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LINEAR POLARIZATION IN CLOSE BINARIES

S.K. Parman
Milki H.S. School, Milki, Malda, West Bengal - 73221
A. Peraiah
Indian Institute of Astrophysics, Bangalore - 560034, India

ABSTRACT

Theoretical models have been computed for estimating linear polarization from the extended dusty outer layers of the components of close binary stars whose surfaces are distorted by rotation and tidal effects due to the presence of secondary. We have assumed plane-parallel layers of the dusty atmospheres of the components. We have employed a wavelength dependent scattering coefficient, and Rayleigh phase function is used in solving the equation of radiative transfer. It is noticed that polarization increases with decreasing the wavelength and increasing the particle size. Polarization for uniform rotation is larger than that for non-uniform rotation. Polarization for the single stars is always less than that for a binary component.

Subject headings: polarization - radiative transfer - stars: binaries - stars: late-type

INTRODUCTION

It is well known that both early and late type stars are intrinsically polarized. For a long distant star, we measure intensity integrated over the whole surface. So, the intrinsic polarization from a spherical star should lead to zero. The observation of large intrinsic polarization from a star means therefore, that there are large distortions and asymmetries in it's extended atmosphere. Based on geometry Harrington and Collins (1968), Collins (1970) have calculated the intrinsic polarization of a rapidly rotating early type star, and on the theory of Roche equipotential surface Ireland (1967) has presented a model of a rapidly rotating star and Peraiah (1969, 1970) has presented that a close binary system. Based on such geometry and Roche model, Peraiah (1976) has calculated linear polarization of rotating stars. In his paper he has worked on early type close binary stars taking electron and molecular scattering. Similar to this model of early type binary stars, we have made in our paper a model of late type binary stars.

In this paper we have computed theoretical models to find polarization from the extended atmospheres of late type close binary stars whose surfaces are distorted due to rotation and tidal effects, and the outer layers contain dust grains (silicate). The models contain several parameters of (i) wavelength dependent incident light (ii) the size of dust particle, (iii) the mass-ratio of binary components, (iv) type of rotation. The aim of this paper is to see how these parameters can change the linear polarization calculated from our theoretical models of such stars.

In section II we describe the calculation of polarization.

II. THEORETICAL MODELS TO COMPUTE LINEAR POLARIZATION OF A CLOSE BINARY STARS

Let the close binary system has masses m_1 (primary) and m_2 (secondary) where m_1 is more massive. The origin of the rectangular axes is the centre of the primary, the z-axis the primary's axis of rotation (cf. Fig.1) and the x-axis the line joining the centers of the binary. The equatorial planes of the components of the binary are assumed to be coincident with the orbital plane of the system. We consider that the surfaces of the primary are distorted (cf. Fig. 1) due to rotation about it's axis perpendicular to the equatorial plane and from the tidal effect due to the secondary of the system.

(a) The radius, the surface gravity and the surface element of the primary star.

From the assumption of the Roche equipotential surface, the radius of a distorted star is given by Peraiah (1970) as

$$\alpha\rho^7 \sin^6\theta + \beta\rho^5 \sin^4\theta + (\gamma \sin^2\theta+J)\rho^3-(1-Q)\rho+1 = 0 \qquad (1)$$

where

$\rho = r/r_p$ (r is the radial distance of any point P on the equipotential surface and r_p the polar radius).

θ = colatitude of the point P,

$\alpha = (f(x-1)^2/6x^2)(r_p/r_e)^7$

$\beta = (f(x-1)/2x^2)(r_p/r_e)^5$

$\gamma = (f/2x^2)(r_p/r_e)^3$

$J = Q(3 \sin^2\theta \cos^2\phi - 1)$

$Q = (1/2)\mu_1 (r_p/r_e)^3$

$\mu_1 = (m_2/m_1)(r_e/R)^3$

$x = \Omega_e/\Omega_p$ (the ratio of the angular velocity at the equator to that at the pole).

