

# Hybrid Viscosity and Magnetoviscous Instability in Hot, Collisionless Accretion Disks

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**Summary.** We aim to illustrate the role of hot protons in enhancing the magnetorotational instability (MRI) via the “hybrid” viscosity, which is due to the redirection of protons interacting with static magnetic field perturbations, and to establish that it is the only relevant mechanism in this situation. It has recently been shown by Balbus [1] and Islam & Balbus [11] using a fluid approach that viscous momentum transport is key to the development of the MRI in accretion disks for a wide range of parameters. However, their results do not apply in hot, advection-dominated disks, which are collisionless. We develop a fluid picture using the hybrid viscosity mechanism, that applies in the collisionless limit. We demonstrate that viscous effects arising from this mechanism can significantly enhance the growth of the MRI as long as the plasma  $\beta \gtrsim 80$ . Our results facilitate for the first time a direct comparison between the MHD and quasi-kinetic treatments of the magnetoviscous instability in hot, collisionless disks.

**Key words:** MHD: Viscosity - Instability in Hot - Accretion Disks

## 1 Introduction

The microphysical source of viscosity in accretion disks has been a long-standing puzzle. Since the early 1990s, there has been a growing consensus that magnetic fields generated by the magnetorotational instability (MRI) are key to providing the required viscosity in cold accretion disks ([2] (and references therein), [4], [14]). The standard treatment of the MRI is valid only for collisional plasmas, which can be described in the MHD approximation. However, the plasmas comprising hot, two-temperature accretion flows, like those described in [8] and [19] (hereafter SLE) are clearly collisionless. This is also the case for the radiatively inefficient, advection-dominated accretion flows (ADAFs) treated in [15] and [16]. Filho ([9]), Kafatos ([12]) and Paczyński ([17]) had initially suggested that viscosity due to collisions between hot protons might be important in two-temperature accretion flows, although

the effects of an embedded turbulent magnetic field were not included in their treatments. Subramanian, Becker & Kafatos ([23]; hereafter SBK96) proposed that a *hybrid* viscosity, due to protons colliding with magnetic scattering centers, might be the dominant viscosity mechanism in such accretion disks. In this paper we investigate the implications of the hybrid viscosity for the development of the MRI in hot disks. In particular, we show that this mechanism can be used to establish an interesting connection between the fluid models and the quasi-kinetic treatments used by previous authors to study the viscous enhancement of the growth rate during the early stages of the MRI.

## 2 MVI in hot accretion disks

Balbus ([1]) and Islam & Balbus ([11]) employed an MHD approach to study the effect of viscosity on the development of the MRI, and discovered a robust instability which they call the magnetoviscous instability (MVI). In the MVI, angular momentum is exchanged between fluid elements via viscous transport, which plays a central role in the development of the instability. Balbus ([1]) does not address the physical origin of the viscosity that is central to the development of the MVI, and therefore his results are stated in terms of an unspecified coefficient of dynamic viscosity,  $\eta$ . Islam & Balbus ([11]) assumed the plain Spitzer (collisional) viscosity due to proton-proton collisions in their treatment of the MVI, but this particular mechanism is not effective in hot, collisionless disks. There have been some recent attempts at quasi-kinetic treatments of MRI-like instabilities in collisionless plasmas (e.g., [18], [20], [21]). It is interesting to note that the pressure anisotropy concept discussed in these papers is somewhat similar to the idea embodied in the hybrid viscosity formalism of SBK96. This suggests that it may be possible to develop a “fluid” picture based on the hybrid viscosity that would be applicable in hot disks, hence bridging the gap between the two paradigms. The hybrid viscosity concept of SBK96 relies only on the momentum deposited by particles propagating along magnetic field lines between adjacent annuli in the disk.

