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Optical colours and polarization of a model reflection nebula. I. Star behind the nebula*

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Abstract. Simple models of a reflection nebula in the form of a plane-parallel slab containing smooth spherical solid particles in submicron size range have been considered. Single scattering has been assumed. The effect of varying the composition and size distribution function of the grains have been brought out in the calculations using Mie theory of scattering. The analytical part of the geometry of the problem has been treated quite rigorously and the resulting expression for nebular intensity has been presented in a somewhat new form. In this paper, the case of the star behind the nebula has been examined.

A comparison of the theoretical results with the observations of the Merope nebula shows that the dirty ice grains with index of refraction about $1\cdot3-0\cdot1i$ and size parameter $a_0 = 0\cdot5\,\mu$ give reasonable agreement with the colours. Simultaneously, the polarization in the visual and blue wavelength bands agree approximately up to offset angle of 6 minutes of arc. The larger offset angles pose an intriguing problem. The general trends of nebular colours and polarization with variation of real and imaginary parts of index of refraction and the size distribution parameter have been tabulated to serve as a guide for further study of reflection nebulae with the star in the rear.

Keywords. Reflection nebula; Scattering of light; UBV colours and polarization; interstellar grains.

1. Introduction

A reflection nebula gives out a continuous spectrum of scattered light related to the material in the nebula, its geometry and the energy distribution of the illuminating star which usually belongs to spectral class B or later. As early as 1912, Slipher was able to deduce for the first time the particulate nature of the material within the Merope Nebula. The observed colours and polarization as function of wavelength and offset angle can be compared with a variety of theoretical models in order to infer the nature of the interstellar grains within the nebula. Several attempts along this line have been made in the past. A reference is made to the survey by Johnson (1968) on the quantitative colorimetric and polarimetric observations of reflection nebulae obtained by photographic, photoelectric and spectrophotometric techniques. A pioneering theoretical study on reflection

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nebula was undertaken by Schalén (1945, 1953) using a distribution of metallic particles. Recent review articles by Greenberg (1968), Lynds and Wickramasinghe (1968), Vanýsek (1969), Wickramasinghe and Nandy (1972) and Aannestad and Purcell (1973) on interstellar grains cover the observational and theoretical aspects of the reflection nebulae. Greenberg and Roark (1967), Greenberg and Hanner (1970), Hanner (1971), Vanýsek and sŏlc (1973) and Zellner (1973) have given calculations of models of reflection nebula for a variety of geometrical parameters and indices of reflection. However, as mentioned by Aannestad and Purcell (1973), the observed range in colours across a nebula is larger than predicted for particles of either graphite, silicates or ice.

It seems that there are certain gaps and drawbacks both in the observational data as well as the theoretical models because there is really not a single model that can simultaneously satisfy all the available observational criteria for a reflection nebula. The observations should be improved as well as extended to far ultraviolet and far infrared, preferably with narrow band photometry.

The exact scattering volume element within the nebula, found in the present treatment is dependent among other things on the angular dimension of the aperture of the telescope, offset (viewing) angle and the ratio of the size of the elementary slab to its distance from the observer. The scattering centre within the elementary volume is located with reasonable symmetry and is specified explicitly. Here we have also taken into account the variation of intensity due to change of distances from the observer to various nebular volume elements along the line of sight. In what follows, a general expression has been derived for nebular intensity with precise consideration of the above features. This may be applied in other areas of astronomy also. For instance, x-ray scattering by circumstellar and/or interstellar grains is of current interest because it may provide some additional clue to the nature of interstellar grains. The effects of variation of complex index of refraction of the grain material and size distribution of the grains have been examined for the case of *star behind the nebula.*

2. Model reflection nebula with star behind the nebula

2.1 *Description of the geometry and useful relations*

We have considered the reflection nebula in the form of a plane-parallel slab perpendicular to the line joining the observer and the star. The star is behind the nebula. The finite aperture of the telescope has been included in the theoretical formulation of the model. The number density of the grains has been assumed to be uniform throughout the nebular region of interest. Single, independent scattering has been assumed. The frequency of the wave before and after the scattering process remains the same. The two states of polarization in the orthogonal reference planes have been considered separately. The empirical correction for the extinction of light within the nebula has been taken into account on the basis of the observations.

The symbols and notations have the following meaning (see figure 1):