$f = \Omega_e^2 r_e^3/Gm_1$

(the ratio of the centrifugal to the gravitational forces at the equator).

R being the distance between the centers of the masses m_2 and m_1, r_e/r_p (r_e and r_p are the equatorial and the polar radii) being given by

$$(r_e/r_p)^3 - u(r_e/r_p)^2 - (1/2)\mu_1 = 0 \qquad (2)$$

$$u = 1 + f(x^2+x+1)/6x^2 + (1/2)\mu_1(3\cos^2\phi - 1) \qquad (3)$$

So the ratio $\rho = r/r_p$ (i.e. the radial distance of a point P in the units of polar distance) can be calculated from equations (1) and (2) for a given set of parameters x, f, θ, ϕ in the case of a single star and adding parameters m_2/m_1 and r_e/R in the case of the components of a close binary system. If binary stars rotate synchronously, then

$$f = (1+(m_2/m_1))(r_e/R)^3 \qquad (4)$$

The total surface gravity g :

$$g = (Gm_1/r^2)\{[1 - f_r(\theta)\sin^2\theta - 2J\rho^3]^2 + [f_r(\theta)\sin\theta\cos\theta + 6Q\rho^3\sin\theta\cos\theta\cos^2\phi]^2 + [36Q^2\rho^6\sin^2\theta\sin^2\phi\cos^2\phi]\}^{1/2}. \qquad (5)$$

where

$$f_r(\theta) = 2\gamma\rho^3 + 4\beta\rho^5 \sin^2\theta + 6\alpha\rho^7 \sin^4\theta \tag{6}$$

The surface element as:

$$ds = gr^2 \sin\theta\, d\theta\, d\phi/g_r \tag{7}$$

(b) The Radiative Transfer

We assume that the rotational and tidal effects are small over a sphere of radius a (cf. Fig. 1), where $a << r_p$ and this is always set to be the inner radius of the spherical shell (where $\tau = T$, where τ is the optical depth at any point and T is the total optical depth). Now, we calculate the distribution of the emergent intensities along the surface. To find such intensities we shall apply the idea proposed by Peraiah (1976). According to the idea the specific intensities I_l and I_r at any point $\rho(r,\theta)$ on the distorted surface of the star are calculated by solving the equation of transfer in spherical symmetry corresponding to radius r. The equation of radiative transfer for spherical symmetry can be written as (Peraiah 1976):

$$\frac{\mu}{r^2}\frac{\partial}{\partial r}\{r^2 I(r,\mu)\} + \frac{1}{r}\frac{\partial}{\partial\mu}\{(1-\mu^2)I(r,\mu)\} + \sigma(r)I(r,\mu)$$

$$= \sigma(r)\{[1-\omega(r)]b(r) + \frac{\omega(r)}{2}\int_{-1}^{+1} P(r,\mu,\mu')I(r,\mu')d\mu'\} \tag{8}$$

and

$$I(r,\ ) = \begin{vmatrix} I_l(r,\mu) \\ I_r(r,\mu) \end{vmatrix} \tag{9}$$

where $I_l(r,\ )$ and $I_r(r,\ )$ refer respectively to the states of polarization in which the electric vector vibrates along and perpendicular to the principal meridian, $\omega(r)$ the albedo for single scattering, σ the extinction coefficient, $b(r)$ the source inside the medium and $P(r,\mu,\mu')$, the Rayleigh's phase function given by

$$P(r,\mu,\mu') = \frac{3}{4}\begin{bmatrix} 2(1-\mu^2)(1-\mu'^2) + \mu^2\mu'^2 & \mu^2 \\ \mu'^2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_{11}(\mu,\mu') & P_{12}(\mu,\mu') \\ P_{21}(\mu,\mu') & P_{22}(\mu,\mu') \end{bmatrix} \qquad (10)$$