## 3 Applicability of the hybrid viscosity

Paczyński ([17]) and SBK96 noted that the presence of even a very weak magnetic field can effectively “tie” protons to magnetic field lines. Paczyński argued that in this situation the ion-ion collisional mean free path is much larger than the proton Larmor radius and therefore the effective mean free path is equal to the proton Larmor radius. This led him to conclude that the viscosity would effectively be quenched in such a plasma. However, the protons in hot accretion disks are typically super-Alfvénic, especially in the initial stages of a magnetic field-amplifying instability such as the MRI, when

the plasma  $\beta$  parameter is quite large. This reflects the fact that the ratio of the proton thermal speed to the Alfvén speed is equal to  $(3\beta/2)^{1/2}$ . Since the magnetic field evolves on Alfvén timescales, it can be considered to be static for our purposes. The motion of collisionless, charged particles propagating through a static, tangled magnetic field has been explored extensively in the context of cosmic ray propagation (e.g., [5], [6], [10]). It has been conclusively established that the particle transport does not obey Bohm diffusion for a wide range of rigidities and turbulence levels (see, e.g., Fig. 4 of [5] and Fig. 4 of [6]). In particular, the low rigidity, low turbulence level case appropriate for our situation obeys the predictions of quasi-linear theory quite well, and the mean free paths are much larger than the Larmor radius as expected.

Under these conditions, SBK96 demonstrated the importance of a new kind of viscosity called the “hybrid viscosity,” in which angular momentum is transported via collisions between protons and static irregularities (“kinks”) in the magnetic field. In this picture, a proton spirals tightly along a magnetic field line until its gyro-averaged guiding center motion (and hence its gyro-averaged momentum) is changed via an encounter with a kink. During the encounter the proton therefore exchanges angular momentum with the field, which transfers the resulting torque to the plasma. The effective mean free path used in the computation of the viscosity is set equal to the distance between the magnetic kinks (i.e., the field coherence length). We express the hybrid viscosity mechanism in terms of a pressure anisotropy in § 6.1.

Here we examine the implications of the hybrid viscosity for the development of the MVI in hot, two-temperature accretion disks around underfed black holes. We assume that the accreting plasma is composed of fully ionized hydrogen. The physical picture involves the perturbation of an initially straight magnetic field line that eventually leads to the instability (see, e.g., Fig. 1 of [1]). Since the proton Larmor radius is negligible in comparison to a macroscopic length scale, we can effectively think of the proton as sliding along the field line like a bead on a wire. The proton is forced to change its direction upon encountering the kink associated with the initial field perturbation. In such a situation, the effective mean free path,  $\lambda$ , used in the description of the hybrid viscosity should be set equal to the wavelength of the initial perturbation. We demonstrate that the hybrid viscosity is the principle mediator of the MVI during the early stages of the instability.

#### 4 Hybrid viscosity in hot accretion disks

The structure of hot, two-temperature accretion disks was first studied in detail by SLE, and later by Eilek & Kafatos ([8]) and SBK96. The closely related advection-dominated accretion flows were analyzed by Narayan & Yi ([15]), Narayan, Mahadevan, & Quataert ([16]), and many subsequent authors. In this section, we investigate the nature of the viscosity operative in hot, two-

temperature accretion disks based on a simplified set of model-independent relations that are applicable to both ADAF and SLE disks.

The gas in the disk can be considered collisionless with respect to the protons provided

$$\lambda_{ii} > H, \quad (1)$$

where  $H$  is the half-thickness of the disk, and the ion-ion Coulomb collisional mean free path,  $\lambda_{ii}$ , is given in cgs units by (SBK96)

$$\lambda_{ii} = 1.80 \times 10^5 \frac{T_i^2}{N_i \ln A}, \quad (2)$$

for a plasma with Coulomb logarithm  $\ln A$  and ion temperature and number density  $T_i$  and  $N_i$ , respectively. We can combine equations (1) and (2) to obtain

$$\frac{\lambda_{ii}}{H} = 1.20 \times 10^{-19} \frac{T_i^2}{\tau_{es} \ln A} > 1, \quad (3)$$

where the electron scattering optical thickness,  $\tau_{es}$ , is given by

$$\tau_{es} = N_i \sigma_T H, \quad (4)$$

and  $\sigma_T$  is the Thomson scattering cross section. Equation (3) can be rearranged to obtain a constraint on  $\tau_{es}$  required for the disk to be collisionless, given by

$$\tau_{es} < \frac{1.20 \times 10^5 T_{12}^2}{\ln A} \sim 4 \times 10^3, \quad (5)$$

where  $T_{12} \equiv T/10^{12}$  K and the final result holds for  $\ln A = 29$  and  $T_{12} \sim 1$ . This confirms that tenuous, two-temperature disks with  $T_i \sim 10^{11}$ – $10^{12}$  K will be collisionless for typical values of  $\tau_{es}$ .

