Indian J. Phys. **70B** (6), 531–536 (1996)

NOTE

IJP B
— an international journal

Compressional Alfvén surface waves with inclined magnetic fields

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Received 23 July 1996, accepted 14 October 1996

Abstract : The properties of the Alfvén compressional waves are discussed for a cold plasma when the magnetic field is inclined at an angle to the interface where there is a sharp change in the densities of the plasma. The condition for the existence of surface waves in terms of the densities ρ_1 and ρ_2 is derived. This, is in contrast to the incompressible Alfvén surface waves which exist at any density discontinuity.

Keywords : Alfvén surface waves, cold plasma, inclined magnetic fields

PACS No. : 52.35.Bj

Ever since MHD waves were predicted a few decades ago, Alfvén waves have been candidates for coronal heating. By Alfvén waves, we mean the noncompressive intermediate MHD mode. They propagate along the magnetic field B with a speed equal to $v_A = B/(4\pi\rho)^{1/2}$, where ρ is the density of the medium. An excellent review on Alfvén waves and their importance to coronal heating is given in Hollweg [1].

Several authors have studied Alfvén waves, in coronal loops, flux tubes and other regions of the solar atmosphere. Uberoi and Somasundaram [2,3] have studied Alfvén surface waves along cylindrical plasma columns. Alfvén surface waves along coronal streamers have been studied by Satya Narayanan and Somasundaram [4]. Roberts [5] discusses MHD waves in the solar atmosphere. Surface waves along moving cylindrical plasma colums was studied in the context of coronal loops by Somasundaram and Satya Narayanan [6]. Uberoi [7] discusses the resonant absorption of compressional Alfvén surface waves. Resonant behaviour of MHD waves on magnetic flux tubes has been considered by Goossens *et al* [8]. Recently, Erdelyi and Goossens [9] discussed resonant absorption of Alfvén surface waves in coronal loops with visco-resistive MHD. In all these studies, the magnetic field was along or perpendicular to the interface of discontinuity. However, Satya Narayanan [10] studied a two layered model wherein the magnetic field was inclined at an angle to the upper fluid of the interface while the lower fluid was field free. In this study, we extend the work of Satya Narayanan [10] and assume the magnetic field to be nonzero in both the layers. Moreover, the inclination of the magnetic field on either side of the interface in general, need not be the same.

To begin with, we discuss the dispersive characteristics of Alfvén surface waves when the magnetic field is not inclined to the interface. Consider a compressible fluid with mass density $\rho(x)$ in the presence of a uniform magnetic field B_0 in the z direction. The wave equation got from the linearized magnetohydrodynamic equations can be written as

$$\frac{d}{dx} R(x) \frac{d\xi_x}{dx} - \varepsilon\xi_x = 0, \tag{1}$$

where $$R(x) = \varepsilon\alpha B_0^2/\alpha k_\perp (B_0^2 - \varepsilon),$$

$$\alpha(x) = 1 + \frac{\omega^2 v_s^2}{v_A^2(\omega^2 - k_\parallel^2 v_s^2)}, \tag{2}$$

and $$\varepsilon(x) = \left[\omega^2\mu_0\rho(x) - k_\parallel^2 B_0^2\right]. \tag{3}$$

Here, ξ_x is the fluid displacement in the x direction, v_s is the sound speed and v_A is the Alfvén speed. $k = (k_\parallel, k_\perp)$ is the wavevector in the (z,y) plane. The simplest model of the wave equation is for the case of cold plasma. In this case v_s can be neglected compared to v_A which reduces eq. (1) to

$$\frac{d}{dx} R \frac{d\xi_x}{dx} - \varepsilon\xi_x = 0. \tag{4}$$

In terms of the z component of the magnetic field b_z, eq. (4) can be written as

$$\varepsilon\left[\frac{d^2 b_z}{dx^2} + \left(\varepsilon/B_0^2 - k_\perp^2\right)\right] = \frac{d\varepsilon}{dx}\frac{db_z}{dx}. \tag{5}$$