We shall set $\omega(r) = 1$, since we are considering pure scattering. The total optical depth T is given by

$$T = L\sigma d \ , \qquad (11)$$

where

$$\sigma = \pi \ell^2 Q_{scat} \ , \qquad (12)$$

$$Q_{scat} = (8/3)(2\pi\ell/\lambda)^4 ((m^2-1)/(m^2+2))^2, \text{ (HULST, 1957)} \qquad (13)$$

where d is the number of dust particles per unit volume, L the total path length, ℓ the radius of the dust particle, m the refractive index of the particle, λ the wavelength of the incident light. The total path length L has been taken as the length between the inner radius and the outer radius of a star.

(c) The Surface Integrated Linear Polarization

From equation (9), we have

$$I(\mu) = \begin{vmatrix} I_1(\mu) \\ I_r(\mu) \end{vmatrix} \qquad (14)$$

where $I_1(\mu)$ and $I_r(\mu)$ (cf. equation (9)) are parallel and perpendicular components to the electric vector of the emergent intensity at $P(r,\theta,\phi)$ (cf. Fig. 2).

Let $\underline{i},\underline{j},\underline{k}$ are the unit vectors along x-, y-, z-axes respectively and $\underline{e}_r$, $\underline{e}_\theta$, $\underline{e}_\phi$ unit vectors along the r,θ,ϕ directions. Then

$$\underline{e}_r = \underline{i}\ \sin\theta \cos\phi + \underline{j}\ \sin\theta \cos\phi + \underline{k}\ \cos\theta \tag{15}$$

$$\underline{e}_\theta = \underline{i}\ \cos\theta \cos\phi + \underline{j}\ \cos\theta\ \sin\phi - \underline{k}\ \sin\theta \tag{16}$$

$$\underline{e}_\phi = -\underline{i}\ \sin\theta + \underline{j}\ \cos\phi \tag{17}$$

Following the idea of Harrington and Collins II (1968), we take two rotations (cf. Fig. 2). One is rotation of x-y plane about the z-axis through an angle ψ' and the other rotation of x'-z' plane about y' axis through an angle $\pi/2-i$, where i is termed as ecliptic angle. Then

$$\underline{x}'' = \underline{i}\ \cos\psi'\ \sin i + \underline{j}\ \sin\psi'\ \sin i + \underline{k}\ \cos i, \tag{18}$$

$$\underline{y}'' = -\underline{i}\ \sin\psi' + \underline{j}\ \cos\ \psi' \tag{19}$$

$$\underline{z}'' = -\underline{i}\ \cos\ \psi'\ \cos i - \underline{j}\ \sin\psi'\ \cos i + \underline{k}\ \sin i, \tag{20}$$

If $\underline{x}''$ be taken as the line of sight, then the observer will see the unit vector $\underline{n}$ projected on the y" - z" - plane. Let α denotes the angle between this projection and the z"-axis. Let $\underline{\ell}$ is a unit vector in the line of sight. Then,

$$g\ \underline{n}.\underline{\ell} = \underline{g}.\underline{\ell} = (g_r \underline{e}_r + g_\theta\ \underline{e}_\theta + g_\phi\ \underline{e}_\phi).$$

or
$$\underline{n}.\underline{\ell} = (g_r\ \underline{e}_r + g_\theta\ \underline{e}_\theta + g_\phi\ \underline{e}_\phi).\underline{\ell}/g \tag{21}$$

= the component of $\underline{n}$ along the line of sight.

