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Optical colours and polarization of a model reflection nebula. III. Composite and mixture of grains in the nebula with the star in the rear

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Abstract. The colour differences between the star and the associated reflection nebula and polarization caused by core mantle grains and mixture of grains have been given. They are based on homogeneous plane-parallel slab-model of the nebula with the star in the rear. The composite particles in the form of concentric spheres consist of homogeneous core of graphite, silicate or SiC and homogeneous mantle of ice. The effect of varying the core and mantle radii has been studied. The mixtures of grains composed of ice, silicate, graphite and SiC in various proportions have also been considered. Each of these grain species has been considered with $\exp(-a^{3})$ type of size distribution function. The wavelength dependent indices of refraction have been used throughout.

Keywords. Reflection nebula; UBV colours and polarization; composite and mixture of grains.

1. Introduction

A reflection nebula is thought to be caused by the diffuse scattering of light from the associated star, which, in general, belongs to the spectral type B or later. The scattering is attributed to the presence of some kind of solid submicron particles referred to as interstellar grains. Several attempts (Greenberg and Roark 1967, Zellner 1974, Shah 1974, 1977) have been made in the past to identify these particles on the basis of observations and theoretical models. There is, however, no unique answer. It has not been possible to predict unambiguously even the observed range in colours and simultaneous polarization across a reflection nebula (cf. Aannestad and Purcell 1973).

Shah (1974, hereafter referred to as paper I) investigated the variation of nebular colours and polarization with changes in the real (m') and imaginary (m'') parts of index of refraction ($m = m' - im''$) and the size distribution parameter a_0. The independent scattering and monodispersion of each type of grains have been considered. The results for the case of star behind the nebula have shown certain systematic trends. The increase in m', m'' or a_0 has the uniform qualitative effect of decreasing the colour differences $(B-V)_{*-N}$ and $(U-B)_{*-N}$. When m' increases, the polarization in V, B or U band goes down. However, the increase in m'' shows the opposite trend. The effect of increasing a_0 on polarization is similar to m' except that the curves for graphite grains may show reverse variation below certain offset angle where the curves for different a_0 cross over.

The purpose of this paper is to report on nebular colours and polarization caused by composite grains consisting of homogeneous core made of graphite, enstatite silicate or silicon carbide and homogeneous mantle composed of ice. In another category of calculation, we have considered the mixture of grains composed of ice, graphite, silicate and silicon carbide taken two or three of them at a time in various proportions. The wavelength dependent refractive indices have been used throughout the present work. Note that composite and mixture of grains in reflection nebula have not been considered in the literature so far.

2. Theoretical consideration

We consider the reflection nebula in the form of homogeneous plane-parallel slab with the illuminating star behind the nebula. The analytical formulation has been adopted along the lines given by Greenberg and Roark (1967) as in paper I. The definitions for colours and polarization are also as given in these references. The exact electromagnetic scattering theories by Mie (cf. van de Hulst 1957) for homogeneous single spheres and by Güttler (1952) for concentric spheres have been used throughout. The material compositions of the grains are taken to be a combination of dirty ice, graphite, enstatite silicate and silicon carbide; the wavelength dependent indices of refraction for these materials have been chosen from Greenberg (1967), Philips and Taft (1962), Huffman and Stapp (1971) and Greenberg (1975), respectively. In the case of ice, however the imaginary part of the index of refraction in the visual wavelength is not accurately known. We have assumed it to be $m'' = 0{\cdot}02$ for $\lambda = 0{\cdot}3\ \mu m$ to $0{\cdot}66\ \mu m$.

The notation and relevant geometrical parameters are as follows:

SBN = the case of star behind the reflection nebula,
T = the thickness of the nebula = 1·0 pc,
D = the distance of the illuminating star from the observer = 160 pcs,
H = the perpendicular distance from the star to the front surface of the nebula = 1·25 pc,
ϕ = the offset angle, i.e. angle away from the direction of the observer to star,
β = the semi-vertex angle of the telescope = 0·0015 radian,
λ = the wavelength of light, in μm,
a_{oi} = parameter in the size distribution function for ith component of the mixture,
$n_i(o)$ = the number density of the grains per cm^3 for a given species of the mixture.

Subsequently, for brevity in the text, we shall denote a_{oi} and $n_i(o)$ by a_i and n_i, respectively. $P_X(\phi)$ = the degree of polarization as function of offset angle ϕ and for a given wavelength band $X(= U, B \text{ or } V)$. The colour difference is denoted by $(I-J)_{*-N}$ i.e. the nebular colour difference between bands I and J is subtracted from that of the star. The band I can be B (or U) corresponding to $J = V$ (or B). A minus value for the quantity $(B-V)_{*-N}$ indicates reddening i.e. the nebula is redder compared to the star and vice versa.

The size distribution function for each component in a mixture of grains in properly normalized form (Shah 1974) is given by $n_i(a) = \{Cn_i(o)/a_{oi}\}$ exp $[5(-a/a_{oi})^3]$

where a_{oi} is the size parameter for *i*th mixture component with grain number density $n_i(o)$, and C is the constant of normalization. The size parameter a_{oi} has been maintained uniform throughout for a given component. In particular, we have chosen $a_o = 0{\cdot}5$ μm for ice and 0.25 μm for graphite, silicate as well as silicon carbide. The proportion of grains for ice has been varied in the range $n = 0, 1, 2(2)10$ and for the rest of the materials $n_i = 0(2)10$. Intensities I_1 and I_2 for two orthogonal states of polarization for each grain component are calculated as function of λ and ϕ as in paper I. They are weighted by appropriate n and summed up for all components of the mixture to give the net average values $\langle I_1 \rangle$ and $\langle I_2 \rangle$. These intensities have been further used to compute the nebular colours and polarization for different bands. The extinction within the nebula has been considered on the basis of the observations by Boggess and Borgman (1964). The empirically determined values of extinction are chosen as $A_V = 1{\cdot}2$, $A_B = 1{\cdot}66$ and $A_U = 1{\cdot}86$ in the unit of mag/pc for V, B and U filter bands, respectively. The filter response functions have been adopted from Allen (1973).