D = the distance from the observer to the illuminating star,

T = the thickness of the nebula,

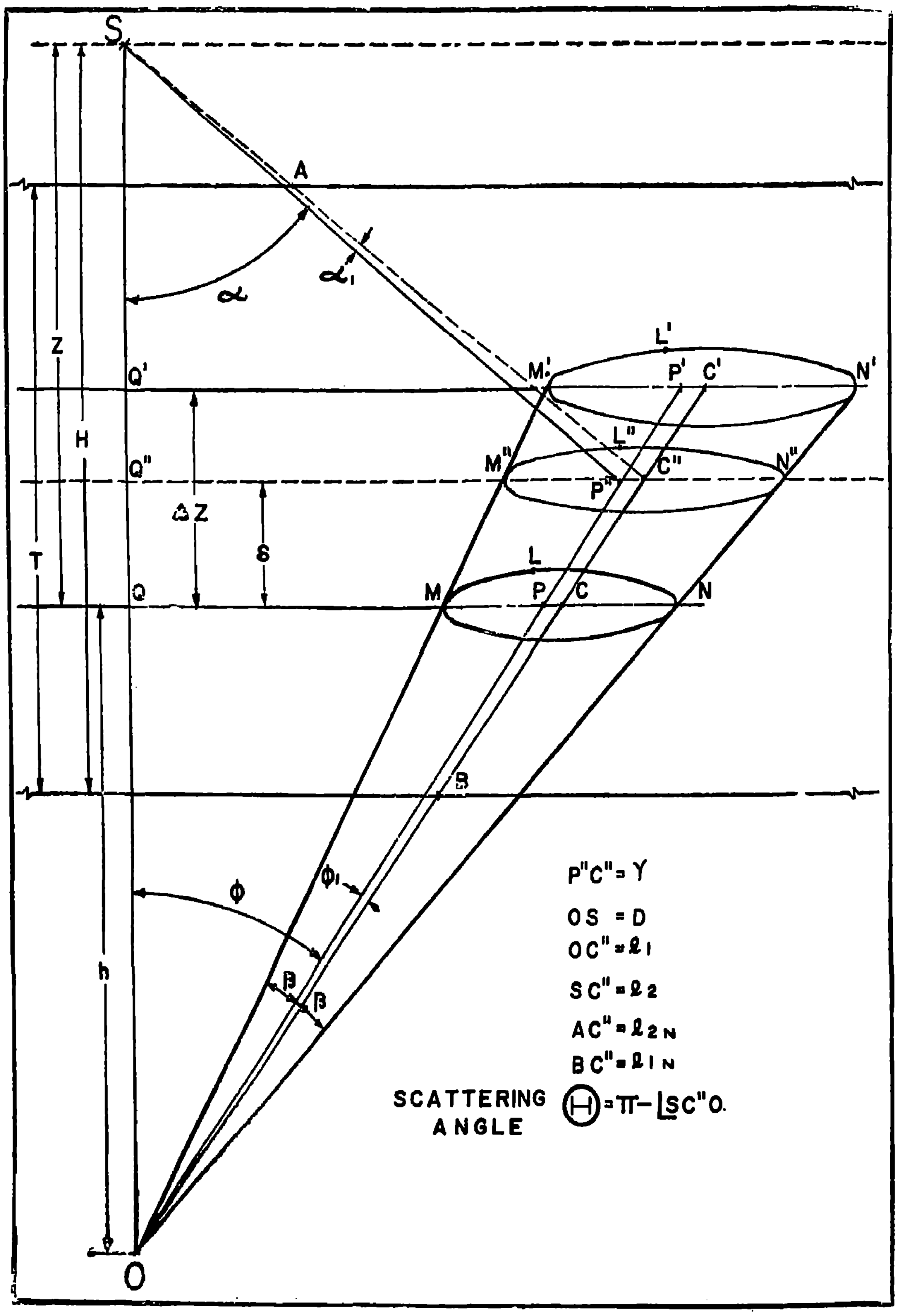


Figure 1. Schematic representation of plane-parallel slab model of a reflection nebula.

H = the perpendicular distance from the star to the front surface of the nebula,

ϕ = the offset angle, *i.e.*, the angular separation between the star and the nebular region as viewed by the observer,

β = semi-vertex angle of the telescope,

h = the distance of the bottom of a typical elementary slab from the observer,

$\triangle\Omega$ = the solid angle of the telescope
= $\pi \tan^2\beta$,

S = the location of the star,

O = the location of the observer.

Let the slab be divided into N (an odd number) parallel elementary slabs each of width $\triangle Z$. The observations are carried out with a telescope having certain fixed cone of view with a semi-vertex angle β. The axis of the telescope, OP, makes an angle ϕ with the line OS. The intersection of each slab with the extension of the telescope is a truncated frustum of cone as shown in figure 1. Consider a typical elementary slab between lines M′N′ and MN. In general, the top (M′L′N′) and the bottom (MLN) of the frustum will be elliptical cross-sections with centres at C′ and C, respectively. To locate the scattering centre with reasonable symmetry, one can consider a section M″L″N″ parallel to MLN such that volumes corresponding to MNN″M″ and M″N″N′M′ are equal. We will assume that all the grains within the elementary volume M′N′NM are located at C″, the centre of the ellipse L″M″N″. The telescope axis cuts the lines MN, M′N′ and M″N″ in P, P′ and P″, respectively. We define δ to be the perpendicular distance between the elliptical cross sections L″M″N″ and LMN. Some useful distances and angles have been denoted as follows:

$$\mathrm{OC''} = l_1, \quad \mathrm{SC''} = l_2, \quad \mathrm{OQ} = h$$

$$\mathrm{P''C''} = \gamma, \quad \alpha = \angle \mathrm{OSP''}, \quad \alpha_1 = \angle \mathrm{P''SC''}$$

$$\phi = \angle \mathrm{SOP''}, \quad \phi_1 = \angle \mathrm{P''OC''}.$$

The semi-major and semi-minor axes (a, b) and (a', b') of the ellipses LMN and L′M′N′, respectively, can be expressed as functions of h, β and ϕ. For ellipse LMN,

$$a = \frac{h \tan\beta}{\cos^2\phi - \sin^2\phi \tan^2\beta} \tag{1}$$

$$b = \frac{h \tan\beta}{(\cos^2\phi - \sin^2\phi \tan^2\beta)^{\frac{1}{2}}} \tag{2}$$

It can be shown that for ellipse L′M′N′ one has the following exact proportionality

$$\frac{a'}{a} = \frac{b'}{b} = 1 + \frac{\triangle Z}{h} \tag{3}$$

The exact volume of the elementary frustum MNN′M′ is then given by

$$\triangle V = h^2 \triangle Z \triangle \Omega \frac{\left\{1 + \frac{\triangle Z}{h} + \frac{1}{3}\left(\frac{\triangle Z}{h}\right)^2\right\}}{[\cos^2\phi - \sin^2\phi \cdot \tan^2\beta]^{3/2}} \tag{4}$$

If $\beta \simeq 0^\circ$ and $(\triangle Z/h) \ll 1$, eq. (4) approximates to the expression given by Greenberg and Roark (1967).

From elementary geometrical considerations one can further derive the following useful relations:

$$\gamma = \mathrm{P''C''} = \frac{(h+\delta)\tan\phi\,\tan^2\beta}{\cos^2\phi - \sin^2\phi\,\tan^2\beta} \tag{5}$$

$$\delta = QQ'' = h\left[\left\{\frac{1+\left(1+\frac{\triangle Z}{h}\right)^3}{2}\right\}^{1/3} - 1\right] \tag{6}$$

$$\tan(a + a_1) = \frac{\gamma + (h+\delta)\tan\phi}{Z - \delta} \tag{7}$$

$$\tan(\phi + \phi_1) = \frac{\gamma + (h+\delta)\tan\phi}{h+\delta} \tag{8}$$

$$l_1 = (h+\delta)\sec(\phi + \phi_1) \tag{9}$$

$$l_2 = (D - h - \delta)\sec(a + a_1) \tag{10}$$

The scattering angle at C″ is then given by

$$\Theta = a + a_1 + \phi + \phi_1 \tag{11}$$

2.2 *Expression for nebular intensity*

The model reflection nebula as described above will give the following intensity at the location of the observer:

$$I_J^N(\phi,\lambda) = \frac{CN_0 I_s^*(\lambda)\, r^2 \lambda^2 \triangle\Omega}{8\pi^2 a_0 (\cos^2\phi - \sin^2\phi\,\tan^2\beta)^{3/2}}$$
$$\times \int_{Z=H-T}^{H} \int_{a_{\min}}^{a_{\max}} \frac{h^2\left\{1 + \frac{\triangle Z}{h} + \frac{1}{3}\left(\frac{\triangle Z}{h}\right)^2\right\}}{l_1^2\, l_2^2}$$
$$\times F_J(\Theta, m, a, \lambda)\exp[-\kappa(\lambda)(l_{1N} + l_{2N})]$$
$$\times \exp[-5(a/a_0)^3]\, da dZ \tag{12}$$

where

N_0 = number density of grains,

$I_s^*(\lambda)$ = surface light intensity of the star,

r = radius of the star,

λ = wavelength of light,

a_0 = parameter in the size distribution function,

$F_J(\Theta, m, a\lambda)$ = the intensity scattering phase functions corresponding to two orthogonal states of polarization, $J = 1$ or 2,