Similarly, the components of $\underline{n}$ along the y"-axis $\underline{n}.\underline{y}''$ and along the z"-axis $\underline{n}.\underline{z}''$ can be written. The ratio of the y" and z" components is the tangent of the angle α:

$$\tan\alpha = (\underline{n}.\underline{y}'')/(\underline{n}.\underline{z}'') \tag{22}$$

Therefore, the polarization (p) along the line of sight is given by

$$p = \frac{\int_0^{\pi/2}\int_{-\pi/2}^{\pi/2} (I_r - I_l) r^2 \cos 2\alpha\ \sin\theta (g\underline{n}.\underline{\ell})/g_r\ d\phi\ d\theta}{\int_0^{\pi/2}\int_{-\pi/2}^{\pi/2} (I_r + I_l) r^2\ \sin\theta (g\underline{n}.\underline{\ell})/g_r\ d\phi\ d\theta} \tag{23}$$

where the limits of integrations for ϕ and θ have been taken for symmetry.

III. COMPUTATIONAL PROCEDURE

Firstly, we have to find out r_e/r_p from equation (2) and then ρ from equation (1). To find r_e/r_p and ρ we have used Newton-Raphson method. In both cases 1.2 can be taken as the starting value. Then for each ρ, we have to solve the radiative transfer equation (8) to find I_l and I_r of equation (9). The inner polar radius of the spherical shell is taken to be 10^{12} cm. The medium above this shell is divided into 200 layers. The equation of transfer is solved by employing the discrete space theory (Peraiah 1984). From equation (23) we have calculated polarizations for wavelengths 5000 A to 10000 A for (i) dust particles ℓ = .02, .025, .03 micron (cf Fig. 3). (ii) m_2/m_1 = 0, .5, .9 (cf. Fig. 4), (iii) ratio of the equatorial to the polar angular velocities x = 1,5, 10 (cf. Fig. 5). Other parameters for the figures have been mentioned in the figures. We have taken synchronous rotation of binaries. For simplicity, we have set $\phi^o = 0$ for all calculations.

IV RESULTS AND DISCUSSIONS

We have set $2\pi\ell/\lambda$ less than 0.4, since the equation (13) for Q_{scat} should be taken for $2\pi\ell/\lambda$ less than .5 (Hulst 1957). For this we have taken small dust particles not greater than .03 micron for the wavelengths 5000 A^o to 10000 A^o. We have taken refractive index of dust particle (silicate) 1.45 and the number density d equal to 4 per c.c. From the figures 3-5, it is clear that polarization decreases with the increase of wavelength like molecular scattering (Peraiah 1976). This indicates the nature of Rayleigh scattering but this nature has been deviated for the dust particle ℓ = .03 micron in the interval of wavelength 5000 A^o to 6500 A^o (cf. Fig.3). This is because the formula for Q_{scat} (cf. equation (13)) has not worked well in the interval for the dust particle .03 micron or larger than this. In the next work we shall use Shah (1977)'s programme to find Q_{scat} that works for any size of dust particles and for any combination of scattering and absorption which are reasonable for the stars of interest. With increase of particle size (cf. Fig. 3), polarization has increased within the interval of wavelength. This is due to the fact that larger particles can create the outer layers and surfaces of a star more a-symmetry and distorted.

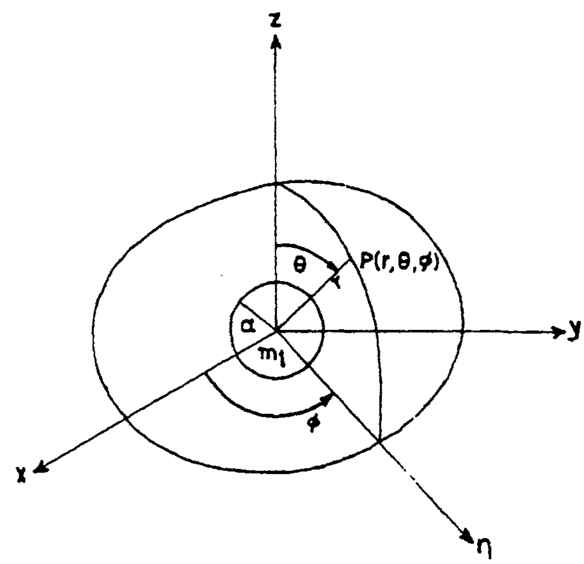

Fig.1 Distorted surface of the primary star m_1 with core radius a.

