Indian J. Phys. **66B** (5 & 6), 569 – 582 (1992)

Self-organization processes in astrophysics

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Abstract : Formation of ordered structures of definite sizes and shapes in an otherwise turbulent and seemingly disordered system is discussed using statistical description of a nonlinear system with a large number of degrees of freedom. Use of variational principle facilitates to fathom the field forms. Two examples from astrophysics : first the granulation patterns on the solar surface and second the patterns of galaxy distribution in the form of superclusters and giant-clusters, are presented here to illustrate the potential of this way of describing a turbulent medium.

Keywords : Self-organization processes, turbulence, astrophysics, statistical mechanics.

PACS Nos. : 95.30.Lz, 98.65.-r, 05.20.Gg, 47.27.-i

1. Introduction

A fluid is said to be turbulent, when in a random state of motion, due to the excitation of instabilities, it exhibits motion on many spatial and time scales, capable of exchanging energy among themselves. Generally, the boundaries and the buoyancy constrain the motion on the largest scales and dissipation at the smallest scales. The presence of turbulence affects the transport of mass, energy, electromagnetic fields, concentration of species, mixing of elements and chemical reactions in an anomalous manner. The transport could become exceptionally efficient if the turbulent system supports the formation of large organized structures. In the following sections, the conditions which facilitate the formation of ordered structures in fields describing the system will be explored and then applied to understand the solar granulation and clustering of galaxies, the two apparently disconnected phenomena [1-5].

2. Characterization of turbulence

In a turbulent fluid, the velocity field $U(x,t)$ can take any value in a completely random fashion. Since $U(x,t)$ is a random function, its values must be distributed according to some definite probability laws which can be determined from the experimental data of the problem. A knowledge of $U(x,t)$ for every point (x,t) constitutes a realization of the turbulent field. In general, there exists a statistical connection between the values of $U(x,t)$. The probability laws describe this statistical connection and one is able to determine the average values of the various quantities of interest. For spatially homogeneous turbulence all regions of space are equivalent. Thus averaging over large number of realisations or ensembles is same as averaging over a large region of space for one realisation. But what one measures in a laboratory experiment is the time variation of $u(x,t)$ at a fixed space point and obtains a time average. With the help of Ergodic theory, one arrives at the equivalence of the time average and the probability average which is further equal to the spatial average for homogeneous turbulence.

3. Quantification of turbulence

Mean values of the products of field variables (like velocity, magnetic field etc) and their derivatives form the fabric of a turbulent medium. Out of these, the two-point correlation function $R_{ij}(r)$ defined as :

$$R_{ij}(r) = \langle u_i(x)\, u_j(x+r) \rangle \quad (1)$$

is the most important. Here angular brackets represent the space average. For homogeneous and stationary turbulence $R_{ij}(r)$ depends only on the configuration and not on its location. For example, the total kinetic energy of a fluid is given by $R_{ij}(o)$. The Fourier transform of $R_{ij}(r)$ can be difined as :

$$R_{ij}(r) = \int \phi_{ij}(K)\, e^{ikr} d^3k$$

$$\phi_{ij}(K) = \frac{1}{(2\pi)^3} \int R_{ij}(r)\, e^{-ikr} d^3r$$

and since $R_{ij}(o) = \langle u_i(x) u_j(x) \rangle = \int \phi_{ij}(k) d^3k,$

$\phi_{ij}(k)$ represents a density in wavenumber space of contributions to $\langle u_i(x)\, u_j(x) \rangle$ which determines the energy associated with the various components of the velocity. Thus $\phi_{ij}(k)$ represents distribution of energy in K space and the total kinetic energy per unit mass of the fluid is given by :

$$E = \frac{1}{2} \langle u_i(x)\, u_i(x) \rangle = \frac{1}{2} \int \phi_{ii}(k)\, d^3k = \int E(k)\, dk.$$

Now, by definition, a turbulent motion is rotational, therefore the vorticity $\omega = \nabla \times u$ and the helicity $\gamma = u.\omega$ must be specified too. Their correlation functions are related to the two-point velocity correlation function $R_{ij}(r)$ and its derivatives. Conditions like incompressibility further restrict the amount of data required. The hope is that the action of Naiver-Stokes equation of motion would direct the random velocity field into a manageable statistical state [6,7].