$\kappa(\lambda)$ = the extinction coefficient within the nebula,

l_{1N} = the portion of the ray trajectory l_1 intercepted by the nebula,

l_{2N} = the same for l_2.

a_{min}, a_{max} = minimum and maximum sizes of the grains,

and

C = dimensionless constant derived in the next section.

I_1 and I_2 represent the components of the incident radiation with the electric vectors (polarization direction) perpendicular and parallel, respectively, to the plane of scattering. F_1 and F_2 are the corresponding intensity scattering phase functions. Further,

$$F_J(\Theta, m, a, \lambda) = |S_J(\Theta, m, a, \lambda)|^2 \tag{13}$$

Where S_J $(J = 1, 2)$ are the scattering amplitude functions expressed in terms of Mie coefficients (*see* van de Hulst 1957). The mean intensity without regard to state of polarization is obtained from

$$I = (I_1 + I_2)/2.$$

2.3 *Size distribution function*

The size distribution function of the interstellar grains included in the previous section has the differential form

$$\mathrm{d}n(a) = \frac{CN_0}{a_0} \exp[-5(a/a_0)^3]\,\mathrm{d}a \tag{14}$$

The normalization condition is given by

$$\int_0^\infty \mathrm{d}n(a) = N_0 \tag{15}$$

The dimensionless constant, C, in this particular case is found from the following equation

$$C = a_0 \left[\int_0^\infty \exp\{-5(a/a_0)^3\}\,\mathrm{d}a\right]^{-1} \tag{16}$$

which leads to

$$C = 5^{\frac{1}{3}}/\Gamma(4/3) \simeq 1{\cdot}914$$

The above size distribution function has the same exponential factor as that given by Greenberg (1968). The coefficient containing a_0 makes the difference. This has an additional effect that the relative number distribution of the grains for varying size parameter will be scaled by a factor a_0. This point has been neglected in the past by most workers.

2.4 *Definitions of colours and polarization*

The flux of the illuminating star received by the observer without intervening obscuration in the nebula is denoted by $I^*(\lambda)$.

The stellar radiation in the U, B and V wavelength bands taking into account the extinction in the nebula can then be expressed as

$$I_J^*(\mathrm{X}) = \int_{\lambda_1}^{\lambda_2} I_J^*(\lambda)\, Q_{\mathrm{X}}(\lambda) \exp\{-\tau(\lambda)\}\,\mathrm{d}\lambda \tag{17}$$

where X = U, B or V,

$Q_x(\lambda)$ = the filter transmission function for colour band X.

$\tau(\lambda)$ = total optical depth for the portion of the nebula between the star and the observer; and

λ_1, λ_2 = the lower and the upper wavelength limits for particular colour filter.

The nebular intensity integrated within a colour band can be expressed by

$$I_J^N(\phi, X) = \int_{\lambda_1}^{\lambda_2} I_J^N(\phi, \lambda)\, Q_x(\lambda)\, d\lambda \tag{18}$$

Now (B–V) or (U–B) colour differences between the star and the nebula for an offset angle ϕ can be expressed in the following manner

$$(B - V)_{*-N} = 2\cdot 5 \log \left[\frac{I^*(V)\, I^N(\phi, B)}{I^*(B)\, I^N(\phi, V)}\right] \tag{19}$$

$$(U - B)_{*-N} = 2\cdot 5 \log \left[\frac{I^*(B)\, I^N(\phi, U)}{I^*(U)\, I^N(\phi, B)}\right] \tag{20}$$

The degree of polarization (p) as function of offset angle is given by

$$p_X(\phi) = \left[\frac{I_1^N(\phi, X) - I_2^N(\phi, X)}{I_1^N(\phi, X) - I_2^N(\phi, X)}\right] \tag{21}$$

with X = U, B or V. To obtain per cent polarization, one has to multiply this expression by 100.

3. Computed colours and polarization and discussion

The geometrical parameters of the model nebula have been chosen as follows:

$T = 1\cdot 0$ pc

$D = 160$ pcs

$H = 1\cdot 25$ pc

$\beta = 0\cdot 0015$ radian.

These values are held constant throughout the present work. The empirical values of extinction within the nebula, based on observations by Boggess and Borgman (1964) are set at $A_U = 1\cdot 86$, $A_B = 1\cdot 66$ and $A_V = 1\cdot 2$ mag/pc for U, B and V filter bands, respectively. Although nebular intensity has been integrated with respect to wavelength, the extinction within each band is held constant. This is justified because of small variation in λ^{-1} as well as extinction coefficient within each band. Besides, calculations on some models, using the empirical extinction curve, agree well with the corresponding results in the present scheme.

The choice of the size parameter (a_0) is somewhat arbitrary. However, $a_0 = 0\cdot 5\,\mu$ for pure dielectric grains and $a_0 = 0\cdot 2\,\mu$ for pure silicate and graphite are likely to be nearer to the actual cases of the theoretical fit to the average interstellar extinction curve. In general, one needs to reduce a_0 as one goes to higher values of index of refraction to satisfy the observational criteria including interstellar extinction and polarization. The peculiar features in the observations such as kinks and hump in the extinction curve, infrared absorption bands, etc., should be kept in mind in more refined treatment to pinpoint the right kind of materials and the sizes of the grains. In choosing the materials and sizes, we have been guided by some current ideas on the interstellar grains in the literature. Although the shape effect can be an important factor, we shall restrict the study to smooth spherical particles in order to understand the effects of other physical parameters such as the real and imaginary parts of the index of refraction and the size distribution of the interstellar grains.