The collisionless nature of hot two-temperature accretion flows established by equation (5) strongly suggests that the plain Spitzer viscosity is not going to be relevant for such disks, although the answer will depend on the strength of the magnetic field. The hybrid viscosity will dominate over the Spitzer viscosity provided the ion-ion collisional mean free path  $\lambda_{ii}$  exceeds the Larmor radius,  $\lambda_L$ , so that the protons are effectively “tied” to magnetic field lines. We therefore have

$$\lambda_{ii} > \lambda_L, \quad (6)$$

where the Larmor radius is given in cgs units by (SBK96)

$$\lambda_L = 0.95 \frac{T_i^{1/2}}{B}, \quad (7)$$

where  $B$  is the magnetic field strength. Whether the disk is of the SLE or ADAF types, it is expected to be in vertical hydrostatic equilibrium, and therefore

$$H \Omega_K = c_s = \sqrt{\frac{kT_i}{m_p}}, \quad (8)$$

where  $\Omega_K = (GM/r^3)^{1/2}$  is the Keplerian angular velocity at radius  $r$  around a black hole of mass  $M$ ,  $c_s$  is the isothermal sound speed, and  $k$  and  $m_p$  denote Boltzmann's constant and the proton mass, respectively.

We can utilize equation (6) to derive a corresponding constraint on the plasma  $\beta$  parameter,

$$\beta \equiv \frac{8\pi N_i kT_i}{B^2}, \quad (9)$$

such that the hybrid viscosity dominates over the Spitzer viscosity. By combining equations (2), (4), (6), (7), (8), and (9), we find that

$$\beta < 3.71 \times 10^{32} \frac{T_{12}^{9/2} R^{3/2} M_8}{\tau_{es} (\ln A)^2}, \quad (10)$$

where  $M_8 \equiv M/(10^8 M_\odot)$  and  $R \equiv rc^2/(GM)$ . The minimum possible value of the right-hand side in equation (10) is obtained for the maximum value of  $\tau_{es}$ , which is given by equation (5). We therefore find that

$$\beta < 3.09 \times 10^{27} \frac{T_{12}^{5/2} R^{3/2} M_8}{\ln A}. \quad (11)$$

This relation is certainly satisfied in all cases involving the accretion of plasma onto a black hole, even in the presence of an infinitesimal magnetic field. We therefore conclude that the protons will be effectively tied to the magnetic field lines in two-temperature accretion disks around stellar mass and supermassive black holes, which implies that the hybrid viscosity dominates over the Spitzer viscosity in either SLE or ADAF disks.

The results of this section confirm that protons in two-temperature accretion disks rarely collide with each other, and are closely tied to magnetic field lines, even for very weak magnetic fields. If a field line is perturbed, a typical proton sliding along it will follow the perturbation, and will thus be effectively redirected. This is the basic premise of the hybrid viscosity concept, which we will now apply to the development of the MVI.

## 5 MVI driven by the hybrid viscosity

Figure 1 of [11] shows that magnetoviscous effects significantly enhance the MRI growth rates in the parameter regime

$$X \lesssim x, \quad Y \gtrsim y, \quad (12)$$

where  $x \sim 1$ ,  $y \sim 1$ , and

$$X \equiv \frac{2.0 (k_Z H)^2}{\beta}, \quad Y \equiv \frac{1.5 \eta k_\perp^2}{N_i m_p \Omega_K}, \quad (13)$$

with  $\eta$  denoting the coefficient of dynamic viscosity and  $k_Z$  and  $k_\perp$  representing the  $z$  and transverse components of the field perturbation wavenumber, respectively. The maximum MVI growth rate is  $\sqrt{3} \Omega_K$ , which is  $4/\sqrt{3} \sim 2.3$  times larger than the maximum MRI growth rate of  $(3/4) \Omega_K$ . The conditions in equation (12) are derived from the dispersion relation given in equation (33) of [11], which is general enough to accommodate different prescriptions for the viscosity coefficient  $\eta$ . The condition  $X \lesssim x$  implies a constraint on  $\beta$  given by

