Heating in Intense Flux Tubes

S.S. Hasan
Indian Institute of Astrophysics
Bangalore 560 034, India

Abstract. Energy transport is considered in an intense flux tube on the Sun, with the aim of examining the role of various heating mechanisms and delineating their contributions to the energy budget. Radiative energy transport is modelled by solving the transfer equation in a generalised version of the Eddington approximation. Convective energy transport is treated within the framework of mixing length theory. Dynamical effects are also incorporated in the analysis by allowing mass motions within the tube. The temporal behaviour of an intense flux tube is studied, when its equilibrium is perturbed by a small downflow. It is found that overstable oscillations with a characteristic time of some 600s are set up. The computed temperature structure appears to be compatible with observations in the upper layers of the tube. However, close to $\tau = 1$, the tube has a somewhat lower temperature than the one inferred from semi-empirical models. Radiative transport dominates the energetics in the photosphere, but in the lower layers the main contribution is from enthalpy transport. It also appears that overstable oscillations are probably not important in heating the chromosphere and corona.

1. Introduction

It is generally believed, on the basis of observations (see Stenflo, 1989 for a review), that most of the magnetic flux at the photospheric level is in the form of vertical magnetic flux tubes. Despite the availability of sophisticated observational techniques, our knowledge of physical conditions inside the tubes is far from complete. Owing to the small horizontal dimensions (typically ≤ 1 arc sec), the internal structure of flux tubes is usually inferred indirectly using empirical methods (see Solanki 1990 and references therein). Recently, several empirical models of flux tubes have been constructed (e.g. Zayer et al., 1990; Keller et al. 1990), providing information on various quantities such as magnetic field strength, temperature and velocity at different depths. They provide fairly stringent constraints on theoretical models, which are ultimately needed to understand the physical processes occurring in flux tubes.

A number of theoretical studies have focused on examining the formation and evolution of flux tubes with the aim of developing self-consistent models. Using the thin flux tube approximation, the formation of intense flux tubes due to convective collapse was studied by Hasan (1983,84,85), the latter hereafter designated Paper I. The results of these time dependent non-linear calculations, clearly demonstrated the existence of oscillatory behaviour in the final state. Numerical simulations on flux tubes have been carried out in 2-D by Deinzer et al. (1984), Knölker et al. (1988), Knölker and Schüssler (1988) and Grossmann-Doerth et al (1989) and in 3-D by Nordlund (1983) and Nordlund and Stein (1989).

The present study is a continuation of earlier work on quasi-1-D models of intense flux tubes by the author. This investigation differs from the previous ones through the inclusion of a more realistic energy equation, which treats both radiative and convective transport of energy. Preliminary results based upon such an approach can be found in Hasan (1990). These calculations have been further refined in two important ways: firstly, an open upper boundary condition is used to allow upward transport of energy and secondly, the convective energy flux is not assumed constant with time. Another difference is that a stronger magnetic field (closer to the observed values) is used in the initial equilibrium state. It should be emphasised that the aim of the present analysis is not to examine the convective collapse phenomenon, but rather to examine the oscillations set up in an intense flux tube, when its equilibrium is perturbed.

2. Model and mathematical Aspects

2.1 Equations

Let us consider a vertical flux tube of circular cross-section threading the photosphere and convection zone on the Sun and adopt a cylindrical co-ordinate system (r,θ,z). In view of the small horizontal scale of the tube, it is convenient to use the thin flux tube approximation (Defouw, 1976; Roberts and Webb, 1978). The MHD equations for an axially symmetric tube, in the lowest order of this approximation, only involve one spatial variable, z. At the interface between the tube and the ambient medium, pressure balance is assumed. In the present study, the same set of equations as those used in Hasan (1985) are used, apart from the energy equation, which is

$$\rho C_v(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial z}) = -\rho C_v(\gamma-1)T\Delta\frac{\chi_\rho}{\chi_T}+4\pi\kappa\rho(J-S)-\frac{\partial F_c}{\partial z} \tag{1}$$

