A SIMPLE MODEL FOR CORONAL LOOPS AND THEIR STABILITY

A. Satya Narayanan
Indian Institute of Astrophysics, Bangalore - 560 034, India. e-mail: satya@iiap.ernet.in

ABSTRACT

The solar corona is highly inhomogeneous, and structures with a variety of shapes and sizes are seen to pervade it. The basic structural component of the solar corona is the coronal loop. Montgomery and his co-workers have developed a frame work to describe the steady state of a turbulent magneto fluid, without the usual recourse to linearization. Thus, the magnetic and velocity fields emerge in their fully nonlinear form as a consequence of the selective decays of the invariants of the system. Using the statistical theory of magneto hydrodynamic turbulence, the pressure, magnetic field and the flow fields of a solar loop have been determined. The spatial and temporal profiles of the loop is derived. In order to study the stability of these loops, we have resorted to the properties of a dynamical system. Whenever this system has a stagnation point which is hyperbolic in nature, the system admits instability. The advantage in this analysis is that one need not resort to the usual eigenvalue problem approach which is quite tedious and complicated.

Key words: Coronal Loops, Stability.

1. INTRODUCTION

The solar corona is highly structured with coronal loop as one of the basic structures. Structures in corona are of various sizes and shapes. There are open radial rays along which the plasma flows into the solar wind which are termed coronal holes and are found near the poles. The loops link magnetically active regions of different polarity. The typical temperature will be of the order of $(2-3) \times 10^6$ K and density $10^{-15} gcm^{-3}$.

Coronal loops or arch-like structures of the active regions of the Sun have been observed in the emission at the UV, FUV, and X-ray wavelengths (Foukal (1978), Levine and Withbroe (1977), Vaiana and Rosner (1978), Bray et.al. (1991)). The number of loops in a single system may vary from one to about ten or so. Plasma flows have also been observed in the loops.

Although there is continuous pumping of magnetic and velocity field fluctuations into the coronal plasma, the loops exhibit a fairly stable and well configured geometry. The steady state pressure structure is the result of the various manifestations of the balance of the inertial and magnetic forces. Krishan (1983) and Krishan (1985) discussed a steady state model of active region coronal loops using the statistical theory of incompressible magneto hydrodynamic turbulence described by Montgomery et al. (1978). The main features of the theory consists of using the MHD equations for an incompressible fluid. The magnetic and velocity fields are expressed in terms of Chandrasekar-Kendall (hereafter referred to as C-K) functions which is a complete system. The pressure profile is derived as a function of the velocity and magnetic fields in the form of a Poisson equation. The pressure structure of the coronal loops in three dimensions was derived by Sreedharan et al. (1992). Sasidharan et al. (1995) studied the temporal behavior of pressure in coronal loops. Modeling of a solar coronal loop in terms of MHD equations has been given in a good review by Krishan (1996). In this study, we present the pressure profile of the coronal loop in three dimensions as well as the temporal structure. In the above studies, the stability of the loop has not been considered. In the present study, we deal with the stability of the loop. We resort to the results of dynamical systems and apply it to the loops.

2. SPATIAL PRESSURE PROFILE

Let the coronal loop be represented by a cylindrical column of length 'L' and radius 'R'. The mechanical pressure P is expressed as a function of velocity $\vec{V}$ and magnetic field $\vec{B}$ using the MHD equations given by

$$\rho[\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}] = -\nabla P + \vec{J} \times \vec{B}, \quad (1)$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J}, \quad (2)$$

$$\nabla \cdot \vec{B} = 0, \quad (3)$$

$$\nabla \cdot \vec{B} = 0, \quad (4)$$

Proc. 9th European Meeting on Solar Physics, 'Magnetic Fields and Solar Processes', Florence, Italy, 12-18 September 1999 (ESA SP-448, December 1999)

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}). \quad (5)$$

It is easy to see that there are three types of equilibria for the above system.

(i) for $\vec{V} = 0, \nabla P = \vec{J} \times \vec{B}$;

(ii) for $\frac{\partial \vec{V}}{\partial t} = 0, \rho(\vec{V} \cdot \nabla) = -\nabla P$ either for $\vec{B} = 0$ or for $\vec{J} \times \vec{B} = 0$

(iii) for $\vec{V} = 0$ and $\vec{J} \times \vec{B} \approx 0, \nabla P \approx 0$.