We consider surface waves when there is a sharp discontinuity in the density. Assume that $\rho = \rho_1$ in the region $x < 0$ and $\rho = \rho_2$ for $x > 0$. The equation for b_z reduces to

$$\frac{d^2 b_z}{dx^2} + \left(\varepsilon/B_0^2 - k_\perp^2\right) = 0, \tag{6}$$

since $d\varepsilon/dx = 0$. Eq. (6) has solutions given by

$$b_z(x) = B_1 e^{K_{1x}} x < 0, \tag{7}$$

and $$b_z(x) = B_2 e^{-K_{2x}} x > 0, \tag{8}$$

where $$K_1 = \left(-\varepsilon_1/B_0^2 + k_\perp^2\right)^{1/2}, \tag{9}$$

and $$K_2 = \left(-\varepsilon_2/B_0^2 + k_\perp^2\right)^{1/2}. \tag{10}$$

Introducing the boundary conditions at the interface given by

$$b_z\big|_{x=0^+} = b_z\big|_{x=0}$$

and
$$(1/\varepsilon_1)\frac{db_z}{dx}\bigg|_{x=0^+} = (1/\varepsilon_2)\frac{db_z}{dx}\bigg|_{x=0^-}, \tag{11}$$

the dispersion relation for the surface waves can be written as

$$K_1\varepsilon_2 + K_2\varepsilon_1 = 0. \tag{12}$$

The above equation will have real roots only when ε_1 and ε_2 have opposite sign. Introducing $\rho_c = k_\parallel^2 B_0^2/\omega^2\mu_0$, as the critical density at which the Alfvén wave speed equals the phase speed, we can see that one of the density must be above the critical density while the other must be below it. Moreover, K_1 and K_2 should both be positive for the roots to represent surface waves. Taking $k_\perp/k_\parallel = \tan\theta$, the required relationship for the existence of the surface wave in terms of the densities can be written as

$$\left(1 - \frac{\rho_1}{\rho_c}\cos^2\theta\right)\left(1 - \frac{\rho_1}{\rho_c}\right)^{-1} = \left(1 - \frac{\rho_2}{\rho_c}\cos^2\theta\right)\left(1 - \frac{\rho_2}{\rho_c}\right)^{-1}. \tag{13}$$

On writing $x = [1 - (\rho_1/\rho_c)]$ and $y = [(\rho_2/\rho_c) - 1]$, eq. (13) can be written as

$$x = \frac{y\sin\theta}{-y\cos\theta + \sin^2\theta}, \tag{14}$$

which gives

$$\frac{\rho_2}{\rho_c} = 1 + \left[\left(1 - \frac{\rho_1}{\rho_c}\right)\sin^2\theta\right]\left(1 - \frac{\rho_1}{\rho_c}\cos^2\theta\right)^{-1}. \tag{15}$$

It is interesting to note that for any given ratio between $k_\perp$ and $k_\parallel$ in the plane of the magnetic field, the furface waves will exist only for values of ρ_1 and ρ_2 which satisfy (15). However, incompressible Alfvén waves can exist at any density discontinuity.

We generalise the above result to inclined magnetic fields. Let the magnetic field be of the form

$$B_{01,2} = (0, B_{01,2}\cos\gamma_{1,2}, B_{01,2}\sin\gamma_{1,2}) \tag{16}$$

and the wave vector to be

$$K = (0, k\sin\theta, k\cos\theta). \tag{17}$$

Here, $B_{01,2}$ denotes the strength of the magnetic field on either side of the interface and $\gamma_{1,2}$ are the inclination angles of the magnetic field in the regions, respectively. The dispersion relation by solving the linearized magnetohydrodynamic equations and applying the boundary conditions, can be written as [11]

$$\tau_1\varepsilon_2 + \tau_2\varepsilon_1 = 0, \tag{18}$$

where
$$\varepsilon_1 = \rho_1\left(-\omega^2 + k^2 v_{A1}^2 \sin^2(\theta+\gamma_1\right), \tag{19}$$

$$\varepsilon_2 = \rho_2\left(-\omega^2 + k^2 v_{A2}^2 \sin^2(\theta+\gamma_2\right), \tag{20}$$

and
$$\tau_1 = \left(k^2 - \frac{\omega^2}{v_{A1}^2}\right)^{1/2},$$

$$\tau_2 = \left(k^2 - \frac{\omega^2}{v_{A2}^2}\right)^{1/2}. \tag{21}$$

τ_1 and τ_2 have the above form because we are considering cold plasma. Substituting (19), (20) and (21) into eq. (18) and simplifying yields

$$\left(\frac{\rho_1}{\rho_c} - 1\right)^{1/2} \left\{-\frac{\rho_2}{\rho_c} + \sin^2\left(\theta+\gamma_2\right)\right\} = -\left(\frac{\rho_2}{\rho_c} - 1\right)^{1/2} \times\left\{\frac{-\rho_1}{\rho_c} + \sin^2\left(\theta+\gamma_1\right)\right\} \tag{22}$$