where ρ is the mass density, T is the temperature, v is the vertical component of velocity, $\Delta = (\nabla.\mathbf{v})_{r=0}$, C_v is the specific heat at constant volume,γ is the ratio of specific heats, κ is the Rosseland mean opacity, S is the source function, F_c is the vertical component of the convective flux (we implicity assume that the strong magnetic field suppresses horizontal convective energy transport) and z is measured positive into the Sun. Expressions for χ_ρ, χ_T can be found in Paper I. The first term on the right hand side of Equation 1 denotes the contribution due to compressional heating, whereas the second and third terms correspond to energy deposited by radiation and convection respectively. The second term can be related to the radiative flux, defined as $\mathbf{F}_{\mathbf{rad}} = -(4\pi/3\kappa\rho)\nabla J$, through the relation

$$\nabla.\mathbf{F}_{\mathbf{rad}} = 4\pi\kappa\rho(S-J) \tag{2}$$

2.2 Initial state

We assume that at $t = 0$, the tube is in hydrostatic and energy equilibrium. Both the external and internal atmospheres were constructed iteratively. Details of the method are described in Hasan (1988). Briefly, the external atmosphere resembles the combined models of Spruit (1977) for the convection zone along with the Vernazza et al. (1976) model for the overlying layers. To obtain the atmosphere within the tube, the static equations of MHD in the thin flux tube approximation were solved, keeping β_0 ($\beta_0 = 8\pi p_0/{B_0}^2$) fixed at 1.0, where the subscript o refers to the top boundary.

2.3 Numerical technique and boundary conditions

The equilibrium described in the previous section was perturbed by introducing a small downflow velocity (< 50 m s^{-1}) and the subsequent time evolution of the tube was followed by numerically solving Equations 1,2 along with the momentum and continuity equations using an explicit finite difference scheme based upon the Flux Corrected Transport algorithm of Boris and Book (1976). A finite length of tube, with upper and lower boundaries at $z = -1000$ km and $z = 2000$ km was used. The level $z = 0$, corresponds to $\tau_e = 1$, where τ_e is the continuum optical depth in the external atmosphere.

The method of charactersitics was used to implement the boundary conditions. A transmitting upper boundary condition was used at the upper boundary. During the downflow phase of the oscillations, the pressure and density were kept constant and the velocity was determined from the characteristic equations. For upflow, the velocity was assumed constant along the C_- characteristic. At the lower boundary, a no flow condition along with constant density were assumed. The pressure was calculated from the characteristic equations.

3. Results

Fig. 1 shows the time variation of the vertical component of velocity v at three different depths in the tube corresponding to z=0 (solid line), -500km (dashed line) and -900km (dotted line). Positive values of v denote a donwflow. We find that the flow exhibits oscillatory behaviour with a period of some 600s. The pattern of the flow is not a simple sinusoidal one, but somewhat complex, with the upflow and downflow phases not being symmetric. The amplitudes of the oscillations increase with time, indicating overstable behaviour. We also find that the oscillations at different heights are not in phase and indicate upward wave propagation.

Fig. 2 depicts the variation of the temperature as a function of vertical optical depth. The curve marked e corresponds to the external temperature profile. Curves marked 1,2 and 3 correspond to t =0, 356s and 698s respectively. At the initial instant, the flux tube is hotter than the ambient medium at equal optical depths, because it is evacuated with respect to its surroundings. During the downflow phase of the oscillation, the upper layers are heated due to an increase in the vertical radiative flux. In the sub-photospheric layers, which are superadiabatic, the tube is cooled by the downflow. The opposite effect occurs during the upflow. It is useful to compare these theoretical curves with those from other models. The dashed curve corresponds to the semi-empirical plage model of Solanki (1986), with the vertical bars corresponding to the temperature range (deduced from different observations) found by Zayer et al. (1990). Shown in dotted curves, are also the temperature profiles calculated theoretically by Deinzer et al. (1984) for different parameters, using a 2-D simulation. The time-averaged theoretical profile, obtained from the present calculation, appears to be broadly in agreement with semi-empirical models in