Cases (ii) and (iii) lead to the force free condition $\vec{J} \times \vec{B} = 0$. A static equilibrium with constant pressure and force-free magnetic field, is described by

$\nabla \times \vec{B} = \alpha \vec{B}$

Montgomery et al (1978) have shown using the selective decay hypothesis that the total energy decays to a minimum value while the magnetic helicity H_M and the cross-helicity H_c remain more or less constant. Here

$H_M = \int \mathbf{A}_B \cdot \vec{B} d^3r$

$H_c = \int \vec{V} \cdot \vec{B} d^3r$

where $\mathbf{A}_B$ is the vector potential. One can write a variational principle through which an organized state of magnetic and velocity field emerges. Using the variational principle one can write

$$\delta E - \lambda_M \delta H_M - \lambda_c \delta H_c = 0 \quad (6)$$

from which one finds that

$$\int [\delta(\vec{V} - \lambda_c \vec{B}) + \delta \mathbf{A}_B \cdot (\nabla \times \vec{B} - 2\lambda_m \vec{B} - \lambda_c \nabla \times \vec{V})] d^3r = 0. \quad (7)$$

Here λ_c and λ_m are the Lagrange multipliers. The above equation can be simplified to yield

$$\nabla \times \vec{B} = \alpha \vec{B} \quad (8)$$

$$\nabla \times \vec{V} = \alpha \vec{V} \quad (9)$$

$= \frac{2\lambda_m}{1-\lambda_c^2}$

$= \lambda_c \vec{B}$

uation (8) describes a force-free field while (9) describes a Beltrami flow.

coronal structure that evolves continuously in response to its highly variable environment may not generally be in the minimum-energy state. It is quite likely that a coronal loop exists in a state consisting of a superposition of the force-free magnetic fields and the Beltrami flows. A single C-K function represents a force-free field. The magneto fluid in the coronal loop is believed to be in an approximate state of the force-free fields with small departures from the current-free fields of the photospheric fluid. It is reasonable to assume that the coronal loop fields and flows to have departures from the strictly force-free conditions. Superposition of CK functions may give a better picture of the departure from the force-free state. The velocity field $\vec{V}$ and the magnetic field $\vec{B}$ are expanded in terms of the CK functions as follows:

$$\vec{V} = \sum_{n,m} \lambda_{nm} \eta_{nm}(t) \mathbf{A}_{nm}(\mathbf{r}) \quad (10)$$

$$\vec{B} = \sum_{n,m} \lambda_{nm} \xi_{nm}(t) \mathbf{A}_{nm}(\mathbf{r}) \quad (11)$$

$$\mathbf{A}_{nm} = C_{nm} \mathbf{a}_{nm}(\mathbf{r}) \quad (12)$$

C_{nm} is a normalizing constant and

$\int \mathbf{A}^*_{nm} \cdot \mathbf{A}_{n'm'} d^3r = \delta_{nn'} \delta_{mm'}$

Here,

$$a_{nm}(\mathbf{r}) = \hat{e}_r a_1 \psi_{nm} + \hat{e}_\phi a_2 \psi_{nm} + \hat{e}_Z a_3 \psi_{nm} \quad (13)$$

where

$a_1 = \{\frac{im}{r} + \frac{ikn}{\lambda_{nm}} \frac{\partial}{\partial r}\}$

$a_2 = \{-\frac{\partial}{\partial r} - \frac{mkn}{r\lambda_{nm}}\}$

$a_3 = \frac{\lambda_{nm}^2 - k_n^2}{\lambda_{nm}}$

$\psi_{nm} = J_m(\gamma_{nmr}) exp(im\phi + iknZ)$,

$\lambda_{nm} = (\gamma_{nm}^2 + k_n^2)^{1/2}, k_n = \frac{2\pi n}{L}$,

$n = 0, \pm 1, \pm 2, ..., m = 0, \pm 1, \pm 2, ...$

The functions $\mathbf{a}_{nm}(\mathbf{r})$ are the solutions of the equation

$$\nabla \times \mathbf{a}_{nm}(\mathbf{r}) = \lambda_{nm} \mathbf{a}_{nm}(\mathbf{r}) \quad (14)$$