4. Cascading characteristics of turbulence

The direction of flow of energy in a turbulent medium containing several spatial scales is determined by the nonlinear interactions between the fluid elements of various sizes. It is also well known that the direction of cascade or the transfer of energy depends very essentially on the dimensions of the system. Thus, a two-dimensional system behaves in an entirely different manner from a three-dimensional one. Since it is almost impossible to completely specify a turbulent state, often Kolmogorov's law [7] is used to find the spectral characteristics in the inertial range which is a range in wave-number with no sources and no sinks. In this range, the spectrum transfers smoothly in a stationary state with energy flow rate equal to the dissipation rate. In the absence of sources and sinks, a turbulent system can be specified by certain time-independent quantities, called the invariants of the system. By applying Kolmogorovic arguments [7] to these invariants, one can get their spectral transfer laws.

4.1. *Three-dimensional hydrodynamic system* :

An incompressible hydrodynamic system is described by the Navier-Stokes equations :

$$\rho\left[\frac{\partial}{\partial t} + V.\nabla\right]V = -\nabla p + \nu\nabla^2 V \qquad (2)$$

and $\nabla V = 0$ (3)

where p is the pressure, ρ the density and ν is the kinematic viscosity.

The total energy defined as :

$$E = \int \frac{1}{2}\rho V^2 d^3 r \qquad (4)$$

is the only quadratic invariant of a 3-*D* system. If the Fourier amplitude of the velocity field is V_k, then the rate at which the spectrum cascades is given by (KV_k), (the convective derivative). Kolmogorov [7] stated that in a quasi-steady state, there should be a stationary flow of energy in *K*-space from the source to the sink, *i.e.* the energy density flow rate (KV_k) (ρV^2_k) should be constant and equal to the dissipation rate ε of the energy density at the sink. Thus

$$\rho K V_k^3 = \varepsilon. \qquad (5)$$

If $W(k)$ is the omnidirectional energy spectrum then the total energy E is given by :

$$E = \int W(k)dk$$

or $KW(k)$ has the dimensions of V_k^2. Therefore :

$$W(k) = C\left(\frac{\varepsilon}{\rho}\right)^{2/3} k^{-5/3} \qquad (6)$$

where C is a universal, dimensionless constant, determined experimentally and lies in the interval 1.4-1.8 Landau [in 11] however, pointed out a contradiction. Kolmogorov assumed an average constant dissipation rate ε, whereas the dissipation can be equally well described by the local rate $\varepsilon(r,t) = \frac{1}{2}\nu\left[\frac{\partial v_i}{\partial x_i} + \frac{\partial v_j}{\partial x_j}\right]^2$, which is a fluctuating quantity, as confirmed by experiments. Notwithstanding this, no experiment has indicated any deviations from Kolmogorov law, though these have been observed for higher order velocity moments. This state of affairs is a pointer to the intermittent nature of small scale turbulence. In a 3-D system, the energy cascades to small spatial scales where it suffers dissipation.

4.2. *Two-dimensional hydrodynamic system :*

In a 2-D incompressible and ideal system, there are two invariants in the inertial range, the total energy E and the enstrophy U difined as :

$$U = \int \frac{(\nabla \times V)^2}{2} d^3 r \tag{6a}$$

Therefore one expects, two types of inertial ranges, one for the energy and the other for the enstrophy. The enstrophy density is given by $\rho k^2 V_k^2$ and the inertial range for enstrophy is determined by demanding :