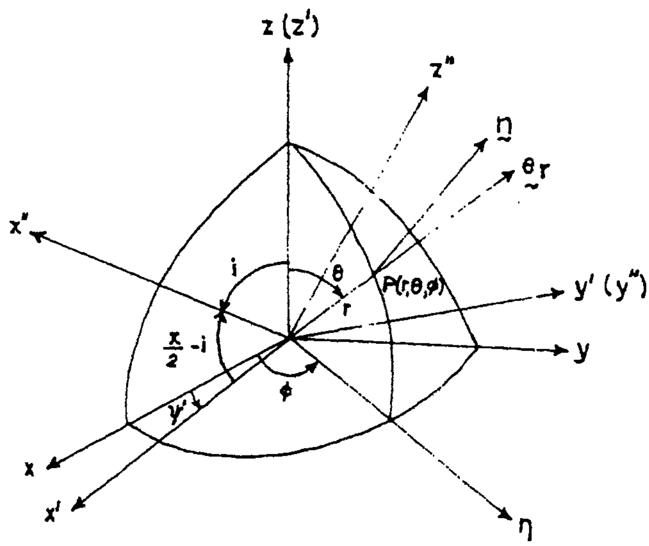

Fig.2

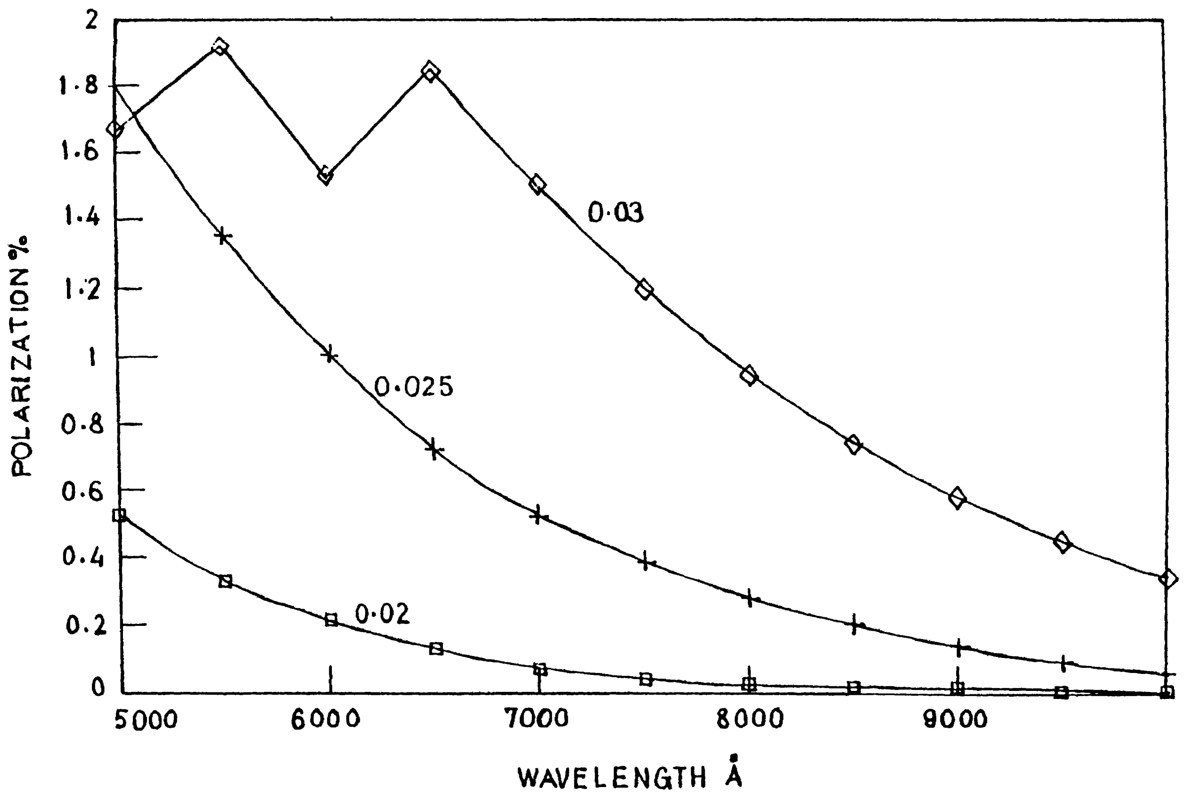

Fig.3 Polarization versus wavelength for $l = \cdot 02\mu$, $\cdot 025\mu$, $\cdot 03\mu$ (mentioned on the graph). The other parameters are: $x = 1$, $m_2/m_1 = \cdot 5$, $r_e/R = \cdot 5$, $m = 1{\cdot}45$, $\psi' = 0^\circ$, $i = 90^\circ$