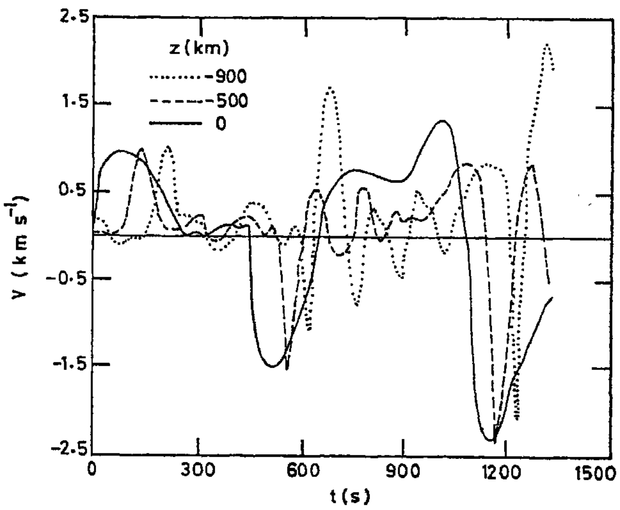

Figure 1. Variation of v with t at z=0 (solid line), -500km (dashed line) and -900km (dotted line).

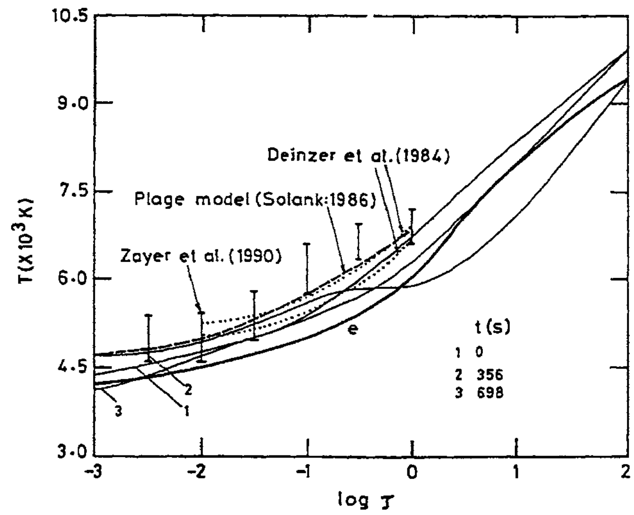

Figure 2. Variation of T with τ at t=0 (curve 1), 356s (curve 2) and 698s (curve 3). The external temperature is denoted by e. The dashed lines correspond to the semi-empirical plage model of Solanki (1986) and the vertical bars denote the temperature range based on the Zayer et al. (1990) model. The dotted curves denote the temperature profiles from Deinzer et al. (1984).

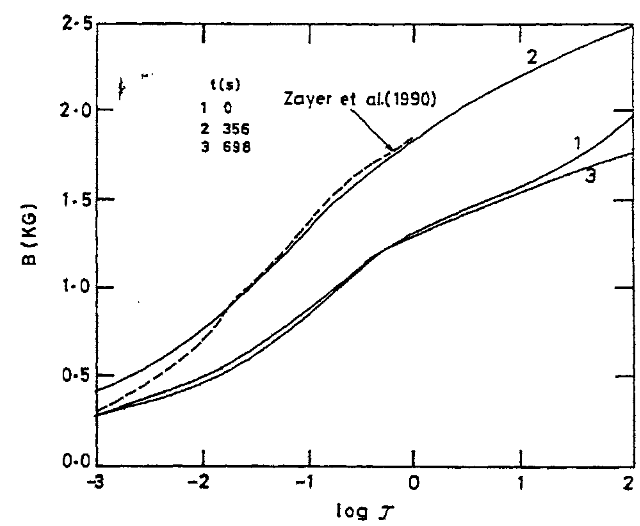

Figure 3. Variation of B with τ at t=0 (curve 1), 356s (curve 2) and 698s (curve 3). The dashed line correspond to the lower limit of the field strengths from the semi-empirical model of Zayer et al. (1990) model.

the upper layers of the tube, but somewhat cooler near $\tau = 1$. This could be because the actual tube is more evacuated than the one considered in this model.