$$(KV_k)(\rho k^2 V_k^2) = \text{Constant} = \varepsilon'$$

Using $KW(k) = V_k^2$, one finds

$$W(k) = C\left(\frac{\varepsilon}{\rho}\right)^{2/3} K^{-3} \tag{7}$$

If $W(k) \propto K^{-3}$, there is no energy cascade and if $W(k) \propto K^{-5/3}$, there is no enstrophy cascade. Hence a source at $K = K_s$ will set up two inertial ranges given by eqs. (6) and (7). Since enstrophy, because of its larger K dependence $(K^2 V_k^2)$ is dissipated at a rate faster than energy, the $K > K_s$ region is expected to be the inertial range for enstrophy and $K < K_s$ for energy. Thus energy is expected to cascade towards large spatial scales and an inverse cascade is set up. This has been further confirmed by the mode-mode coupling consideration of Hasegawa [8]. Here, a source K_s splits into two modes K_1 and K_2 such that the total energy and enstrophy are conserved. The process can continue for K_1 and K_2 until one finds that the energy condensates to the largest allowed spatial scale determined by boundaries. The inverse cascade of energy in a 2-D system has been substantially confirmed by experiments and the numerical solution of the Naiver-Stokes equations [9,10].

Question : Can inverse cascade of energy occur in a 3-D system ?

4.3. *Inverse cascade in 3-D :*

One learns from a 2-D system that it is the incompatibility in the inertial range spectra of the two invariants, that led to the energy cascade towards large spatial scales. So, if one had one more invariant in a 3-D system, perhaps inverse cascade of energy could occur. Besides, the appearance of large scale structures in 3-D atmospheres of planets compells us to look for the possibility of inverse cascade in 3-D. Specifically, the observations of helical type of flow structures in circumstances varying from oceans to cloud complexes, brought to the fore the importance of helicity in turbulent fluids. The fundamental idea that needed to be appreciated was that the large helicity fluctuations always exist in a turbulent medium even if the average helicity vanishes. It was shown that the fluctuating topology of the vorticity field in turbulent flows can be characterised by the statistical helicity invariant I represented by conserved mean square helicity density [11] :

$$I = \int < \gamma(x) \ \gamma(x+r) > d^3r = \int I(r) d^3r \tag{8}$$

where $< \gamma(x) \ \gamma(x+r) > = \lim_{V_2 \to \infty} \frac{1}{V_2} \int \gamma(x)\gamma(x+r) d^3x$

$$I(r=0) = < \gamma(x) \ \gamma(x) > = \int I(k) \ d^3k \tag{9}$$

and $\gamma = V.(\nabla \times V)$.

Here, $I(k)$ represents a density in wave number space of contributions to $< \gamma(x) \ \gamma(x) >$ which is the mean square helicity density. Thus E an I are the two invariants of a 3-D system. For a quasi-normal distribution of helicities, I can be written as :

$$I = A \int [W(k)]^2 \, dk \tag{10}$$

where A is a constant.

Using Kolmogorovic arguments for the I invariant, one can determine the spectrum in the inertial range as :

$$(KV_k) \ (kW_k^2) = \text{constant}$$

$$W_k \propto K^{-1} \tag{11}$$

and the total energy density is

$$E = \int E \ (k) \ dk \quad \propto \ \log \frac{L(t)}{l}. \tag{12}$$

Here $L(t)$ is the transient largest scale excited at time t. One observes that the energy grows very slowly as the spatial scale $L(t)$ grows. So, practically, there is little transfer of energy towards large scales. But, what happens is that as the correlation length of helicity fluctuations increases, the velocity and vorticity become more and more aligned and as a consequence the nonlinear term $(V. \nabla) \ v$ of the Navier-Stokes equation decreases and retards the flow of energy towards small scales. On the other hand, the growth of correlation length cannot go on indefinitely. Especially if the medium is restricted in the vertical direction by gravity or buoyancy as is true of atmospheres of any celestial object, may it be a planet or a star. Under such circumstances, the correlation length continues to grow in the horizontal plane and the system becomes more and more anisotropic. In addition, since $I(k) = [E(k)]^2$ and $E(k) \propto K^{-5/3}$, in analogy with the 2-D case one expects $I(k)$ to be dominant at small k while $E(k)$ will be larger at large K and this itself is a pointer to the inverse cascade of I.