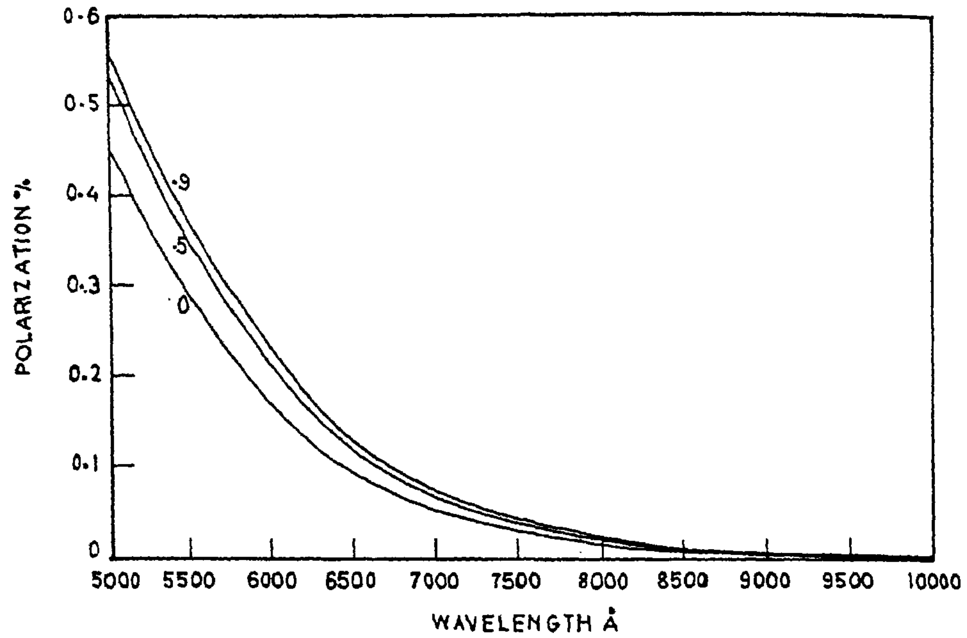

Fig.4 Polarization versus wavelength for $m_2/m_1 = 0, \cdot 5, \cdot 9$ (mentioned on the graph)
The other parameters are: $l = \cdot 02\,\mu$, $x = 1$, $r_e/R = \cdot 5$, $m = 1\cdot 45$, $\psi' = 0^\circ$, $i = 90^\circ$