Putting $\frac{\rho_1}{\rho_c} - 1 = x$ and $\frac{\rho_2}{\rho_c} - 1 = y$, eq. (22) can be simplified to yield

$$y = \lambda + \frac{\left(x_1 + (x_1^2 + x_2 x_3)^{1/2}\right)}{2x_3}, \tag{23}$$

where
$$\lambda_1 = \sin^2(\theta+\gamma_1), \qquad \lambda_2 = \sin^2(\theta+\gamma_2), \qquad \lambda = \lambda_2/\lambda_1;$$
$$\lambda_3 = \cos^2(\theta+\gamma_1), \qquad \lambda_4 = \cos^2(\theta+\gamma_2), \qquad x_1 = -(1-x)^2\lambda_1,$$
$$x_2 = 4x^4(1-x)^2\lambda_4, \qquad x_3 = \lambda_3 + (1-x)^2\lambda_1.$$

Eq. (22) puts more restriction on the existence of Alfvén surface waves as a function of the densities ρ_1 and ρ_2 and the angles $\gamma_{1,2}$.

It is clear from the expression (22) that ρ_2/ρ_c depends on θ, γ_1 and γ_2. We are forced to study for restricted values of these parameters. Figure 1 presents the behaviour of ρ_2/ρ_c as a function of ρ_1/ρ_c for various values of $\theta = 0°$, $10°$, $40°$, $60°$ and $80°$ and for a given value of $\gamma_1 = 70°$, $\gamma_2 = 45°$. The variation of ρ_2/ρ_c is significantly less compared to the case when the inclination angles γ_1 and γ_2 have the value $\pi/2$. It is interesting to note that ρ_2/ρ_c can take values less than 1 which was not the case when $\gamma_1 = \gamma_2 = \pi/2$. The condition for the existence of surface waves in this case is that the density ρ_2 can also be less than the critical density ρ_c wherein the phase speed of the surface wave equals the Alfvén speed in the medium 1. Figures 2 and 3 present the behaviour of ρ_2/ρ_c as a function of ρ_1/ρ_c for $\gamma_1 = 70°$ and $\gamma_2 = 60°$ and $80°$, respectively. There is a drop in the value of ρ_2/ρ_c for $\gamma_1 = 70°$ and $\gamma_2 = 60°$ compared to $\gamma_1 = 70°$ and $\gamma_2 = 45°$. Also the variation in ρ_2/ρ_c for $\theta = 80°$ is quite significant compared to Figure 1 while the variation is more or less the same for other values of the angle θ. The value of ρ_2/ρ_c increases monotonically as the wave propagation angle is increased.

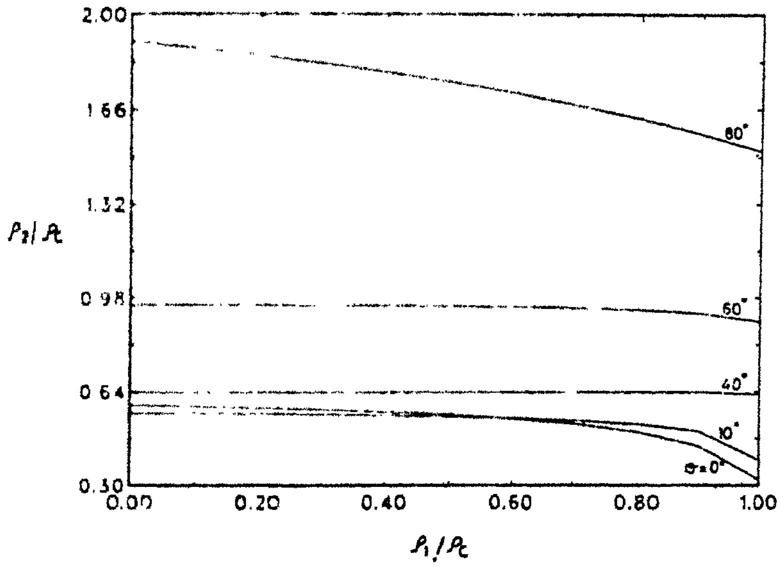

Figure 1. The behaviour of the plasma densities on either side of the density discontinuity plotted as a function of θ for $\gamma_1 = 70°$, $\gamma_2 = 45°$.