The filter transmissivity functions $Q_x(\lambda)$ used in the calculations of colours and polarization are identical both for theoretical and observational treatments. For simplicity, we have assumed that the star and the nebula have nearly similar spectral intensity distributions. This enables one to use the same effective monochromatic wavelengths in either of the two cases of the star and the nebula. The functions $Q_x(\lambda)$ for the colour filters X = U, B and V and other data are adopted from Allen (1973) and Greenberg and Roark (1967). We have used observations of Merope Nebula in the Pleiades clustar. This nebula is illuminated by Merope, a star of spectral type B6 IV with $B - V = -0{\cdot}06$. The data on $I^*(\lambda)$ are based on observations of α Eri and HD 188350 (Willstrop 1965). The observations pertaining to colour difference and polarization of Merope Nebula have been taken from Elvius and Hall (1967) and Roark (1966). Elvius and Hall have described the following conspicuous features of this nebula: (*a*) Filamentary structure of the nebulocity, (*b*) Polarization vectors of small magnitude in the vicinity of the star occur perpendicular to the radius vector from the star to the nebular element, (*c*) As one moves farther out in the south, this correlation disappears but curiously, polarization vectors increase in magnitude and tend to become parallel to the filaments implying that there is some sort of magnetic alignment of non-spherical or anisotropic grains, (*d*) In the vicinity of the star, the nebula is bluer; far out it is relatively redder.

Barring some irregular appearance, the nebulosity surrounding Merope is much less complex compared to other reflection nebulae. Besides, relatively high brightness, larger angular extent and known distance make this object interesting for representative study.

In what follows, all the results are for the case of star behind the nebula. The abscissa represent offset angle in minutes of arc in all figures. The dashed curves correspond to a smooth run through appropriate observations on colours or polarization. As far as we know, no observations of U-band polarization as a function of offset angle have been reported so far. However, we have also included the theoretical results on U-band polarization for future comparison. The index of refraction is denoted by $m = m' - im''$.

The numerical techniques and the Fortran Computer program for Mie theory of scattering by smooth spheres have been adopted from Shah (1975).

The colour differences $(B\text{-}V)_* - (B\text{-}V)_N$ (hereafter abbreviated as $(B\text{-}V)_{*-N}$ with similar notation for U and B colours) are plotted in figure 2 *a* for non-absorbing materials except for curve 5. A positive value of $(B\text{-}V)_{*-N}$ implies that the nebula is bluer compared to the star. The indices of refraction for curves 1 through 5 correspond to dielectric material, fused silica, silicate, silicon monoxide and

Table 1. Representative indices of refraction

Material	m'	m''	References
Pure dielectric ice ..	1·3	0	Irvine and Pollack (1968)
	1·33	0	Greenberg and Shah (1966)
Dirty Ice ..	1·3	0·02	Greenberg and Shah (1971)
	1·33	0·05	Shah (1972)
Fused Silica SiO_2 ..	1·46	0	HCP[a]
Enstatite silicate (Mg, Fe) SiO_3	1·66	0	HCP[a]
			Huffman and Stapp (1971)
Silicon monoxide SiO ..	1·7821	0	Hacskaylo (1964)
Obsidian silicate ..	1·48	2×10^{-4}	Pollack *et al* (1971)
			Bromage *et al* (1971)

[a] *Handbook of Chemistry and Physics 1956* (Cleveland: Chemical Rubber Co.).
Note:—The imaginary part (m″) has been varied in the range 0 (0·05) 0·1 for dielectric, fused silica, enstatite silicate and silicon monoxide.