What is achieved by the growth of correlation length of helicity fluctuations is the anisotropy in the system which can now be approximated to a quasi 2-D system. Here, the horizontal scale is much larger than the vertical scale and the vertical velocity V_z is much smaller than the horizontal velocities (V_x, V_y). This decouples the horizontal and vertical motions. V_z becomes independent of (x,y,z) and (V_x, V_y) independent of (z) leading to

$\omega_{x,y} = (\nabla \times V)_{x,y} = 0$. The invariant I becomes:

$$I = \int <(V_z\omega_z)^2> \; dx\,dy\,dz$$

$$\simeq L_z < V_z^2 > K^2 V_k^2 K^{-2} \propto V_k^2 = KW(k) \propto L^{2/3}$$

and $\qquad I(k) \propto K^{-5/3}. \qquad (13)$

Here, L is the largest length scale in the horizontal plane. Thus $I(k)$ spectrum coincides with energy spectrum of 2-D turbulence $E(K) \propto K^{-5/3}$ corresponding to the inverse cascade. One expects that an increasing fraction of energy is transferred to large spatial scales as the anisotropy in the system increases. This can go on until coriolis force begins to be effective. The length scale L_c where the non linear term of the Navier-Stokes equation becomes comparable to the coriolis force, can be determined from

$$(V.\nabla)^{*}V = 2(V \times \Omega) - \Omega \times (\Omega \times r) \qquad (14)$$

$$\text{or } L_c \approx \frac{V}{\Omega} \qquad (15)$$

where Ω is the angular velocity. Given sufficient energy, structures of size L_c must form. At these large spatial scales, the system simulates 2-D behaviour and enstrophy conservation begins to play its role. One may consider scales $L \geq L_c$ as a source of vorticity injection into the system. The enstrophy then cascades towards small scales with a power law spectrum given by

$$W(k) \propto K^{-3} \text{ and } E \propto L^2. \qquad (16)$$

Thus there is a break in the energy spectrum as energy must cascade to larger spatial scales as $L^{2/3}$ and to small scales as L^2. Therefore the energy must accumulate at $L \sim L_c$ and pass on to the highest possible scales of the general circulation of the atmosphere. The complete energy spectrum of a hydrodynamic turbulent medium is given in Figure 1.

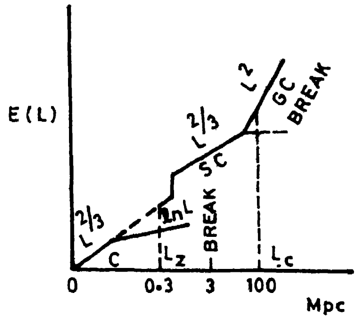

Figure 1. Turbulent energy spectrum. L_z = scale of the first break due to buoyancy, L_C = scale of the second break due to the Coriolis force, C = cluster, SC = supercluster and GC = giant cluster.

5. Self-organization or negative temperature

Using the two invariants, the energy E and the enstrophy U of a two dimensional incompressible fluid, the Gibbs distribution function is written as

$$P = \text{Constant} \times \exp[-\alpha E - \beta U] \tag{17}$$

where α and β are constants. In wave-number Fourier space, $U_k = K^2 W_k$, one finds the model energy distribution in wave-number space to be:

$$< W_k > = \frac{1}{\alpha + \beta k^2} \tag{18}$$

Thus $< W_k >$ 0 even for $\alpha < 0$ and therefore, temperature $\equiv \alpha^{-1}$ can be negative and a self-organized state with energy accumulating at large spatial scales due to inverse cascade forms.

Analogous to 2-D, one can write the Gibbs distribution function for a 3-D system using energy E and mean square helicity I as the two constants of motion:

$$P = \text{Constant} \ \mathrm{X} \exp[-\alpha E - \beta I]. \tag{19}$$

In the wave-number Fourier space, $I(k) = [W(k)]^2$, the modal energy distribution in wave-number space is given by

$$\langle W_k \rangle = \frac{\int_0^\infty e^{-b(W_k + a/2b)^2} W_k dW_k}{\int_0^\infty e^{-b(W_k + a/2b)^2} dW_k}$$
$$= \frac{-\alpha}{2b} + \frac{\exp\left[-\alpha^2/4b\right]}{2b\sqrt{\frac{\pi}{\beta}}\left\{1 + \phi\left(\alpha/2\sqrt{b}\right)\right\}} \tag{20}$$