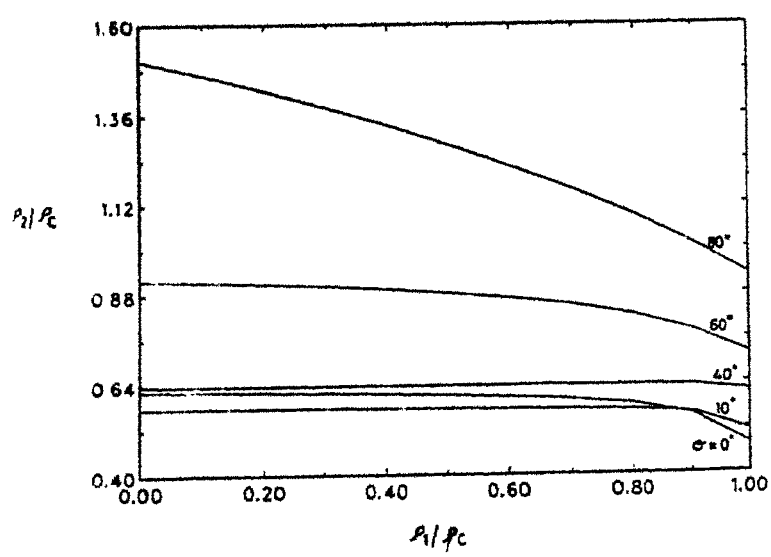

Figure 2. The behaviour of the plasma densities on either side of the density discontinuity plotted as a function of θ for $\gamma_1 = 70°$, $\gamma_2 = 60°$.

Figure 3 depicts ρ_2/ρ_c as a function of ρ_1/ρ_c for $\gamma_1 = 70°$ and $\gamma_2 = 80°$. We see a dramatic change in behaviour of ρ_2/ρ_c in this case. The variation of ρ_2/ρ_c is quite significant for $\theta = 60°$ and $80°$ while it is not much for other values of θ. Also in this case, the monotonicity is lost. There is a crossing of values of ρ_2/ρ_c when $\rho_1/\rho_c > 0.6$. The value of ρ_2/ρ_c is less for $\theta = 0°$. It is interesting to note that in this case, ρ_2 is always less than ρ_c except when $\theta = 80°$ and $\rho_1/\rho_c \approx 0$.

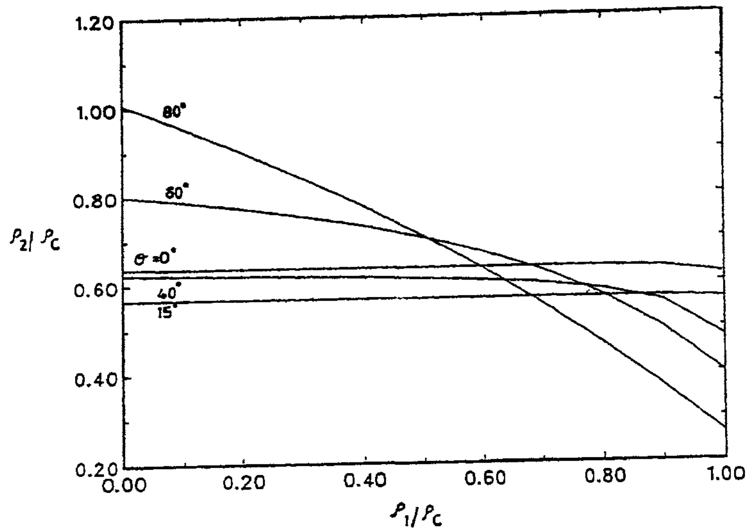

Figure 3. The behaviour of the plasma densities on either side of the density discontinuity plotted as a function of θ for $\gamma_1 = 70°$, $\gamma_2 = 80°$.

This study presents the dispersive characteristics of Alfvén surface waves for a cold plasma when the magnetic field is inclined at an angle to the interface of density discontinuity. The density surfaces depend greatly on the inclination angle of the magnetic field. The phase speed of these waves are also greatly dependent on the angle γ. However, the effect of magnetic field ratio is felt for angles θ lying between 10° and 80°. More realistic geometries will have to be considered. This will be carried out in future studies.

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