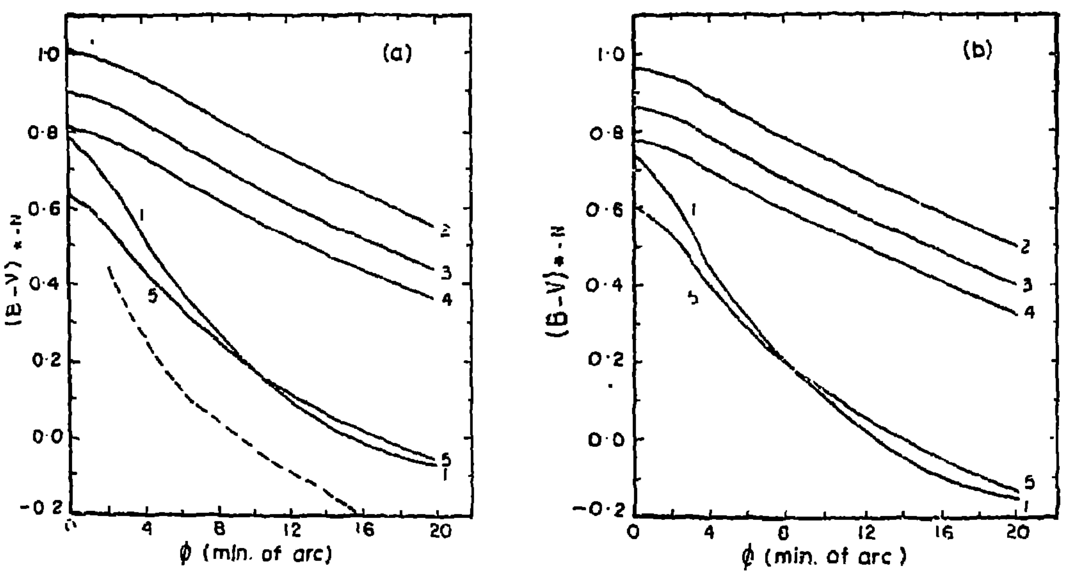


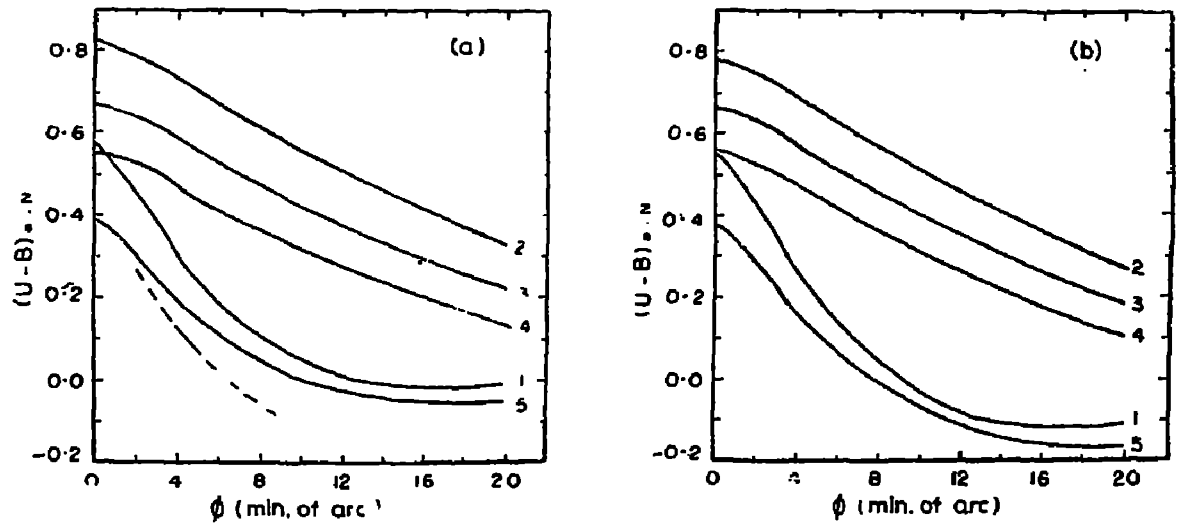
Figure 2*a*. (B–V) colour difference between star and nebula for the case SBN (Star behind the nebula). Imaginary part (m'') of index of refraction is zero or nearly so. Size distribution parameter (a_0) and real part (m') of index of refraction in respective orders are 0·5 μ and 1·3 for curve 1; 0·2 μ and 1·46 for curve 2; 0·2 μ and 1·66 for curve 3; 0·3 μ and 1·7821 for curve 4; and 0·4 μ and 1·48 for curve 5. The dashed curve corresponds to a smooth curve through observations by Greenberg and Roark (1967) and Elvius and Hall (1967).

Figure 2*b*. Same as in figure 2*a* but imaginary part (m'') of refractive index is 0·05 for all curves.

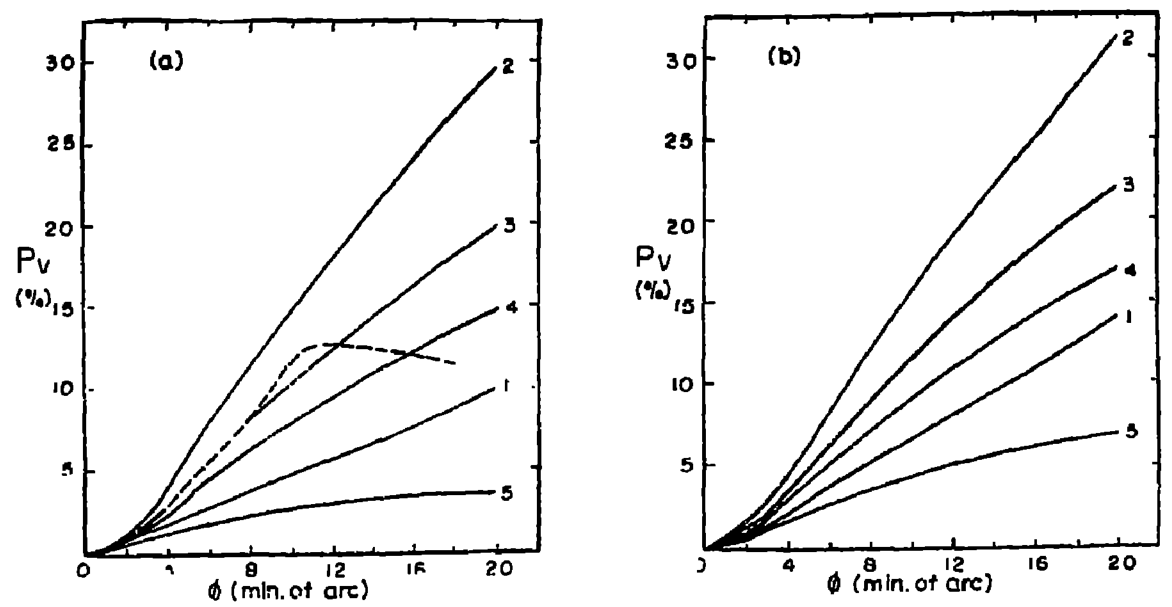
obsidian, respectively. The choice of indices of refraction for these and other materials are based on those given in table 1. Here one can see the effect of varying the real part of the index of refraction. The set of curves of figure 2 *b* has imaginary part $m'' = 0{\cdot}05$, the real parts m' being as given in figure 2 *a*. Similarly figures 3 *a* and 3 *b* display the $(U{-}B)_{*-N}$ colours.

The colour differences $(B{-}V)_{*-N}$ and $(U{-}B)_{*-N}$ both drop off monotonically as the offset angle (ϕ) increases. The higher the real part of refractive index the smaller the value of colours for a given size parameter (a_0) and offset angle (ϕ) as can be seen by comparison of curves 2, 3 and 4 in figures 2 *a*, *b* and 3 *a*, *b*. Addition of small imaginary part has the effect of slightly lowering the curve. This can be seen by comparison of curves 2, 3 or 4 in figures 2 *a* and 2 *b* or in figures 3 *a* and 3 *b*. The increase in a_0 from $0{\cdot}2\mu$ to $0{\cdot}4\mu$ (*see* curves 2 and 5 in figures 2 *a*, 2 *b*, 3 *a* or 3 *b*) dramatically reduces the colours for all offset angles.

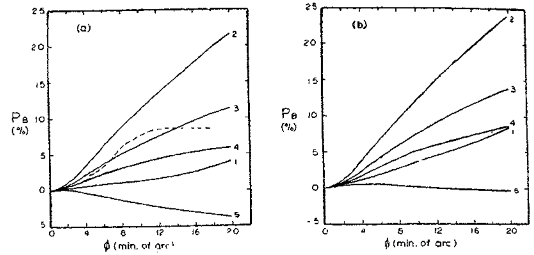
For all indices of refraction and sizes used in figures 2 *a*, *b* and 3 *a*, *b* the nebula is bluer than the star except for curves 1 and 5 for which the colour differences change sign near $\phi = 16'$ in the case of $(B - V)_{*-N}$ and near $\phi = 10'$ (curve 5)



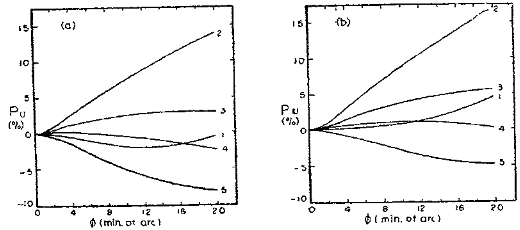
Figures 3 *a*, *b*. Same as in figures 2 *a* and *b*, respectively, but ordinates represent (U–B) colour difference.