For a rigid, perfectly conducting, impenetrable wall at radius $r = R$, i.e. for an isolated loop with no exchange through its surface, the boundary conditions are

$$V_r(r = R) = 0,$$

$$B_r(r = R) = 0, \quad (15)$$

The two ends of a coronal loop are anchored in the sub photospheric region, where they undergo small twisting motions. The sub photospheric region contains a high β plasma, where β is the ratio of the gas kinetic pressure to the magnetic pressure. As a result, the magnetic field lines move on a time scale much longer than the coronal time scales. This line-tying reduces the region of unstable excitations, especially those of long wavelength. It is reasonable to assume that nearly identical conditions prevail at the end of the two foot points, at least for symmetric loops. We shall assume periodic boundary conditions in the Z directions with a period equal to the length L of the loop. This gives $k_n = 2\pi n/L$. The condition (15) gives for $m^2 + n^2 > 0$

$$Rk_n\gamma_{nm}J_m^{'}(\gamma_{nm}R) + m\lambda_{nm}J_m(\gamma_{nm}R) = 0. \quad (16)$$

where primes on the Bessel functions J_m denote their derivatives. For $m = n = 0$, we observe that $a_r = 0$. Thus, for the lowest mode $m = n = 0$, we need to resort to some additional constraint. It can be checked that for the (0,0) mode, the ratio of the toroidal flux ϕ_t to the poloidal flux ϕ_p is given by

$$\frac{\phi_t}{\phi_p} = \frac{-RJ_0^{'}(\gamma_{00}R)}{LJ_0(\gamma_{00}R)} \quad (17)$$

for $\lambda_{00} > 0$, where

$\phi_t = \frac{1}{2}\int_0^L dz \int_0^R rdr \int_0^{2\pi} B_z d\theta = const$ at $r = R$,

$\phi_p = \int_0^L dz \int_0^{2\pi} d\theta A_{BZ} = const$ at $r = R$.

Thus λ_{00} is determined for a given value of ϕ_t/ϕ_p.

The pressure profile of the cylindrical column of plasma is given by

$$\frac{\nabla P}{\rho} = \frac{(\nabla \times \vec{B}) \times \vec{B}}{\rho} - (\vec{V}\cdot\nabla)\vec{V} - \frac{\partial \vec{V}}{\partial t}, \quad (18)$$

or

$$\nabla\{\frac{P}{\rho} + \frac{1}{2}V^2\} = \frac{(\nabla \times \vec{B}) \times \vec{B}}{\rho} - (\vec{V} \times \nabla) \times \vec{V} - \frac{\partial \vec{V}}{\partial t} \quad (19)$$

and

$$\nabla \times (\vec{V} \times \vec{B}) - \frac{\partial \vec{B}}{\partial t} = 0. \quad (20)$$

The complete dynamics of the loops can be described by a set of infinite coupled nonlinear ordinary differential equations, which are of first order in time for the expansion coefficients of velocity and magnetic fields and it is a formidable task to find solutions to these equations. Thus we choose to represent the fields by superposition of the three lowest order CK functions. The reason being that these lowest order functions represent the largest spatial scales and therefore may be the most suitable. In the triple-mode system,

$$\vec{V} = \lambda_a\eta_a(t)\mathbf{A}_a + \lambda_b\eta_b(t)\mathbf{A}_b + \lambda_c\eta_c\mathbf{A}_c \quad (21)$$

$$\vec{B} = \lambda_a\xi_a(t)\mathbf{A}_a + \lambda_b\xi_b(t)\mathbf{A}_b + \lambda_c\xi_c(t)\mathbf{A}_c \quad (22)$$

The dynamics can be described by taking the inner products of the curl of the equations (18) and (20) and integrating over the volume. The resulting six coupled coupled nonlinear ordinary differential equations for the triple mode system are given by

$$\frac{\partial\eta_a}{\partial t} = \frac{\lambda_b\lambda_c}{\lambda_a}(\lambda_c - \lambda_b)I[\eta_b\eta_c - \xi_b\xi_c/\rho] \quad (23)$$

$$\frac{\partial\eta_b}{\partial t} = \frac{\lambda_c\lambda_a}{\lambda_b}(\lambda_a - \lambda_c)I^*[\eta_c^*\eta_a - \xi_c^*\xi_a/\rho] \quad (24)$$

$$\frac{\partial\eta_c}{\partial t} = \frac{\lambda_a\lambda_b}{\lambda_c}(\lambda_b - \lambda_a)I^*[\eta_a\eta_b^* - \xi_b^*\xi_a/\rho] \quad (25)$$

$$\frac{\partial\xi_a}{\partial t} = \lambda_b\lambda_c I[\eta_b\xi_c - \eta_c\xi_b] \quad (26)$$

$$\frac{\partial\xi_b}{\partial t} = \lambda_c\lambda_a I^*[\eta_c^*\xi_a - \eta_a\xi_c^*] \quad (27)$$

$$\frac{\partial\xi_c}{\partial t} = \lambda_a\lambda_b I^*[\eta_a\xi_b^* - \eta_b^*\xi_a] \quad (28)$$

where $I = \int \vec{A}_a^*\cdot(\vec{A}_b \times \vec{A}_c)d^3r$ and the (n,m) values of the modes (a,b,c) satisfy the conditions $n_a = n_b + n_c$ and $m_a = m_b + m_c$.

