EMISSION ALTITUDES IN RADIO PULSARS WITH TRIPLE PROFILES

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Abstract

We propose here an independent method to understand emission altitudes in radio pulsars showing triple profiles. The centers of core and conal components in triple profiles are often non coincident in longitude. Treating these offsets as a generic property we seek an explanation by attributing different emission altitudes to these components. The offsets are then easily explained as resulting from aberration, retardation and magnetic field line sweepback. This immediately allows determination of the difference between core and conal altitudes which are much less uncertain than the individual altitudes derived by other methods and leading to many interesting conclusions. Individual altitudes can also be determined by focussing on the relation between emission altitudes and polar flux tube filling factors of these components. For this purpose we propose an empirical '1/3 rule' to concisely describe the triple pulse morphologies. We are led to generally higher emission altitudes and an interesting variation of filling factors with them.

1 Core-cone phase offsets

The knowledge of emission altitudes of radio pulses is of great importance for an understanding of the pulsar magnetospheric and emission physics. Various methods for estimating emission altitudes have been used in the past (for a review see Cordes, 1992). The mean pulse profiles of many radio pulsars consist of three components. In these triple profiles the components are designated as core and conal having differing characteristics (Rankin 1990 (R4), 1993(R6)). Previous data analyses, using the dipole geometry for the stellar magnetic field, generally indicate that the core emission emanates from the full polar cap at an altitude $r_{core} \simeq R_*$, the stellar radius (usually taken as 10 km) and the conal emission comes from a hollow cone at an altitude, $r_{cone} \simeq 10 - 20\,R_*$. A general maximum figure suggested is of the order of 10% of the light cylinder radius $r_L = (cP/2\,\pi)$, where P is the pulsar period.

We propose here an independent method to find emission altitudes by attributing the often observed phase offsets between centres of the core components and mid-points of the conal pairs in triple profiles to a difference in their altitudes (Kapoor and Shukre 1999, 2002, 2005 (KS99, KS02, KS05 respectively). We call this offset in terms of arrival times, a **lead** or a **lag**, depending on whether the core component is placed earlier or later in comparison with the midpoint of the conal pair. Both cases are seen (typical offsets $\simeq \pm 2°$).

These offsets find a natural explanation in terms of differential aberration, retardation and the magnetic field line sweepback *mfs*. At all altitudes, aberration and retardation advance the pulse arrival and the *mfs* tends to delay the arrival. We combine them here appropriately and *separately* for the core and conal components (observed at one and the same frequency).

The aberration, retardation and *mfs* are described in Kapoor and Shukre (1998), Dyks et al. (2004) and Shitov (1983, 1985) respectively. Denoting the

Table 1: Phase offsets for some pulsars

SN	Pulsar	Type	Δ_{cc} (deg.)	P (s)	α (deg.)	β (deg.)	Δr (R_*)	Δv	ν_{obs} (MHz)
1	0329+54	T	-2.0^a	0.715	30.8	+2.9	60.0	0.009	408
2	0450-18	T	0.0^b	0.549	31.0	+4.9	—	—	408
3	0523+11	T	$+1.0^c$	0.354	49.1	+4.6	14.5	0.007	1418
4	1917+00	T	$+0.6^d$	1.27	78.2	-1.2	30.9	0.005	1400
5	2045-16	T	-2.1^b	1.96	37.0	+1.1	171	0.01	408
6	0531+21	$T_{1/2}$	$+22^e$	0.033	90.0	—	32.0	0.20	318
7	0823+26	$T_{1/2}$	$+30^d$	0.531	90.0	—	700	0.28	1400
8	1742-30	T	-9^f	0.367	57.0	—	137	0.07	1700
9	1802+03	—	$\leq +4.1^c$	0.219	—	—	38.0	≤ 0.04	1418
10	2020+28	T	$\leq +5^g$	0.343	71.0	+3.6	70.0	0.04	430

References for Δ_{cc} : a - LSG71, b - LM88, c - WCL99, d - RSW89, e - MH76, f - XR91, g - SRW75.

Lorentz factor of the corotation speed by γ, the emission altitude in units of light cylinder radius by ξ, the different contributions for the magnetic axis direction are

$$\varphi_{ab} = tan^{-1}(\gamma\,\xi), \qquad \varphi_{ret} = (d/r_L) - \xi, \qquad \varphi_{mfs} = tan^{-1}\,(\frac{2\sqrt{\pi}}{3}\xi^3 sin^2\alpha). \quad (1)$$

Here, α is the inclination of the dipole with respect to the rotation axis and d the distance from us to the star. For any emission component the magnitude of net advancement is

$$\Delta\varphi(\xi) = \varphi_{ab} - \varphi_{ret} - \varphi_{mfs} \qquad (2)$$

We denote the phase offset between the core peak and the conal midpoint by Δ_{cc} and it is given by,

$$\Delta_{cc} = \Delta\varphi(\xi_{core}) - \Delta\varphi(\xi_{cone}). \qquad (3)$$

In addition, we also define the altitude and corotation speed differences as

$$\Delta r = |r_{core} - r_{cone}|, \qquad \Delta v = |\xi_{core} - \xi_{cone}|\; sin\,\alpha. \qquad (4)$$

2 Comparison with observations

For a first comparison with observations all unnecessary complications can be avoided if we look at the phase shift suffered only by the magnetic axis direction. Also, it suffices to use profiles from which observed values of Δ_{cc} could be obtained by simple visual inspection. From 32 pulsars for which we could collect useful data a sample of 10 pulsars appears in Table 1. Pulsar types and other parameters are as in R4 and Lyne and Manchester (1988). The parameter β is the usual line of sight impact angle.