Figures 4 *a*, *b*. Same as in figures 2 *a* and *b*, respectively, except that ordinates show polarization in V colour band,



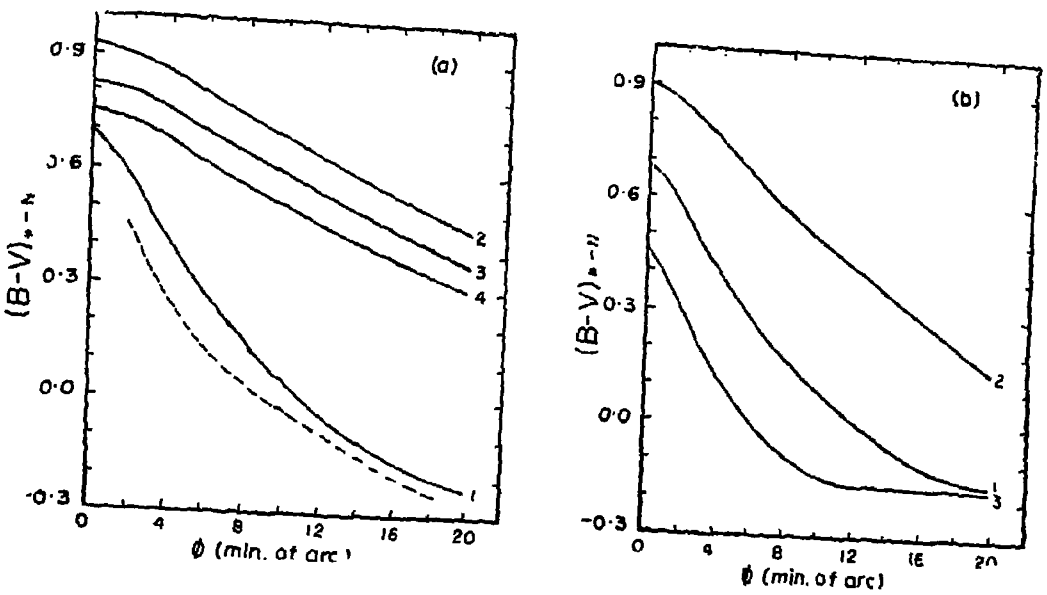
Figures 5 *a*, *b*. Same as in figures 3 *a* and *b*, respectively, but for polarization in B colour band.



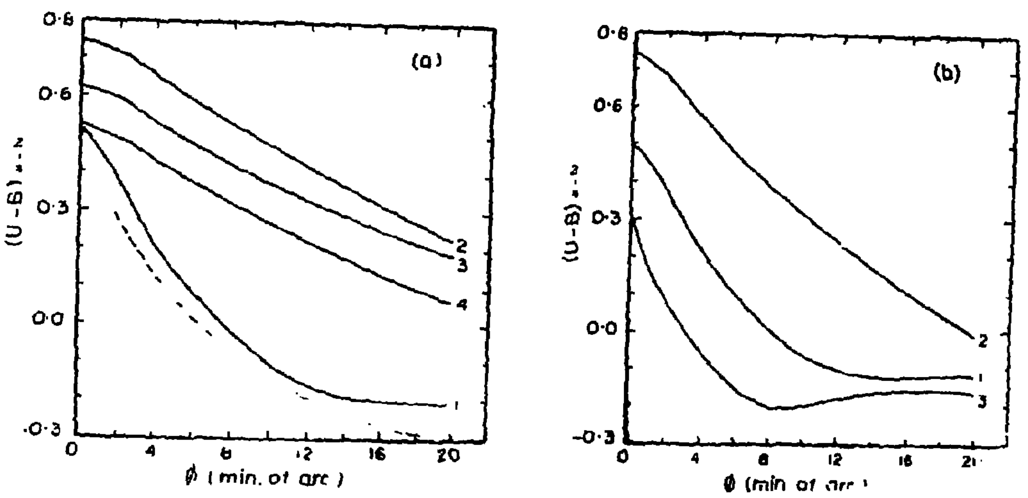
Figures 6 *a*, *b*. Same as in figures 3 *a* and *b*, respectively, but for polarization in U colour band.

or 12′ (curve 1) in the case of $(U-B)_{*-N}$. However, observations show that change of sign should occur at $\phi \simeq 9'$ and 6′ for $(B\text{-}V)_{*-N}$ and $(U\text{-}B)_{*-N}$, respectively. The curves 1 and 5 for dielectric and obsidian grains, respectively, in figures 2 *a*, *b* and 3 *a*, *b*, approach nearer to observations compared to other materials.

In figures 4 *a*, *b*, 5 *a*, *b* and 6 *a*, *b* are shown the polarization in V, B and U bands, respectively. It may be noted that the polarization at $\phi = 0°$ is zero for all m and a_0 because we are considering single scattering of homogeneous smooth spheres. This is evident if one recalls the fact that the forward scattering amplitude functions are identical for scattering angle equal to 0°. For small offset angle in the range $0° < \phi \lesssim 6'$ the observed polarization in V and B bands match approximately with the curves labelled 3 which are for silicate particles. For $\phi \gtrsim 6'$, it is not possible to fit observed polarization with any particular curve. This is contradictory to the case of colour differences where we found that dielectric and obsidian grains are hopeful candidates. Therefore, further experiment on effect of varying m and/or a_0 was necessary to see if the best fits to colour differences as well as polarization could be obtained simultaneously by choosing the same set of model parameters.



Figures 7 *a*, *b*. (B–V) colour difference between star and nebula for the case SBN (*a*) Imaginary part (m'') of index of refraction is 0·1 for all curves. The size parameters (a_0) and real part (m') of index of refraction, in respective orders, are 0·5 μ and 1·3 for curve 1; 0·2 μ and 1·46 for curve 2; 0·2 μ and 1·66 for curve 3; and 0·2 μ and 1·7821 for curve 4. (*b*) Effect of varying size distribution parameter (a_0); $m = 1{\cdot}33 - 0{\cdot}05\,i$ for all curves. The size parameter (a_0) is equal to 0·3, 0·5 and 0·7 μ for curves 2, 1 and 3, respectively.