In Fig. 3, the variation of the B, the vertical magnetic field strength, with τ is shown, at three different times. There is a significant increase of B during the downflow phase. The dashed curve corresponds to the lower limit to the field strength, deduced by Zayer et al. (1990). The value of β used in the present model gives field strengths somewhat lower than those inferred from semi-empirical models.

Fig. 4 shows the vertical radiative and enthalpy fluxes, $F_{\rm rad}$ (solid lines) and $F_{\rm enth}$ (dashes) as a function of time at z=0 (curve 1) and z=-200km (curve 2). All values have been normalised with respect to the normal photospheric flux. Positive values denote an upward flux. In the

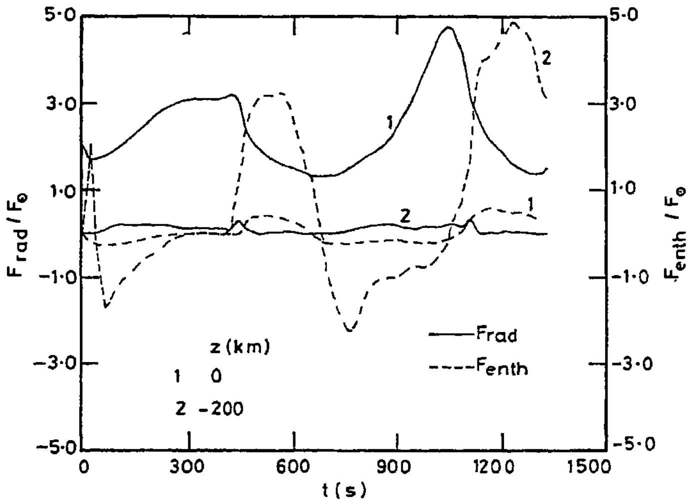

Figure 4. Variation of F_{rad} (solid lines) and F_{enth} (dotted line), in units of $F_{\odot}$, as a function of time at z=0 (curve 1) and z=-200km (curve 2). Positive values denote an upward flux.

photosphere and above, the energetics in the tube is dominated by radiative transport, whereas in the lower layers, it is the enthalpy flux which is much larger. In the layers close to the top boundary, it is found that F_{enth} is very small. The kinetic energy and convective energy fluxes have not been shown, since they are very small.

4. Summary and conclusions

The results indicate that intense flux tubes exhibit oscillatory behaviour, with a period of some 600s and with the amplitudes of the oscillations increasing in time. The upflow and downflow phases do not appear to be symmetric. In the upper layers, the computed temperature profiles are within the observed range. Close to $\tau = 1$, they are somewhat cooler. The major contribution to the energetics within the tube comes from radiative transport in the photospheric layers and above. However, in the subphotospheric layers and below, enthalpy transport dominates. It also appears, that energy transport by overstable oscillations is probably not an effective mechanism for heating the chromosphere and corona.

References

Boris,J.P. and Book,D.L.: 1976, *J. Comp. Phys.* **20**, 397.

Deinzer,W., Hensler,G., Schüssler,M., Weishaar,E.: 1984, *Astron. Astrophys.* **139**, 435.

Defouw,R.J.: 1976, *Ap. J.* **209**, 266.

Grossmann-Doerth,U.,Knölker,M.,Schüssler,M.: in Solar and Stellar Granulation, NATO Advanced Research Workshop, eds. R.J.Rutten and G.Severino (Dordrecht: D.Reidel), p.481.

Hasan,S.S.: 1983, in Solar and Stellar Magnetic Fields, IAU Symp. 102, ed. J.O. Stenflo (Dordrecht: D.Reidel), p.73.

Hasan,S.S.: 1984, *Ap. J.* **285**, 851.

