**Short Research Communications :**

# A NEW METHOD OF IMAGE RESTORATION

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The image r(x) of an object distribution q(x) is given by the convolution equation

$$r(x) = \int_{-\infty}^{\infty} p(x-y)q(y)dy,$$

where p(x) is the known response of the imaging system to a point object. This equation occurs frequently in many branches of physics and astronomy. One has to restore q(x) from a set of noisy data $r(x_m)$, m=1, 2......N. The restoration is complicated because the solution is not unique and is also sensitive to the noise present in the data. From the Fourier transform of the equation (R (u)=P(u) Q(u), where capital letters denote Fourier transform), it can be seen that the equation contains no information on Q(u), in the region where P(u) vanishes, as happens for all apertures of finite size beyond a critical frequency or for missing frequencies in the case of interferometers. Conventional methods assume Q(u) also to be zero when P(u) vanishes to arrive at the " smoothest " possible solution agreeing with the data (the " principal solution " introduced in Bracewell and Roberts 1954). This has two limitations : (a) the resolution is limited as information is suppressed beyond the critical frequency ; and (b) the sharp cut-off in P(u) introduces spurious negative values (sidelobes) in the restored distribution. The latter, in fact, is a violation of our prior knowledge that q(x), being usually an intensity distribution across the object, is always positive. Improvement of the solution is possible only by considering information not present in the original equation, e.g. extrapolation of Q(u) beyond the critical frequency. Positivenes of q is an important prior information and extrapolation incorporating this feature is known to give an improved resolution (e.g. Biraud 1969). Moreover, the noise in the data is stochastic in nature and hence should be treated according to statistical principles (e.g. Frieden 1972). These methods, however, involve nonlinear equations whose convergence is difficult for noisy data.

We report here a relatively simple and more general method, which gives a solution consistent with out prior knowledge, particularly the positiveness, and follows a statistical treatment (least squares method) for noise. We give here a brief summary of the method and apply it for restoring brightness distribution of a radio source from lunar occultation observations. Our results show considerable improvement over a conventional method (Scheuer 1962). Details will be presented elsewhere. For simplicity, one-dimensional problem has been considered throughout.

By choice of a suitable integration formula, the integral equation is reduced to a system of algebraic equations with integration being replaced by a summation. There will thus be N linear equations in K unknowns $q_i$, i=1, 2,.........K, where $q_i$ are the values of the object function sampled at discrete points. These will be an ill-conditioned set of equations whose realistic solutions can be obtained only by imposing smoothness of q explicitly as a constraint (Phillips 1962). Preferably, K is chosen to be less than N so that we have an over-determined system of equations. The initial solution for our method is obtained by minimising a linear combination of two variances—the variance of residuals (difference between computed and observed data) and that of the second differences $\Delta^2 q_i$ of the solution (Phillips 1962 ; Twomey 1963). The solution, in the presence of noise, still retains a large number of unphysical negative values and hence we optimise it by imposing positiveness as another constraint. For this, we minimize a linear combination of the variances mentioned above and a weighted mean of the squares of those $q_i$ which were negative in the initial solution. The weights are proportional to the square of the corresponding $q_i$ from the initial solution. This needs changing only the diagonal elements of the normal equation matrix. The solution thus obtained is improved in the next iteration in a similar way.

This simple procedure, involving only linear equations to solve, was found to be quite efficient in enforcing positiveness on the solution. The convergence is rapid, needing only two or three iterations for most cases. The resolution is significantly improved over the initial solution.

As a test of the merit of this method, we have analysed computer simulations of lunar occultation of a radio source at 327 MHz (operating frequency of the Ooty radio telescope). In this case, p(x) is the Fresnel diffraction pattern of a point source due to a straight edge. The source was chosen to have a Gaussian distribution (amplitude=2.5, and half-power width=1 arc sec), and noise was superposed on the data by taking random numbers distributed normally with unit rms. A total of 32 statistically independent simulations so obtained were analysed with the conventional Scheuer's method (Scheuer 1962) and also with the Optimum Deconvolution Method (ODM), described above. The results are summarized in Table 1. We have also tabulated, for comparison, the expected uncertainties in the source parameters

**Table 1**

**Analysis of 32 independent noisy occultation data**

| Parameter | True value | ODM | | Scheuer's method** | | |
|---|---|---|---|---|---|---|
| | | Value | rms | Value | rms | expected rms |
| Size (arc sec) ... | 1.00 | 1.08 | 0.38 | 0.98 | 0.87 | 0.86 |
| Flux ... | 2.65 | 2.45 | 0.32 | 2.96 | 0.40 | 0.34 |
| Position ... | 0.00 | 0.02 | 0.26 | -0.01 | 0.22 | 0.24 |

** Size in Scheuer's method is obtained as $\sqrt{\beta^2 - \beta^2_r}$, where $\beta_r$ = 2 arc sec is the resolution employed, and $\beta = 2''.23 \pm 0''.51$ is the value obtained for the mean total size in our experiments.

obtained in Scheuer's method. These are calculated according to formulae given in von Hoerner (1964), where our data correspond to a $q_o$=5.3, $q_o$ being the signal-to-noise ratio when the data are averaged over a block of 1 arc sec. Our results for Scheuer's method agree with the formulae for rms errors given by von Hoerner.

Table 1 shows that the acuracy of angular size obtained by our method is markedly superior to that given by the conventional method. This has been achieved without losing accuracy in any other parameter. For a $q_o = 5.3$, we have restored a size of 1 arc sec with an rms error of 35 per cent of the mean, whereas the minimum size for which accuracy is possible in Scheuer's method is about 2 arc sec (according to von Hoerner's equations). There is thus a gain in resolution by almost a factor of two for $q_o$=5. In addition, since it is implicitly assumed in our method that all the noise is present in the data and not in the solution, the restored output is contaminated by noise to a less extent than in conventional methods. This may lead to a more objective interpretation in solutions heavily limited by noise.

The above example from lunar occultation is only illustrative; the method is applicable to any convolution problem. The results may be improved somewhat by using statistical criteria superior to the least squares method. However, the present method has the advantage that all the equations to be solved are linear.

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