Figures 8 *a*, *b*. Same as in figures 7*a* and *b*, respectively, except that the ordinates represent (U–B) colour differences between star and nebula.

The set of curves in figures 7 *a*, 8 *a*, 9 *a*, 10 *a* and 11 *a* represent colours and polarization for a further increase in the imaginary part of the index of refraction to $m'' = 0{\cdot}1$ for all curves. The general trends of the variation in colours or polarization are similar to the cases of $m'' = 0$ and $m'' = 0{\cdot}05$ discussed earlier. But now referring to curve 1 for $m = 1{\cdot}3 - 0{\cdot}1\,i$ and $a_0 = 0{\cdot}5\,\mu$ in figures 7 *a* and 8 *a*, it is seen that calculated $(B–V)_{*-N}$ and $(U–B)_{*-N}$ colours are reasonably close to observations (dashed curves). Further, the polarization in the visual and blue wavelength bands as given by curves 1 in figures 9 *a* and 10 *a*, respec-

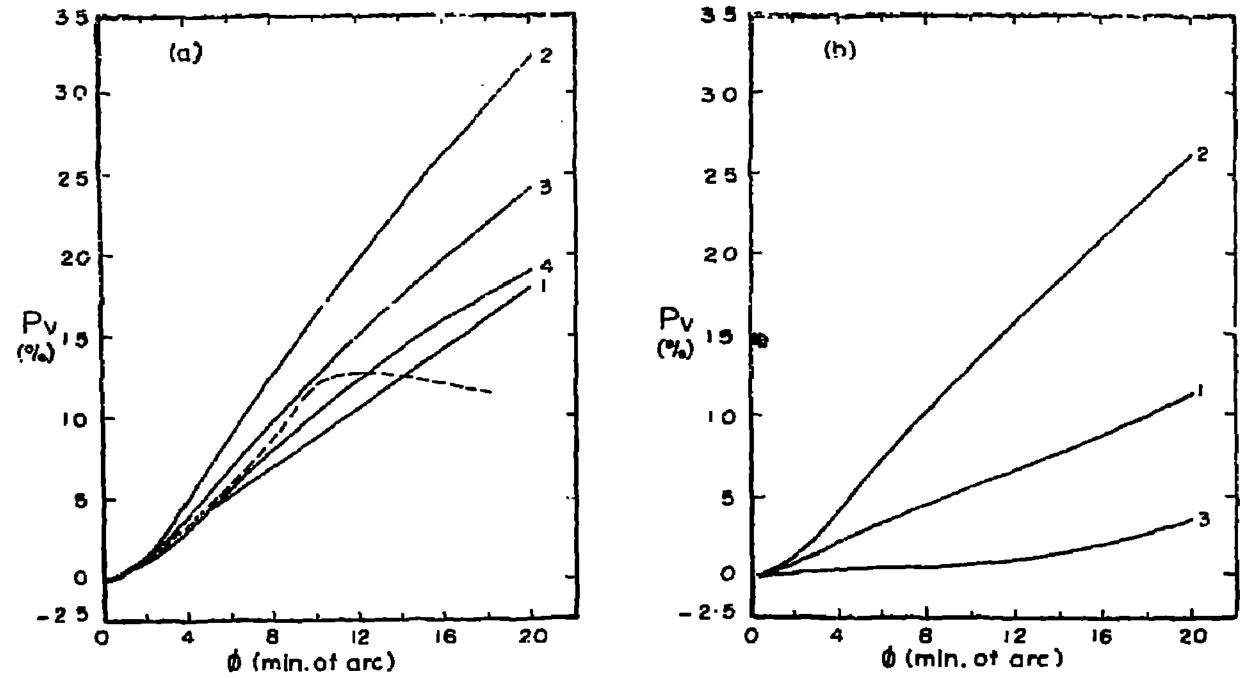
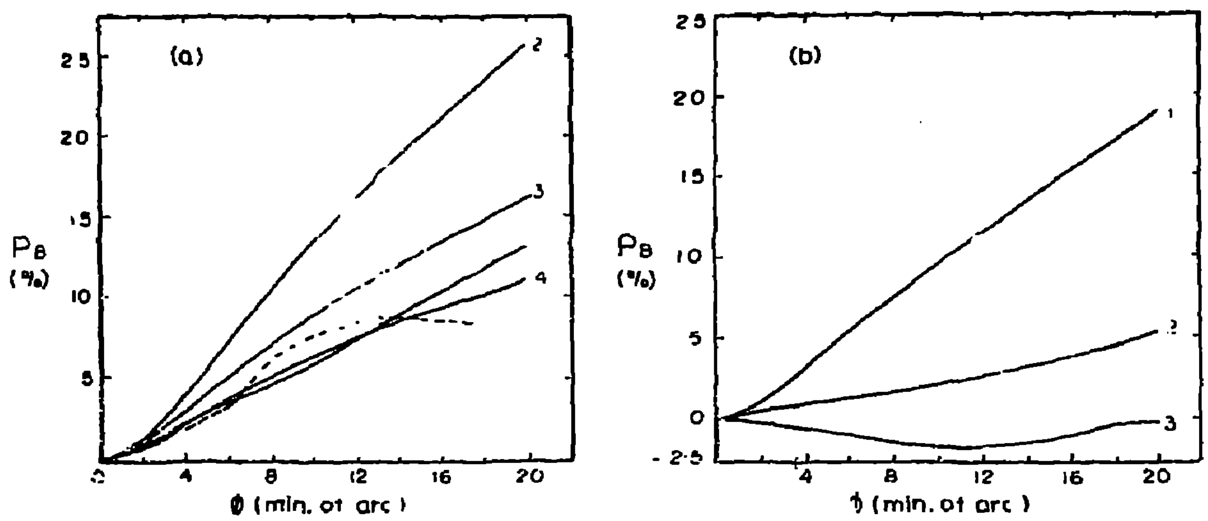
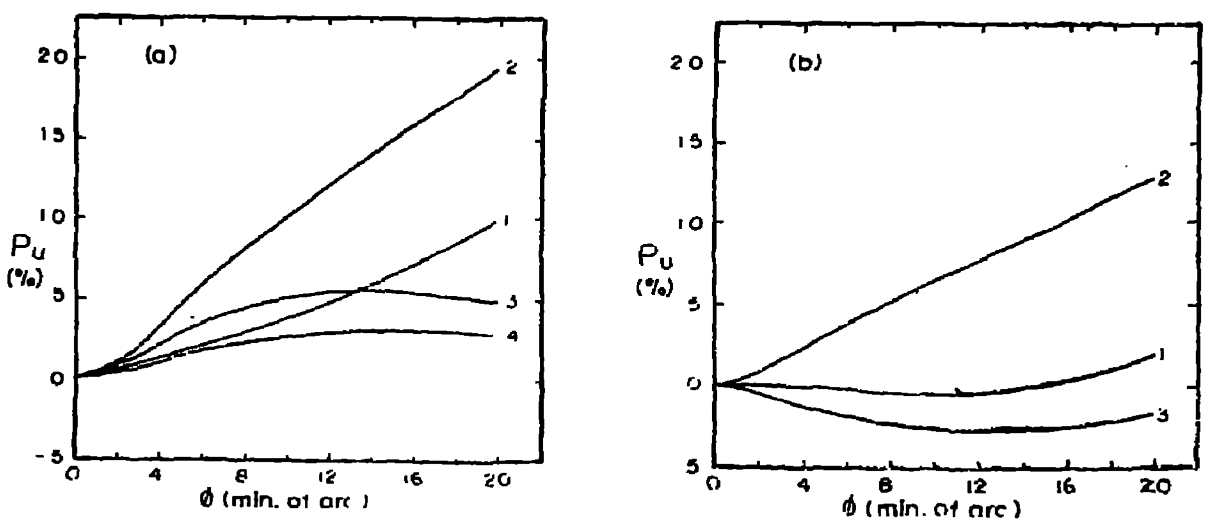