Hasan,S.S.: 1985, *Astron. Astrophys.* **143**, 39.

Hasan,S.S.: 1988, *Ap. J.* **332**, 499.

Hasan,S.S.: 1990, in Physics of Magnetic Flux Ropes, ed. C.T. Russell (AGU Monograph), in press.

Keller,C.U.,Solanki,Steiner,O., S.K.,Stenflo,J.O.: 1990, *Astron. Astrophys.* **233**, 583.

Knölker,M., Schüssler,M., Weishaar,E.: 1988, *Astron. Astrophys.* **194**, 257.
Nordlund,A.: 1983, in Solar and Stellar Magnetic Fields, IAU Symp. 102, ed. J.O. Stenflo (Dordrecht: D.Reidel), p.79.
Nordlund,A., Stein,R.F.: 1989 in Solar Photosphere, Structure, Convection and Magnetic Fields, IAU Symp. 138, ed. J.O. Stenflo (Kluwer), p.191.
Roberts,B., Webb,A.: 1978, *Solar Phys.* **56**, 5.
Schüssler,M.: 1990 in Solar Photosphere, Structure, Convection and Magnetic Fields, IAU Symp. 138, ed. J.O. Stenflo (Kluwer), p.161.
Solanki,S,K.: 1986, *Astron. Astrophys.* **168**, 311.
Solanki,S.K.: 1990 in Solar Photosphere, Structure, Convection and Magnetic Fields, IAU Symp. 138, ed. J.O. Stenflo (Kluwer), p.103.
Spruit,H.C.: 1977, Ph.D thesis (Univ. of Utrecht).
Stenflo,J.O.: 1989, *Astron. Astrophys. Rev.* **1**, 3.
Unno,W., Spiegel,E.A.: 1966, *Pub. Astr. Soc. Japan* **18**, 85.
Vernazza,J.E., Avrett,E., Loeser,R.: 1976, *Ap. J. Suppl.* **30**, 1.
Zayer,I.,Solanki,S.K.,Stenflo,J.O.,Keller,C.U.: 1990, *Astron. Astrophys.*, (in press).

Discussion

M.Ruderman: (a) Did you study magnetic tube stability in the linear approximation, and if you did what were your results?(b) What were the boundary conditions at the ends of the tube?

Answer: (a) Yes, I have studied the linear stability of a tube in the adiabatic approximation and also with later heat exchange. The results can be found in my paper: 1986, *Mon. Not. R. astr. Soc.* **219**, 257. (b) As alrbady stated, a transmitting upper boundary condition and a no flow condition at the lower boundary were used.

A.Sterling: Do your results have any dependence on the location of the lower boundary?

Answer: The results have a weak dependence on the location of the lower boundary, provided it is taken sufficiently below the region of strong superadiabaticity, driving the instability. In the deeper regions, the velocity amplitude is fairly small, owing to the large density, so that taking a no flow condition at the lower boundary is a reasonable approximation.

K.Shibata: What is the maximun velocity at the photospheric base in your flux tube? Does it depend on the initial plasme β? If there is a 1-2 km s^{-1} velocity perturbation at the photosphere, such a perturbation may develop into a large amplitude wave in the upper photosphere. Did you find such behaviour?

Answer: Owing to overstability, the velocity amplitude grows as a function of time. Starting from a small perturbation, flows with amplitudes of around 1 km s^{-1} develop after some 600s at the photospheric base. The calculations clearly reveal that the amplitude of the flow increases with height, owing to the decrease in density. In the present investigation, the dependence of the results on β was not studied, but I expect, on the basis of my earlier calculations, that the effect of changing β is to increase the growth rate of the instability and the amplitude of the oscillation.

B.Roberts: It may be worth stressing that the low energy flux you find from the tube waves applies only to that arising in a tube, when left to itself, but subject to overstability. In other words, other mechanisms (such as buffeting by granules surrounding the tube) will add to the energy flux and to the generation of oscillations in tubes.