It can be shown that

$$(\nabla \times \vec{B}) \times \vec{B} = \sum_{i=a,b,c:j=b,c,a} \lambda_i\lambda_j\xi_i\xi_j(\lambda_i - \lambda_j)\vec{A}_i \times \vec{A}_j \quad (29)$$

$$(\nabla \times \vec{V}) \times \vec{V} = \sum_{i=a,b,c:j=b,c,a} \lambda_i\lambda_j\eta_i\eta_j(\lambda_i - \lambda_j)\vec{A}_i \times \vec{A}_j \quad (30)$$

so that

$$\nabla[\frac{P}{\rho} + (\frac{1}{2})V^2] = \sum_{i=a,b,c;j=b,c,a} \lambda_i\lambda_j(\lambda_i - \lambda_j)$$

$$\{\frac{\xi_i\xi_j}{\rho} - \eta_i\eta_j\}\vec{A}_i \times \vec{A}_j - \frac{\partial\vec{V}}{\partial t} \quad (31)$$

For the steady state, $\partial/\partial t[\eta, \xi] = 0$, and hence one can show from the time dependent equations

$\eta_a = \xi_a/\rho^{1/2}$

$\eta_b = \xi_b/\rho^{1/2}$

and

$\eta_c = \xi_c/\rho^{1/2}$

Thus equation (31) reduces to

$\nabla[P/\rho + (1/2)V^2] = 0$.

i.e., $P/\rho + (1/2)V^2 =$ constant.

If the value of P at the origin ($r = 0, z = 0$) is P_0, then

$$P/\rho = P_0/\rho + (1/2)V_0^2 - (1/2)V^2, \quad (32)$$

3. DISCUSSION

The spatial variation of the pressure is presented for a cylindrical column of plasma for which the ratio of the radius R to the length L is taken to be $R/L = 1/10$ and the ratio of the toroidal to poloidal flux $\phi_t/\phi_p = 0.1$.

We have chosen two triads a, b, c such that they represent the largest possible spatial scales, as well as satisfy the condition $a = b + c$. These are

$a = (1,1), b = (1,0), c = (0,1)$

The corresponding values of $\gamma's$ and $\lambda's$ are found to be

$\gamma_a R = 3.23, \gamma_b R = 3.85, \gamma_c R = 3.85$

$\lambda_a R = 3.29, \lambda_b R = 3.90, \lambda_c R = 3.85$

Figure 1 presents the value of $(p - p_0) \times 10^2$ as a function of $\gamma_a r$ for different values of z with θ the azimuthal component, averaged over a full cycle. It is clear that the pressure at any height increases along the radius towards the surface. The radial variation of pressure is maximum at the foot points of the loop and it is minimum at the apex.

The axial variation of the pressure $(p - p_0 \times 10^2$ for various values of z are presented in Figure 2. The axial variation is maximum at the axis and minimum at the surface. The maximum value is attained near the apex for all values of $\gamma_a r$.

Figure 3 presents the azimuthal variation of the pressure for different values of $\gamma_a r > 2.0$. The pressure exhibits an oscillatory behaviour predominantly near the surface.