Figure 9 *a*. Polarization (P_V) in the visual wavelength band. Refractive index (m) and size parameter (a_0) are as in figure 7 *a*.
Figure 9*b*. Same as in figure 7*b* except for polarization in the visual.



Figures 10 *a*, *b*. Same as in figures 9 *a* and *b*, respectively, except that ordinates represent polarization in the B colour band.



Figures 11 *a*, *b*. Same as in figures 9 *a* and *b*, respectively, except that polarization is in U colour band.

tively, agree reasonably well for $\phi \lesssim 6'$. Contrary to Greenberg and Hanner (1970), it may be noted that the value of $a_0 = 0 \cdot 5\,\mu$ is still the same as in the case of colour differences. For $\phi \gtrsim 6'$, the matching of polarization is difficult. This question will be examined further in other models to be constructed with star within the nebula and with star in front of the nebula.

The effect of varying the size distribution parameter a_0 has been brought out in figures 7 *b*, 8 *b*, 9 *b*, 10 *b* and 11 *b* for dielectric dirty ice grains with $m = 1 \cdot 33 - 0 \cdot 05\,i$. As a_0 increases from $0 \cdot 3\,\mu$ to $0 \cdot 7\,\mu$, the $(B–V)_{*-N}$ and $(U–B)_{*-N}$ colour differences as well as polarization in U, B and V bands decrease for a given offset angle. Note that the polarization in U and B bands in certain range of offset angles has negative values for very large size of the grains given by $a_0 = 0 \cdot 7\,\mu$. P_v is negative for a certain range of ϕ in the case of $a_0 = 0 \cdot 5\,\mu$ also.

At this stage, an important result may be mentioned. Zappala (1973) has observed that the stars in the Pleiades cluster exhibit colour excess in V – [$2 \cdot 2\,\mu$] independent of interstellar reddening. It has been suggested that the comparatively young stars in Pleiades are still surrounded by circumstellar materials of low temperature ($\simeq$ 500 K) and moderate optical depth. Therefore, it may become necessary to examine the question of thermal emission by circumstellar grains around Merope and its role in modifying the colours and polarization of radiation from the reflection nebula.

The fact that the calculated polarization is consistently higher than indicated by observations for large offset angles may be explained by the following argument: If certain component (or fraction) of the grains, producing zero or nearly null polarization, exists within the nebula, it is likely that the degree of polarization may be considerably lowered especially for large offset angles. For instance, this can happen with very large particles. Suppose that the intensities, referred to two reference orthogonal planes, received by the observer are $I_1 + I_1'$ and $I_2 + I_2'$ due to contributions from two types of grains with average characteristic sizes $\bar{a}_1$ and $\bar{a}_2$ such that $\bar{a}_2 > \bar{a}_1$. The primed quantities are related to $\bar{a}_2$. If $\bar{a}_2$ is sufficiently large, one would expect $I_1' \simeq I_2'$. The degree of polarization is then given by $(I_1 - I_2)/(I_1 + I_2 + I')$, where $I' = I_1' + I_2'$. It is evident that the polarization will be over estimated due to neglect of I'. Accordingly, the difference from the observed polarization will be accentuated with increase in the offset angle. This is what is happening actually in the comparison of the calculated model polarization with the corresponding observations. Therefore, a bimodel size distribution possibly along with a mixture of particles deserves consideration in the theoretical models before banking on more complicated models including multiple scattering, shape effect and orientation of the grains.

4. Qualitative summary

Some systematic trends regarding the variation of nebular colours and polarization with changes in m', m'' and a_0, emerging from the present work, have been summarized in table 2. The colours and polarization produced by the size distribution of silicate and graphite grains will be reported elsewhere. However, they have been considered in preparing table 2. It is assumed that only one of the three parameters is changed at a time, the other two being held constant. The increase in m', m'' or a_0 has the uniform qualitative effect of decreasing $(B–V)_{*-N}$ and

Table 2 General trends of nebular colours and polarization with the variation of m', m'' and a_0 for the case of the star behind the reflection nebula.

Calculated quantity	Effect of increase in m'	Effect of increase in m''	Effect of increase in a_0
$(B-V)_{*-N}$	↓	↓	↓
$(U-B)_{*-N}$	↓	↓	↓
P_V ..	↓	↑	↓ **
P_B ..	↓	↑	↓ **
P_U ..	↓	↑	↓ **
	↑ = Increase	↓ = Decrease	

** Graphite is exception in that the curves show opposite trends on the two sides of cross-over point which depends upon a_0 and colour band among other things.

$(U-B)_{*-N}$ colour differences. When m' increases, the polarization in U, B or V band goes down. However, the increase in m'' has the opposite effect on polarization in U, B or V band. The effect of varying a_0 on polarization is similar to that for m' except that the curves for graphite grains show opposite trends on the two sides of the cross-over point. The location of the cross-over point varies with the colour bands and a_0. It is hoped that the summary in table 2 will serve as a useful guide for further study of reflection nebulae with star behind them.

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