release of energy which can escape to infinity and make it luminous. The energy extraction processes are astrophysically insignificant and therefore the most promising source of luminosity for a black hole is accretion of gaseous matter from the surrounding (interstellar) space. The simplest case is spherically symmetric accretion, which calculations reveal might be unpromising. However, in case the matter carries some angular momentum, it will form an accretion disk around the black hole and the resulting luminosity would be greatly increased. Thus, even if the gas density is low, accretion of matter with angular momentum can render a massive ($\geqslant 10^4 M_\odot$) black hole extremely luminous. The main source of energy is the viscous dissipation within the disk by virtue of which the angular momentum of gaseous matter is transported to the outer regions; the gas spirals in and gets sucked into the hole while the outward transfer of angular momentum heats the gas up. With the result that the hot gas emits X-ray radiation, with a power law spectrum of the form $F_\nu \sim \nu^{-\alpha_s}$, α_s being the spectral index. Most of the emission comes from regions close to the horizon. The best place to look for a stellar black hole is a binary system where the hole can swallow enough material from the companion star. There may be 10^{6-8} such systems in the Galaxy and systems like Cygnus X-1, ϵ Aur, β Lyr, BM Ori, HD 72754, Algol and HD 187399, etc., have been suspected of having black holes as one of their components. The X-ray object in Cygnus, Cyg X-1, has been the focus of great attention lately as it seems to be the best candidate for a black hole. There exists ample evidence that it is a binary system consisting of a normal B0Iab supergiant (HDE 226868) and that the secondary is a compact object emitting X-rays (Gursky 1973). A correct estimate of the masses is then necessary to decide whether the X-ray source is a neutron star or a black hole. On the basis of different arguments a number of workers suggest $M_1 \approx$ 20-30 $M_\odot$ for the supergiant and $M_2 \approx (3.4 - 8) M_\odot$ for the secondary (see, e.g., reports of I.D. Novikov and of M. Rees, and comments, p. 343 ff E. P. J. Van den Heuvel in Astrophysics and Gravitation, 1974). But the work of Rhoades and Ruffini (1974) has shown that under very general constraints ($0 \leqslant dp/d\rho \leqslant 1$) on the relation between pressure P and density ρ, the maximum mass of a (non-rotating) neutron star is $\approx 3.2\ M_\odot$. Moreover, all known compact stars rotate so slowly, with periods $P \gtrsim 33$ millisec., that the fractional enhancement of this maximum mass by rotation is entirely negligible: $\delta M/M \lesssim 10^{-4}$. X-ray brightness variations of Cyg X-1 over times ~ 0.1 sec. indicate that the source is smaller than 3×10^4 km. The secondary is thus far too small for a normal star. But it is too massive for either a white hole ($M \lesssim 1.2\ M_\odot$) or a neutron star. We are forced to the only remaining possibility that it is a black hole. An attempt to get around this conclusion by invoking a model with 3 component stars orbiting around one another does not seem to work — there is no evidence for the presence of a third body. The black - hole model of the object seems to be the most likely for Cyg X-1 therefore. In addition to Cyg X-1, three other binary X-ray sources are candidates for black holes. Two of these (2U 1700 -37 and 2U 0900 - 40), in our galaxy, show similarities to Cyg X-1 in their X-ray spectra, although no mass estimates are available. To the third, SMC X1, in the small Magellanic Cloud, is attributed a mass = 10 $M_\odot$ in order that its X-ray luminosity of 10^{39} ergs. sec^{-1} does not exceed the limit set by Eddington, $L_{Edd} = 10^{38}\ M/M_\odot$ ergs. sec^{-1}.

There are other suggestions like looking for a "gravitational lens" effect caused by the presence of a black hole in an eclipsing binary system, as the effect might modify the light curve. Also, massive black holes might be present at the centers of globular clusters, causing the surrounding stars to concentrate more toward the center, giving a bright central spot. However, these effects are yet to be observed.

Lynden Bell's idea of the possible presence of massive black holes at the centers of galaxies is widely known. Supermassive stars with masses $\sim 10^{8-9}\ M_\odot$ might form in the nuclei of galaxies, which at a certain point in their evolution, would undergo gravitational collapse to form gigantic black holes. Accretion of matter from the surrounding regions would then produce high energy radiation. The tendency of accreting matter is to enhance the hole's angular momentum so that it remains almost extreme Kerr (in fact, $a \simeq 0.998$m), according to Bardeen (1970) and Thorne (1974).

Further, if the collapse is nonspherical, one would expect copious amounts of energy to be released in the form of gravitational waves in an anisotropic fashion, carrying away not only mass and angular momentum, but linear momentum too. This would result in the recoil of the black hole, with velocities up to $v = \beta c \approx 10^4$ km sec^{-1}. As it advances through the galaxy, it would accrete gas and stars from its neighbourhood, to ultimately appear as a luminous object. Assuming that the recoil takes place in the center of an elliptical galaxy which can be approximated by an isothermal gas sphere with a central star density n_0 (Stars pc^{-3}), and a scale height a, then the number of stars captured by the hole (mass M) during its flight through the galaxy is

$$N \simeq \frac{\pi R_s^2\, n_0\, a}{\beta^2} \approx 10^{4-5}\ \text{Stars.} \qquad (8)$$

Stellar collisions, substantial gas accretion, tidal interactions between the stars and the black hole would soon introduce radical changes in the cluster of 10^{4-5} stars about the hole. At the time of emergence out of the galaxy, the system might consist of a massive black hole at the center of an accretion disk with a number of stars moving around in low - energy orbits and having a luminosity of $\sim 10^{40-41}$ erg sec^{-1}. It is, therefore, interesting to study models based on this idea which might provide an explanation to the phenomenon of the ejection of compact objects by galaxies, observed by Arp (1974) (cf Rees and Saslaw (1975)).

Gravitational and electromagnetic synchrotron emission from particles spiralling round a gigantic central collapsed object at radii extremely close to the photon sphere (the sphere of unstable, circular orbits for massless particles, r=3m in the case of a Schwarzschild black hole) has lately been of interest for attempts to explain Weber's observations of gravitational waves from the Galactic center (Misner et al. 1972; Breuer et al. 1973; Breuer 1975, p. 8 explains that these attempts fail to yield a palusible astrophysical mechanism because the radiating particle must be injected at relativistic speeds into special, unstable orbits), and the ν^{-1} spectrum common to many of the extragalactic sources and quasars (Chitre et al. 1974,1975). However, the infalling matter must have high enough initial energy to excite high harmonics.

Despite the considerable advances that have been made in observational astronomy over the past decade, an unequivocal proof of the existence of black holes remains yet to be given.

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