$$\beta \gtrsim \frac{2 (k_Z H)^2}{x}. \quad (14)$$

As mentioned earlier, a proton sliding along a given field line is forced to change its direction when it encounters a kink/perturbation in the field line. The effective viscosity arises due to the momentum deposited in the fluid by the proton when it encounters the perturbation. In this picture, the perturbation wavelength plays the role of an effective mean free path. If we consider perturbations along an initially straight field line, as in Figure 1 of [1], then only the transverse component of the perturbation wavelength is relevant, and the effective mean free path for the proton is therefore

$$\lambda = \frac{2\pi}{k_\perp} \equiv \xi H, \quad (15)$$

where  $\xi \leq 1$ , since the perturbation wavelength  $\lambda$  cannot exceed the disk half-thickness  $H$  (SBK96).

In general, the Shakura-Sunyaev ([22]) viscosity parameter  $\alpha$  is related to the coefficient of dynamic viscosity  $\eta$  via (SBK96)

$$\alpha P \equiv -\eta R \frac{d\Omega_K}{dR} = \frac{3}{2} \eta \Omega_K, \quad (16)$$

where  $P = N_i k T_i$  is the pressure in a two-temperature disk with  $T_i \gg T_e$ . By combining equations (8), (13), and (16), we find that the condition  $Y \gtrsim y$  can be rewritten as

$$(k_\perp H)^2 \alpha \gtrsim y. \quad (17)$$

Following Islam & Balbus ([11]), we expect that  $k_\perp \lesssim k_Z$ . By combining equations (14) and (17), we therefore conclude that  $\beta$  must satisfy the condition

$$\beta \gtrsim \frac{2y}{\alpha x}. \quad (18)$$

We can also combine equations (14) and (15) to obtain the separate constraint

$$\beta \gtrsim \frac{79}{\xi^2 x}. \quad (19)$$

Equations (18) and (19) must *both* be satisfied if the MVI is to significantly enhance the MRI growth rates. Hence the combined condition for  $\beta$  is given by

$$\beta \gtrsim \text{Max} \left( \frac{79}{x\xi^2}, \frac{2y}{\alpha x} \right). \quad (20)$$

We can use equation (16) to calculate the Shakura-Sunyaev parameter  $\alpha_{\text{hyb}}$  describing the hybrid viscosity. The associated coefficient of dynamic viscosity is given by

$$\eta_{\text{hyb}} = \frac{\lambda}{\lambda_{ii}} \eta_s, \quad (21)$$

where  $\lambda_{ii}$  is computed using equation (2) and  $\eta_s$  is the standard Spitzer collisional viscosity, evaluated in cgs units using

$$\eta_s = 2.20 \times 10^{-15} \frac{T_i^{5/2}}{\ln A}. \quad (22)$$

The quantity  $\eta_{\text{hyb}}$  defined in equation (21) describes the effect of momentum deposition due to protons spiraling tightly along a magnetic field line over a mean free path  $\lambda$ . It differs from the expression given in equation (2.14) of SBK96 by a factor of 2/15, because we do not consider tangled magnetic fields here. Setting  $\eta = \eta_{\text{hyb}}$  in equation (16) and utilizing equations (2), (8), (15), (21), and (22), we find after some algebra that the expression for  $\alpha_{\text{hyb}}$  reduces to the simple form

$$\alpha_{\text{hyb}} = 1.2\xi. \quad (23)$$

We can now combine equations (20) and (23) to conclude that in the case of the hybrid viscosity, the MVI is able to effectively enhance the MRI growth rates if

$$\beta \gtrsim \beta_{\text{crit}} \equiv \text{Max} \left( \frac{79}{x\xi^2}, \frac{1.7y}{x\xi} \right). \quad (24)$$

In particular, we note that if  $x \sim 1$  and  $y \sim 1$ , then equation (24) reduces to  $\beta_{\text{crit}} = 79\xi^{-2}$ , since  $\xi \leq 1$ . We therefore conclude that magnetoviscous effects driven by the hybrid viscosity will significantly enhance the growth rate (compared with the standard MRI growth rate) until the plasma  $\beta$  parameter reaches  $\sim 80$ , or, equivalently, until the field strength  $B$  reaches  $\sim 10\%$  of the equipartition value. This assumes that the dominant perturbations have  $\xi \sim 1$ , which is expected to be the case during the early stages of the instability. Once the field exceeds this strength, the growth rate of the instability during the linear stage will be equal to the MRI rate.