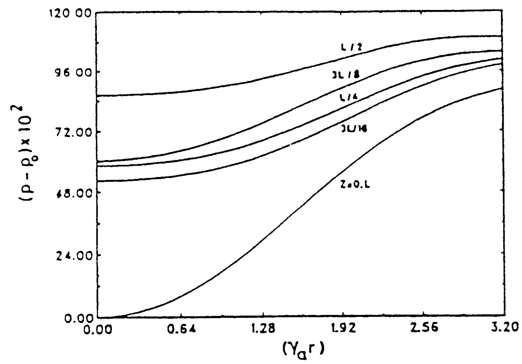

Fig.1 Radial Variation of the pressure P for different z

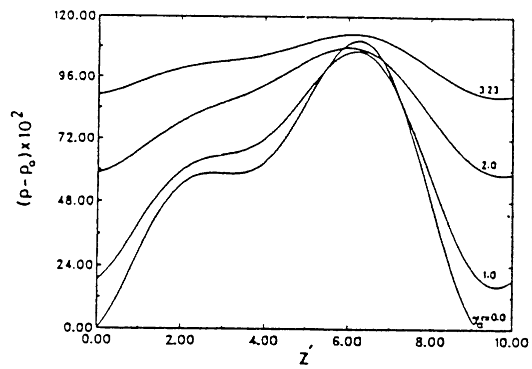

Fig.2 Axial Variation of the pressure P for different r

The contour plot of the pressure as functions of $\gamma_a r$ and z when the pressure is averaged over θ is presented in Figure 4.

There are two physical situations under which Equations (23) to (28) can be solved analytically. (i) the linear case, (ii) the pump approximation.

(i) Linear Case

We study the time evolution of small deviations of the velocity and magnetic fields from their equilibrium values. We assume $\eta = \eta_0 + \eta_1, \xi = \xi_0 + \xi_1$ and that $\eta_0 = \xi_0$, $\eta_1 << \eta_0, \xi_1 << xi_0$ for all modes. Assuming that both $\eta(t)$ and $\xi(t)$ have time dependence through e^{st}, the dispersion relation is obtained as

$$s = \pm i \mid I \mid [\lambda_b^2(\lambda_b - \lambda_c\lambda_a)^2 \mid \eta_{b0} \mid^2 + \lambda_c^2(\lambda_c - \lambda_a\lambda_b)^2$$

$$\mid \eta_{c0} \mid^2 - \lambda_a^2(\lambda_a - \lambda_b - \lambda_c)^2 \mid \eta_{a0} \mid^2]^{1/2} \quad (33)$$

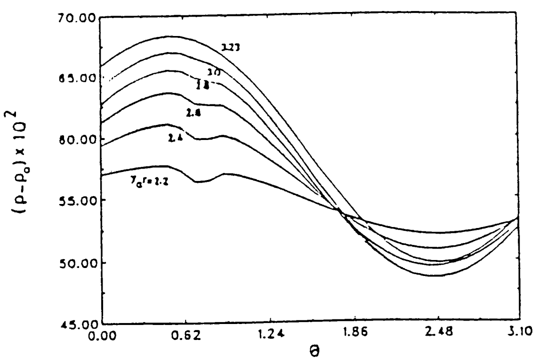

Fig.3 Azimuthal Variation of the pressure for $\gamma_a r$ = 2.2 to 3.23

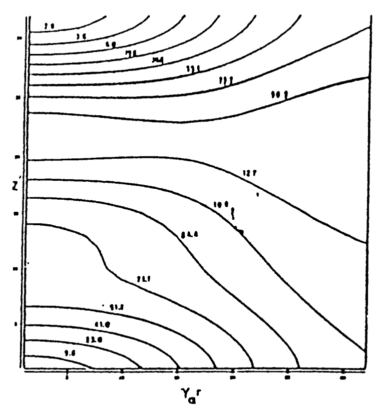

Fig.4 Contour plot of the pressure as a function of $\gamma_a r$ and z when θ is averaged.

(ii) The Pump Approximation

In the pump approximation, one of the three modes is taken to be the strongest. For example, here, since the conservation condition gives $a = b + c$, we can take 'a' to be the dominant mode and call it the pump which share its energy with the other two modes. The time evolution of the two modes does not produce any significant change in the pump mode, and hence we can neglect all time variations in (η_a, ξ_a). The system of equations (23) to (28) can be recast with some algebraic simplifications as

$$\frac{d^2\eta_b}{dt^2} = P_1\eta_b + P_2 \quad (34)$$

$$\frac{d^2\eta_c}{dt^2} = P_1'\eta_c + P_2' \quad (35)$$

where

$\xi_b = \frac{\lambda_b}{\lambda_a - \lambda_c}(\eta_b - I_b)$

$I_b = \eta_{b0} - \frac{(\lambda_a - \lambda_c)}{\lambda_b}\xi_{b0}$

$\xi_c = \frac{\lambda_c}{(\lambda_a - \lambda_b)}(\eta_c - I_c)$

$I_c = \eta_{c0} + \frac{(\lambda_b - \lambda_a)}{\lambda_c}\xi_{c0}$

$P_1 = \lambda_a^2(\lambda_a - \lambda_b - \lambda_c)^2 \mid I \mid^2 \mid \eta_a \mid^2$