where ϕ is the probability function. Here we find α<0 (β>0) is possible. One could in principle, get a state with negative temperature corresponding to E. This is surprising since it was found that it is I that cascades towards large spatial scales and E towards small spatial scales in a 3-D system. Therefore one would expect $\alpha > 0$ and $\beta < 0$ to be the only possibility, which corresponds to negative temperature associated with I instead of E. However, from the fact that cascading of I towards large spatial scales is also accompanied by the alignment of velocity and vorticity which retards the flow of energy to small spatial scales, it is in this sense that α can acquire negative value in a system developing anisotropy.

6. Variational principle for self-organized state

Hasegawa [8] has discussed the earlier work on the formation of ordered structures in a medium that supports at least two differentially decaying ideal invariants. Thus, the minimization of the rapidly decaying quantity $B(V)$ keeping the slowly decaying one $A(V)$ constant leads to a quasi-stationary state in which the random field V, *e.g.* velocity in a turbulent fluid can be described by the equation:

$$\partial A(V) - \lambda \partial B(V) = 0. \tag{21}$$

For a 3-D system, $A(V) = \int (V.\omega)^2 d^3x$ and $B(V) = \int V^2 d^3x$ gives

$$\int \partial V.[2(V\cdot\omega)\omega - \lambda V - V\times \nabla(V.\omega)]\, d^3x$$
$$+ \int \nabla.[(V.\omega)\partial V \times V] d^3x = 0$$

where $\omega = \nabla \times V$ is the vorticity. Applying the divergence theorem and assuming that $\hat{\omega}.\hat{n}$ vanishes on the boundary, the second them in eq. (21) vanishes and the variational equation equation becomes

$$2(V.\omega)\omega - V\times \nabla(V.\omega) - \lambda V = 0. \tag{22}$$

For comparison, the corresponding equation obtained for a 2-D system with enstrophy and energy as its invariants is

$$\nabla\times(\nabla\times V) = \lambda V. \tag{23}$$

Thus, the variational eqs. (22) describing the flow pattern of a 3-D system is nonlinear and enormously more difficult to solve. The self-organized state described by eq. (22) should be a stationary solution of the Navier-Stokes equation (without dissipation) and from it one can determine the temperature distribution

$$\begin{aligned} \rho(V.\nabla)V &= -\nabla p \\ p &= \rho RT(x,y), \end{aligned} \tag{24}$$

where R is ths gas constant. Summarising, the energy spectrum, the velocity flow pattern and the temperature distribution of an organized state in a turbulent fluid are given by eqs. (6,12,16), (22) and (24) respectively.

7. Applications in astrophysics

(i) Solar granulation

Radiation and convection are the two main energy transport processes in the solar interior. The convective transport becomes operative where the temperature and density gradients are such that a fluid element, when displaced from its equilibrium position, continues to move away from it. This stratification through unstable convection produces turbulence in the medium. Fluid eddies of varying sizes then carry energy as they propagate and dissipate. The cellular velocity patterns observed on the solar surface are believed to be manifestations of convective phenomenon occurring in the sub-photospheric layers. The cellular velocity fields are seen prominently on two scales : the granulation and the super granulation, though mesogranulation and giant cells are also suspected to be present. The formation of granules with an average size of 1000 km and a lifetime of a few minutes is understood either from the mixing length [12] or from the linear instability [13] description of the convection in the hydrogen ionisation zone of the sub-photospheric medium. The supergranules with an average size of 30000 km and a lifetime of 20 hours do not have an unambiguous association with a subphotospheric region. The attempts have been to seek an explanation for the energy concentration at the supergranular scale and to identify the region. Simon and Leighton [14] suggested helium ionisation to be responsible for accumulation of energy at supergranular scales. Convective modes with dominant growth rates at the two scales have been favoured by Bogart *et al* [13] and Antia *et al* [15].