## 6 Relation to previous work

It is interesting to contrast our result for the  $\beta$  constraint with those developed by previous authors using different theoretical frameworks.

### 6.1 Hybrid viscosity in terms of pressure anisotropy

Before proceeding on to discussing the result for the  $\beta$  constraint, we first cast the basic hybrid viscosity mechanism in terms of a pressure anisotropy. Several similar treatments appeal to a large-scale pressure anisotropy, rather than an explicit viscosity mechanism (e.g., [18], [20], [21]). It is therefore instructive to show that the hybrid viscosity mechanism we employ can be cast in these terms.

We follow the approach of SBK96 in considering a perturbation in the local magnetic field of an accretion disk. The pressure anisotropy due to the momentum flux carried by the particles can be analyzed in the local region using cartesian coordinates, with the  $\hat{z}$ -axis aligned in the azimuthal (orbital) direction, the  $\hat{y}$ -axis pointing in the outward radial direction, and the  $\hat{x}$ -axis oriented in the vertical direction. The unperturbed magnetic field is assumed to lie in the  $\hat{z}$  direction, and the perturbed field makes an angle  $\theta$  with respect to the  $\hat{z}$ -axis, and an azimuthal angle  $\phi$  with respect to the  $\hat{x}$ -axis. In keeping with the hybrid viscosity scenario, we assume that the particles spiral tightly around the perturbed field line. In this situation, the component of the particle pressure in the direction *parallel* to the magnetic field,  $P_{\parallel}$ , is equal to the  $\hat{z}$ -directed flux of the  $\hat{z}$ -component of momentum,  $P_{zz}$ . Likewise, the total particle pressure *perpendicular* to the field,  $P_{\perp}$ , is equal to the sum of the  $\hat{x}$ -directed momentum in the  $\hat{x}$ -direction and the  $\hat{y}$ -directed momentum in the  $\hat{y}$ -direction, denoted by  $P_{xx}$  and  $P_{yy}$ , respectively. Following the same approach that leads to equation (2.11) of SBK96, we obtain for the parallel pressure

$$P_{\parallel} = P_{zz} = 2 m_p N_i \cos^2 \theta \times \left[ \frac{kT_i}{2m_p} - \left( \frac{2kT_i}{\pi m_p} \right)^{1/2} u'(0) \lambda \cos \theta \sin \theta \sin \phi \right], \quad (25)$$

where  $u(y)$  represents the shear velocity profile and the prime denotes differentiation with respect to  $y$ . Similarly, the total perpendicular pressure is given by

$$P_{\perp} = P_{xx} + P_{yy} = 2 m_p N_i \sin^2 \theta \times \left[ \frac{kT_i}{2m_p} - \left( \frac{2kT_i}{\pi m_p} \right)^{1/2} u'(0) \lambda \cos \theta \sin \theta \sin \phi \right]. \quad (26)$$

Taken together, equations (25) and (26) imply that

$$\frac{P_{\perp}}{P_{\parallel}} = \tan^2 \theta. \quad (27)$$

This result characterizes the pressure anisotropy associated with the hybrid viscosity mechanism. Equation (27) is strictly valid only in the limit of zero



proton gyroradius, which is a reasonable approximation in hot advection-dominated disks. When the field line is unperturbed, so that it lies precisely along the  $\hat{z}$ -direction, then  $\theta = 0$ , and equation (27) indicates that the perpendicular pressure tends to zero; in reality, owing to finite gyroradius effects, the perpendicular pressure would actually be a small, but finite quantity even in this limit. Early in the instability, when the field line is slightly perturbed,  $\theta$  has a small but non-zero value, and equation (27) predicts that the perpendicular pressure starts to increase in relation to the parallel pressure. We have cast the hybrid viscosity mechanism in terms of a pressure anisotropy in this section in order to make contact with that part of the literature in which viscous momentum transport is treated solely in this manner. The quasi-kinetic treatments of Quataert and co-workers rely on a Landau fluid closure scheme for deriving the perturbed pressure. The pressure anisotropy implied by the hybrid viscosity mechanism (eq. [27]) is much simpler than the corresponding result obtained using either the fluid closure scheme of Quataert et al., or the double adiabatic scheme ([7]) adopted by other authors.