The variation of the shift $\Delta\varphi$ with ξ, the emission altitude is shown in Fig. 1. Marking two points apart by an observed Δ_{cc} on the y-axis leads us to

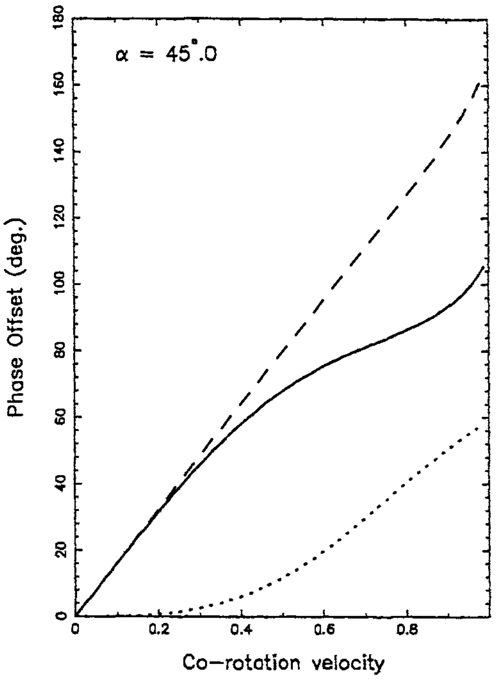

Figure 1: Phase shifts of the emission as a function of the corotation speed for $\alpha = 45^\circ$. Net shift is the solid line, and the contribution $\varphi_{ab} - \varphi_{ret}$ is the dashed line and φ_{mfs} the dotted line.

two points on the x-axis separated by Δr. Clearly several pairs of core/cone altitudes can be derived for a given Δ_{cc} all of which have about the same altitude difference. Values of Δr and Δv derived in this manner are included in Table 1. The individual core and conal altitudes can also be found either if one of them is considered known or by using the pulse width information.

Our considerations provide a simple explanation for Δ_{cc}. For most pulsars $|r_{core} - r_{cone}|$ is small compared to r_L but not necessarily compared to R_*; however for some pulsars $|r_{core} - r_{cone}|$ could be a sizeable fraction of r_L, in marked contrast to altitudes found by other authors. Since the pulse component which is emitted at a higher altitude will always lead with respect to the one at a lower altitude, offsets of either sign imply that each of the possibilities, i.e., $r_{core} <, =, or > r_{cone}$ can occur, especially that the core emission can not come from the surface in all pulsars. Indeed the surprising ones are the leads, because they imply core altitudes larger than conal ones.

Very large values of Δ_{cc} are permitted by Eq. 3. The last five pulsars in the Table 1 illustrate this. A large offset can make the core component merge with one of the conal ones or even displace it outside the conal pair. Such cases are of great interest and may throw much light on pulsar emission altitudes. For such pulsars very large altitudes are called for.

3 Emission altitudes

Can the added input of Δr, the altitude difference, help us in a better understanding of previously determined emission altitudes using the period pulse-width (or component-separation) relation (R4, R6)? Our Δr values are much also shared by the altitudes derived using the offsets between intensity and polarization centroids (BCW offsets, see Blaskiewicz et al. 1991).

Lags : The pulsar emission can at most fill the full cap in the open flux tube, in which case the pulse width gives us a lower limit on the emission altitude. For PSR 0329+54 the conal width is 30° and the core width is 10° in the 408 MHz profile. This gives $r_{cone} \geq 27R_*$ and $r_{core} \geq 3R_*$. Its Δ_{cc} further constrains the lowest admissible altitudes to the pair $63R_*$ and $3R_*$. Generally both core and conal filling can not be maximum.

Zero offsets : In these cases the simplest alternative is that both core and conal altitudes are same. The 1/3 rule implies that $f_{core}/f_{cone} = 1/2$, again indicating non maximal fillings.

Leads : In these cases the core altitude is higher than the conal one. One needs to ensure that placing the core component higher does not broaden it so much, due to the divergence of the open flux tube, that it overlaps the conal outriders. For PSR 1917+00 the minimum possible altitude consistent with the 1400 MHz conal width of 13.5° is $36\,R_*$. Now invoking the 1/3 rule, Eqs. 5 and 6 give us $f_{core}/f_{cone} = 0.35$. The solid angle filling factor ratio will be the square of this. Higher altitude combinations consistent with Δ_{cc} value will further reduce this ratio. *Clearly the full open flux tube must have a large fraction which does not radiate at all for pulsars in which the core leads.*

4 Conclusions

The otherwise difficult to understand, observed longitude offsets between the centers of core and conal components in triple profiles get easily explained in terms of differential aberration, retardation and *mfs* due to their different emission altitudes. Our conclusions are as follows (see also KS02):

- Core emission altitudes, r_{core} may be smaller, larger than or same as r_{cone}, the conal ones. Core emission can not always come from the stellar surface.
- For most pulsars $|r_{core} - r_{cone}|$ is small compared to r_L but not necessarily compared to R_*, however for some pulsars $|r_{core} - r_{cone}|$ could be a sizeable fraction of r_L.
- Both core and conal emissions may not come from the full available part of the polar flux tube and their filling factors vary with the altitudes in a specific manner.

The characteristics of emission altitudes implied by these conclusions, though derived for pulsars showing triple profiles, are expected to hold also for all pulsars. These conclusions can be verified and sharpened further by a detailed analysis of observations using both Δ_{cc} and the BCW offsets (in this connection see Dyks and Harding (2004)), as also the variation of Δ_{cc} with emission frequency (KS05). Such a study in our view holds much promise.

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