The quality observations obtained at Pic-du-Midi indicate the existence of a continuum of sizes instead of one dominant scale in the granulation. The fractal dimension studies reveal that small eddies are more regular than the large ones [16]. Further, a plot of kinetic energy E(K) dK contained in the scales between K and $K+dk$ *versus* the wave number K shows two slopes of ~ (–0.70) and (–1.70) with a break at $K^{-1} \cong 2000$ km [17]. The slope of (–1.70) is very close to the Kolmogorov's law $K^{(-5/3)}$ for homogeneous isotropic turbulence, where large eddies cascade to small eddies, setting up an inertial range. Similar conclusions confirming the turbulent nature of granulation have been arrived at through the SOUP observations by Title *et al* [18]. Prompted by these observations it was suggested that perhaps the energy injection into the solar atmosphere occurs at the supergranular scale and the continuum of granules is formed by the direct cascading of energy from large scales to small scales [19]. Assigning the convective energy transport to supergranules also alleviates the problem of getting too large a measure of vertical velocities and temperature fluctuations if one restricts the energy transport only to granules [20,21]. The large scale motions reported by Ribes *et al* [22] could also be sustained by supergranules or may be a consequence of the instability of the convection zone against large scale disturbances. Mesogranulation refers to the scales lying between the granules and supergranules as reported by November *et al* [23]. Here, we explore if the excitation of random small scale motions can lead to large organized structures which are observed in the form of granules, mesogranules, supergranules and giant cells.

Energetics :

According to the picture [11] presented here, the energy in the larger structures has been inverse cascaded from the smaller structures, if so, then the energy density per unit gram $E(L)$ in the large scale L should not exceed in the small scale (l). From the energy spectrum $E \propto L^{2/3}$, it follows that

$$E(L) \le \left(\frac{E_0(l)}{\tau}\right)^{2/3} L^{2/3} \simeq E_0(l), \tag{25}$$

where τ is the time for which energy injection must occur. If we take τ to be the lifetime of the larger structure then for $\tau \sim 20$ hrs for supergranules and $E_0(l) \cong (0.5)^2$ km^2/sec^2 for energy density/gm in the granules, one gets

$$L \sim L_{SG} \sim 36000 \text{ km}. \tag{26}$$

which is the typical size of a supergranule. The size of a giant cell can be determined from eq. (15) for $\Omega = \left(\frac{2\pi}{27}\right)$ day^{-1} and assuming $V \sim 0.3$ km/sec for supergranular velocity (since they provide the stirring force for turbulence that organises itself into giant cells) one gets :

$$L_C = L_{GC} \sim 1.17 \times 10^5 \text{ km}. \tag{27}$$

Again, the energy content of giant cells should not exceed that of supergranules. Using eq. (16) for the energy spectrum in this region, one gets :

$$E_{GC} \leq \left[E(L_{SG}) \;\Big|\; L_{SG}^{2} \tau \right]^{2/3} L_{GC}^{2} \leq E(L_{SG}) \tag{28}$$

where $\left[\dfrac{\dot{E}(L_{SG})}{L_{SG}^{2}\, \tau} \right]$ is the enstrophy injection rate.

For $L_{GC} \sim 10^5$ km, τ = 30 days = lifetime of a giant cell, $L_{SG} \sim 30000$ km, and $E_{SG} \sim (0.3)^2\, \text{km}^2/\text{sec}^2$. One finds that eq. (28) can be barely satisfied. Furthermore, in the presence of coriolis force, the pressure balance condition becomes :

$$\frac{1}{\rho} \; |\nabla p| \; \cong \frac{V^2}{L_C} + F_C \; = 2V^2/L_C \tag{29}$$

In contrast to the case with no coriolis force where $\frac{1}{\rho} |\nabla p| \cong V^2/L_c$. Thus, one concludes that larger energy density is required to maintain structures at scale L_c. This may be the reason for their rare observability. The appearance of structures at L_c must be accompanied by a corresponding increase in the convective flux and therefore probably of total solar flux. Total solar luminosity changes of 1% have been observed. If we attribute all of this 1% to increase in the convective flux, eq. (29) can be satisfied and structures of size L_c can get excited. The differential rotation of the sun favours the formation of larger structures at the polar regions in comparison to the equatorial regions. This is further substantiated by the fact that the dominantly open magnetic fields in polar regions do not inhibit flow of convective flux. Thus one may look for probable correlation between polar phenomena and solar luminosity enhancements with the appearance of giant cells. A very steep spectrum eq. (16) practically forbids further organisation of turbulence into structures larger than L_C.