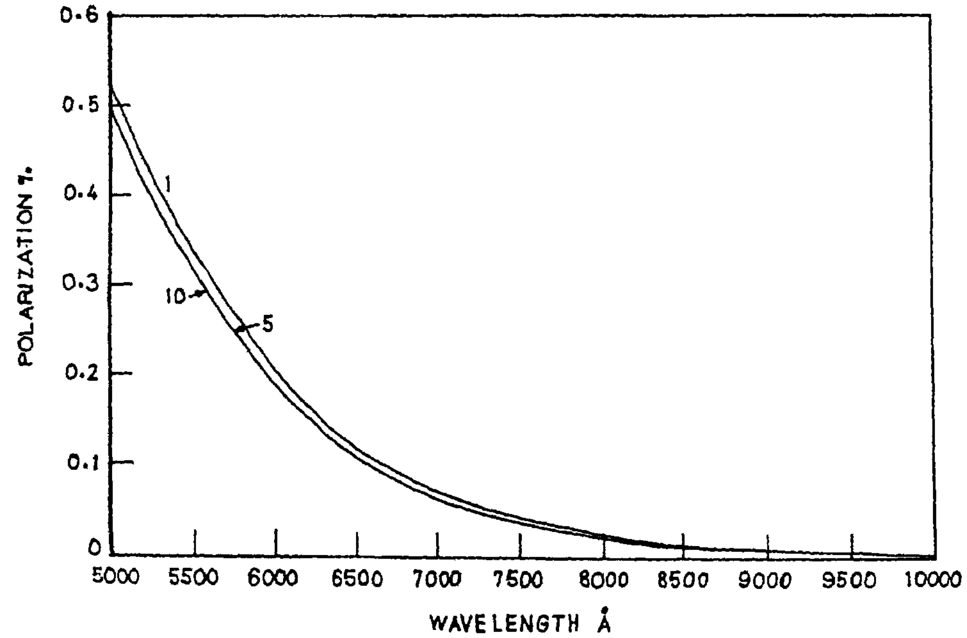

Fig.5. Polarization versus wavelength for $x = 1, 5, 10$ (mentioned on the graph)
The other parameters are: $l = \cdot 02\,\mu$, $m_2/m_1 = \cdot 5$, $r_e/R = \cdot 5$, $m = 1\cdot 45$, $\psi' = 0^\circ$, $i = 90^\circ$

Polarization has increased when mass ratio has increased (cf. Fig. 4). Polarization, when $m_2/m_1 > 0$, is greater than that when $m_2/m_1 = 0$.This is due to the fact that the primary component is becoming more asymmetric due to presence of the secondary component. This is in agreement with Peraiah's work (1976) for electron scattering. It is noticeable that uniform rotation ($x = 1$) about it's axis of rotation has the maximum effect (cf. Fig. 5) on polarization. Polarization for uniform rotation ($x = 1$) is greater than that for non-uniform rotation ($x > 1$). Here we are considering $x > 1$ i.e. the angular velocity at the equator is always larger than that at the pole. In our model we have $r_p < r < r_e$ and this has been possible by taking into account of $x > 1$.

We have done this work to see the effects of some parameters on the polarization by dusty outer envelopes of close binary system in the line of sight. This theoretical model we wish to apply in real situations (in late type binaries) in future. For that case, we have to adjust the parameters, the size of the particle should be increased (so $(2\pi\ell/\lambda)$ may be much greater than 1) and the absorption by dust particle has to be considered. The medium should be taken non-uniform. All these considerations are under study.

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REFERENCES

[1] Collins II,G.W., (1970), In Stellar Rotation, ed.A. Slettebak, p.85.
[2] Harrington, J.P. and Collins II ,G.W. (1968),Astrophys. J.,151,1051.
[3] Hulst, H.C. van de, (1957), Light Scattering by Small Particles, p. 270, John Wiley & Sons, New York.
[4] Ireland, J.G. (1967), Z.Astrophys., 65,123.
[5] Peraiah, A. (1969), Astron. Astrophys.,3,163.
[6] Peraiah, A. (1970), Astron. Astrophys.,7,473.
[7] Peraiah, A. (1976), Astron. Astrophys.,46,237.
[8] Peraiah, A. (1984), In: Methods in Radiative Transfer, p. 281, ed. W.Kalkofen, Cambridge University Press, Cambridge.
[9] Shah, G.A. (1977), Kodaikanal Obs. Bull. Ser. A,2,42.