$P_2 = \lambda_a^2\lambda_b(\lambda_a - \lambda_b - \lambda_c) \mid I \mid^2 \mid \eta_a \mid^2 I_b$

$P_1' = P_1$

$P_2' = \lambda_a^2\lambda_c(\lambda_a - \lambda_b - \lambda_c) \mid I \mid^2 \mid \eta_a \mid^2 I_c$

Integrating equations (34) and (35) we get

$\eta_b = Ae^{\sqrt{P_1}t} + Be^{\sqrt{P_1}t} - P_2/P_1$

$\eta_c = Qe^{\sqrt{P_1}t} + Re^{\sqrt{P_1}t} - P_2'/P_1$

where A, B, Q, R are to be determined by the initial conditions. Thus in the pump approximation analytical solutions to the system can be found.

4. STABILITY

There is a very extensive literature concerning the field of hydrodynamic stability. Bayley (1988) studied the stability of quasi 2-dimensional steady flows via an analysis of a Floquet system of ODE. The stability of certain very special flows which are exact solutions of the Navier-Stokes equations have been considered by Craik and Criminale (1986). Friedlander and Vashik (1991) have discussed the instability criteria of the flow of an inviscid incompressible fluid. They have obtained a geometric estimate from below on the growth rate of a small perturbation of a three dimensional flow. Satya Narayanan (1993) studied self-organization processes in quasi 2-dimensional hydrodynamic flows. Satya Narayanan (1994) considered the stability of quasi 2-dimensional hydrodynamic flows. In this study, we present the analysis of Friedlander and Vashik (1991) and Satya Narayanan (1994) to study the pressure structure of coronal loops.

Consider the dynamical system of ODE's :

$$\dot{\mathbf{X}} = -U(\mathbf{X}) \quad (36)$$

$$\dot{\xi} = [\frac{\partial U}{\partial \mathbf{X}}]^T\xi \quad (37)$$

$$\dot{\mathbf{b}} = -[\frac{\partial U}{\partial \mathbf{X}}]^T\mathbf{b} - [(\nabla \times U) \times \mathbf{b}\cdot\xi]\frac{\xi}{\mid \xi \mid^2} \quad (38)$$

The dot denotes differentiation with respect to time t. The initial conditions at $t = 0$ are

$X = X_0, \xi = \xi_0, b = b_0$

with $\xi_0 \cdot b_0 = 0$.

The quantity $\xi_0 / \mid \xi_0 \mid$ is the direction of the spatial wave vector. The matrix $\partial U/\partial X$ has components $\partial u_i/\partial x_j, i, j = 1, 2, 3.$

The vector $\mathbf{b}(X_0, \xi_0, t)$ is the amplitude of a high-frequency wavelet localized at x_0 . If σ is the growth rate of the perturbation of the equilibrium solution, then it was shown using WKB methods by Friedlander and Vashik (1991) and Satya Narayanan (1994) that

$$\bar{lim}_{t\to\infty}(1/t) InfSup_{|x_0,\xi_0,b_0|} ||b(X, \xi_0, t)_0|| \leq \sigma \quad (39)$$

with the condition

$\mid \xi_0 \mid = 1, \xi_0 \cdot b_0 = 0$

They also showed that at a stagnation point of the flow where $\nabla \times U = 0$, the nature of the stability would depend on the eigenvalues of the matrix given by

$$[\frac{\partial U}{\partial \mathbf{X}}]^T \quad (40)$$

They showed that if one of the eigenvalues of the above matrix is positive, then such a flow admits exponentially growing solutions.

Returning to the problem of coronal loops, it is very clear that the stability of the pressure profile depends on the stability of the velocity profile as is clear from equation (32). It is very clear from the expression for the velocity which is in terms of CK functions which form a complete set that the eigenvalues of the matrix which is given in equation (40) depends on λ and η . It can be shown by straightforward algebra (we skip the details for the sake of brevity) that whenever η_a is greater than η_b and η_c with $\eta_b \approx \eta_c$, the system has one eigenvalue which is negative implying that the loops become unstable. One can consider other possibilities for the triple mode system. However, in this study, we have restricted our calculations to a particular choice of the amplitudes. More work needs to be done in this direction and will be taken up separately.

ACKNOWLEDGMENTS

I am grateful to the organizers of the 9th European Solar Physics Meeting on 'Magnetic Fields and Solar Processes' held at Firenze, Italy from September 12 - 18, 1999, for the financial support which enabled me to attend the meeting.

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