(ii) *Clustering of galaxies :*

The formation of the observed hierarchy of larger scale structures in the universe remains a challenging problem in conventional cosmology. The distribution of galaxies is no longer pictured as a random sprinkling. There are enormous superclusters as well as giant cellular voids, interspersed here and there with long lacy chains of galaxies. The survey by M Geller and J Huchra of thousands of galaxies in a relatively narrow strip of space shows that galaxies follow intricate network of arcs and segments with huge rounded cell [24]. This distribution of galaxies into very ordered large scale formations, far more ordered into self-organised structures than previously thought suggests the operation of well defined physical processes.

It is to be noted that gravity by itself appears to be too weak to form such large structures from the initial density perturbations in time scales comparable with the Hubble age. In other words the conventional hypothetical scenarios in which the density had subtle scale invariant fluctuations which grew in amplitude and in scale with the expansion of the universe to form such large scale structures seem untenable. However, a more detailed recent work indicates that it might be possible to form large structures through gravitational instability picture.

Many galaxies have independent motion (*i.e.* so called peculiar velocities) at odds with the direction and speed of the overall Hubble flow. Looks as if the galaxies are being drawn towards enormous concentrations of matter, high density regions with masses ~ 10^{17} m. For instance the Local Group is moving at ~ 300 km/s towards Virgo cluster in relation to Hubble flow. And as shown by Burstein *et al* [25] galaxies between the local supercluster and the Hydra-centaurus Supercluster share same direction of motion showing peculiar velocities of several hundred kilometres per second, this being the deviation from uniform Hubble flow. This reveals the presence of enormously massive structures powerful enough to draw several clusters towards it. Structures ~ 10^{18} $m_\odot$ seem indicated [26]. More recently the Great wall of galaxies [27] has been identified over scales 100 Mpc! Also recent observations have shown galaxies to already exist at red shifts close to $z = 4$. This suggests the formation of galaxies, which are the smaller sçale structures at $z \geq 5$. Then one is faced with the problem of how the smaller scale structures interacted or what physical processes led to the formation of the larger clusters and superclusters of galaxies from the galactic size smaller structures. In the conventional warm or hot dark matter scenarios (chiefly involving neutrinos with small rest masses as the dark matter), it appears that the largest scale structures formed first. Then one is faced with the problem of accounting for the presence of small scale structures even at $z = 4$. The Zeldovich type pancake fragmentation model also favours initial formation of large scale structures. The cold dark matter scenario has had some measure of success in forming the smaller structures first. But it essentially invokes a particle (*i.e.* the axion) evidence for which seems to be becoming more and more meagre (despite intense recent searches).

We will use ideas developed here to construct a model of cosmic clustering in its entirety from clusters of galaxies to giant clusters of galaxies.

In our model, elementary formation (or vortices) are identified with ordinary galaxies including dwarf galaxies. These would then form clusters by ordinary gravitational clustering as well as by turbulent cascading. We invoke inverse cascading only for the formation of superclusters and giant clusters like the great wall, beginning with clusters of galaxies. Velocities and scale sizes of the large scale structures formed by inverse cascade are consistent with these two types of structures *i.e.* superclusters and giant clusters. For instance, turbulent velocities of 300 km/sec for the clusters give rise to structures on scales ~ Mpc on a time scale of 3 billion years and turbulent velocites of ~10^4 km/sec for superclusters give rise to structures on scales ~ 100 Mpc. There is a gap in the energy spectrum, situated between the clusters of galaxies and superclustes. This is consistent with the observational absence of visible objects between galaxy elements and superclusters [28]. The energy spectrum also shows a discontinuity at a scale where superclusters begin to develop into giant clusters with much steeper energy spectrum which perhaps may explain the rarity of the largest scale structures.