## 6.2 Relation to MVI treatment

In their treatment of the MVI, Islam & Balbus ([11]) parametrized the viscous transport in terms of an unspecified proton-proton collision frequency,  $\nu$ . Their estimates of the growth rates in *collisional* plasmas agree fairly well with those derived using quasi-kinetic treatments. Based on their formalism, they conclude that the  $\beta$  regime within which magnetoviscous effects can significantly impact the MRI growth rates in two-temperature accretion flows extends to  $\beta_{\text{crit}} \sim 1$ . However, as they point out, their approach breaks down in the collisionless limit  $\nu \rightarrow 0$ , which describes the ADAF disks of interest here. It is therefore not surprising that their constraint on  $\beta$  is significantly different from the one we have derived in equation (24).

## 6.3 Relation to quasi-kinetic treatment

Quataert, Dorland & Hammett ([18]) have treated the case of a strictly collisionless plasma using a fairly complex kinetic formalism. Their results suggest that, for the case with  $B_\phi = B_z$  and  $k_r = 0$  (which is the one considered by Islam & Balbus and ourselves), viscous effects will significantly impact the MRI growth rates for values of  $\beta$  that are several orders of magnitude larger than those predicted by our formalism. For example, their analysis predicts that a growth rate of  $1.5 \Omega_K$  can be achieved if  $\beta \gtrsim \beta_{\text{crit}} \sim 10^4$  (see Fig. 4 of [18]) and Fig. 2 of [20]). On the other hand, Figure 1 of [11] indicates that a growth rate of  $1.5 \Omega_K$  can be achieved in the MHD model if  $X \lesssim 0.35$ ,  $Y \gtrsim 12$ , which corresponds to  $x = 0.35$ ,  $y = 12$  in equation (12). Assuming that  $\xi \sim 1$  as before, equation (24) yields in this case the condition  $\beta \gtrsim \beta_{\text{crit}} \sim 225$ . Hence our MHD model based on the hybrid viscosity predicts that viscous effects will enhance the MRI growth rates down to much lower values of  $\beta$  than those

obtained in the quasi-kinetic model. This difference reflects the differing role of the particle pressure in the two scenarios.

In our formulation, the viscosity is expressed by protons that deposit their momentum into the fluid upon encountering kinks in the magnetic field, which is anchored in the local gas. The importance of forces due to gas pressure relative to those due to the tension associated with the magnetic field thus scales as the plasma  $\beta$ . On the other hand, gas pressure forces are only  $\sqrt{\beta}$  times as important as forces arising out of magnetic tension in the quasi-kinetic treatment of [18]. It follows that the value of  $\beta_{\text{crit}}$  computed using our MHD model based on the hybrid viscosity should be comparable to the square root of the  $\beta_{\text{crit}}$  value obtained using the quasi-kinetic model, and this is borne out by the numerical results cited above.

## 7 Conclusions

In this paper we have investigated the role of hot protons in influencing the magnetoviscous instability described in [1] and [11]. We have shown that the only relevant viscosity mechanism in this situation is the “hybrid” viscosity, which is due to the redirection of protons interacting with magnetic irregularities (“kinks”) set up by the initial field perturbations. In particular, we have demonstrated in equation (24) that viscous effects associated with the hybrid viscosity will significantly augment the MRI growth rates for  $\beta \gtrsim 80$ , which corresponds to a magnetic field strength  $B$  below  $\sim 10\%$  of the equipartition value. For smaller values of  $\beta$ , we expect the instability to grow at the MRI rate as long as it remains in the linear regime. This conclusion is expected to be valid in any hot, two-temperature accretion disk, including advection-dominated ones. We have obtained this result using a relatively simple fluid treatment, based upon the general dispersion relation obtained in [11]. Our use of the hybrid viscosity concept alleviates an important drawback in the fluid application made by Islam & Balbus ([11]), because their treatment of viscous transport breaks down in the collisionless plasmas of interest here. The new results we have obtained allow an interesting comparison between the MHD approach and the quasi-kinetic formalism used by other authors. We show that the differences between the predictions made by the two methodologies stem from the differing treatments of the particle pressure.

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