Energetics :

According to the picture [11] presented here, the energy of the larger structures has been inverse cascaded from the smaller structures, if so then the energy density per unit gram $E(L)$ in the large scale L should not exceed that in the small scale E_0 (1). From the energy spectrum $E \propto L^{2/3}$ it follows that

$$E(L_{SC}) \le \left(\frac{E_0(L_C)}{\tau_{SC}}\right)^{2/3} L_{SC}^{2/3} = E_0(L_C), \tag{30}$$

where τ_{SC} is the time for which energy injection must occur, which should be at least the lifetime of the large structure. If we take $L_{SC} \sim 3$ Mpc for a supercluster and $E_0(L_C) \sim (300$ km/sec$)^2$ for turbulent energy of a cluster, we get $\tau_{SC} \sim 3 \times 10^9$ years.

The giant clusters are formed in a turbulent medium being stirred by the random motion of superclusters. The largest horizontal scale is limited by the coriolis force. From eq. (15) for L_{GC} = size of the giant cluster ~ 100 Mpc and a random velocity of superclusters of 10,000 km/sec one finds the angular velocity $\Omega \sim 3 \times 10^{-18}$ radians/sec. The energy content of giant clusters should not exceed that of superclusters. Using eq. (16) for the energy spectrum in this region one gets:

$$E_{GC} \le \left[\frac{E(L_{SC})}{L_{SC}^2\, \tau_{GC}}\right]^{2/3} L_{GC}^2 \le E(L_{SC}), \tag{31}$$

where $[E(L_{SC})/L_{GC}^2\ \zeta_{GC}$ is the enstrophe injection rate. For $L_{GC} \sim 100$ Mpc, $E(L_{SC}) \sim (10^4$ km/sec$)^2$, and $L_{SC} \sim 3$ Mpc, one gets $\zeta_{GC} \cong 10^{11}$ years. It is clear that structures of the size of few hundred Mpc cannot be formed if the random velocity of a supercluster is less than 10,000 km/sec. Furthermore, in the presence of Coriolis force, the pressure balance condition becomes:

$$\frac{1}{\rho}\,|\nabla p| \cong \frac{V^2}{L_{GC}} + F_C \cong \frac{2V^2}{L_{GC}}, \tag{32}$$

in contrast to the case with no coriolis force where $\frac{1}{\rho}\,|\nabla p| \cong V^2/L_{GC}$ Thus, one concludes that larger energy density is required to form or maintain structures at scale L_{GC}. This may be the reason for their rare observability. A very steep spectrum eq. (16) practically forbids further organization of turbulence into structures larger than L_{GC}.

The processes of inverse cascade have been shown to occur in an incompressible turbulent medium. The incompressibility fixes the epoch of formation of large structures before the recombination phase. Since the recombination takes about 20% of the Hubble time, the structures formed then should be detectable, at present. The eddy turnover time for the largest structure of say 100 Mpc with an associated fluid velocity of 10,000 km/sec is 10^{10} yrs which is smaller than Hubble time. This is also true at smaller scales. Within the framework of present theory, the smaller structure are more or less isotropic whereas the larger ones are anisotropic and become nearly two dimensional at the largest scale. One may also note that the turbulent velocity scales as $V \propto L$ at the largest scales instead of $L^{1/3}$ as for Kolmogorovic spectrum (Figure 1). Peebles [29,30] has obtained a non-Kolmogorovic spectrum including effects of expansion. Silk [31,32] has shown that the effects of expansion cause little deviation in turbulence from the incompressible case.

8. Conclusion

The two apparently different phenomena of solar granulation and closuring of galaxies have been modelled invoking the properties of inverse cascade in a hydrodynamically turbulent medium. The proof of the inverse cascade, in addition to other aspects we believe, lies in the energy spectrum, parts of which are available through observations of velocity on the solar surface, whereas such observations for galactic systems are not yet available. In view of the recent "exit of cold dark matter hypothesis" (and the inability of the hot dark matter to form bottom-up structures in addition to its undetectability (even in the laboratory) the inverse cascade mechanism offers a new solution to the large scale structure of the universe.

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