Although the computational results given below are applicable in particular to Merope reflection nebula, they are of general interest in understanding the observations on other reflection nebulae and related theoretical models.

3. Results based on composite grains

We have chosen the core radii in the range $r_1 = 0{\cdot}04$ (0·04) 0·12 μm. The mantle radii are varied such that the ratio of mantle (V_m) to core (V_c) volumes is in the range $V_m/V_c = 0{\cdot}044$, 0·11 (0·11) 0·88. The mantle is composed of ice in all the calculations. For convenience of comparison, the observational colours and polarization for the Merope reflection nebula based on data by Elvius and Hall (1966, 1967) and Greenberg and Roark (1967) are summarized in table 1. The colours and polarization in table 1 should be treated as a rough guide for assessing the theoretical models because they vary from one nebular region to another.

Figure 1 shows the $(B-V)_{*-N}$ colour difference for representative cases. For $r_1 = 0{\cdot}04$ μm and 0·08 μm, $V_m/V_c = 0{\cdot}04$ to 0·88 graphite, silicate and SiC cores give much larger positive colours from offset angle $\phi = 0$ to 20 min. of arc. The negative values of $(B-V)_{*-N}$ at higher ϕ demanded by observations show up mostly

Table 1. Observed colours and polarization of Merope reflection nebula.

ϕ (min. of arc)	$(B-V)_{*-N}$	$(U-B)_{*-N}$	$P_V(\%)$	$P_B(\%)$
2	0·45	0·27	—	0·88
4	0·26	0·13	3·1	1·8
6	0·123	· 0·008	5·7	3·5
8	0·035	−0·065	8·4	6·2
10	−0·026	−0·144	12·0	7·5
12	−0·089	−0·2	12·8	8·4
14	−0·148	−0·240	12·4	8·8
16	−0·209	−0·27	12·0	8·4
18	−0·243	−0·29	11·5	8·4

Sources: Elvius and Hall (1966, 1967) and Greenberg and Roark (1967).

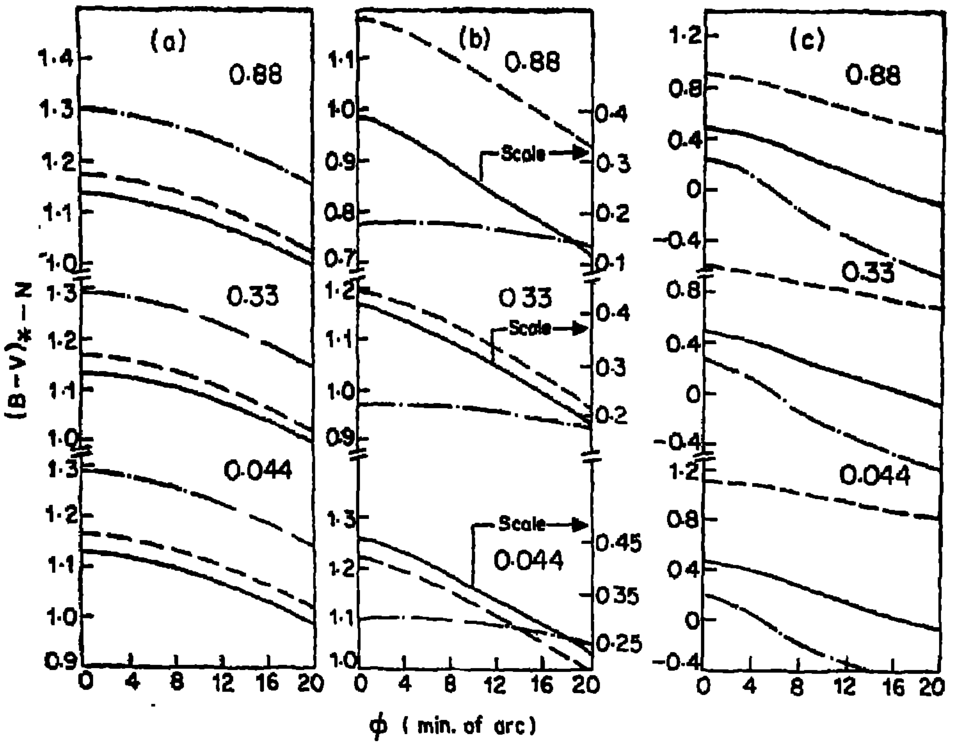

Figure 1 a,b,c. $(B-V)$ colour difference between star and the reflection nebula fo[r] the case of star behind the nebula (SBN) containing core-mantle composite spherical grains. The core radius is $r_1=0{\cdot}04 \mu m$, $0{\cdot}08 \mu m$ and $0{\cdot}12 \mu m$ in (a), (b) and (c) respectively. The number above a set of three curves gives the ratio of mantle-to-core volumes. Note that the scale for some of these curves is as indicated by arrow, i.e. on the right hand side of the figure. Mantle is ice in all the cases, and core is graphite ——; enstatite silicate — —; and silicon carbide — . —.

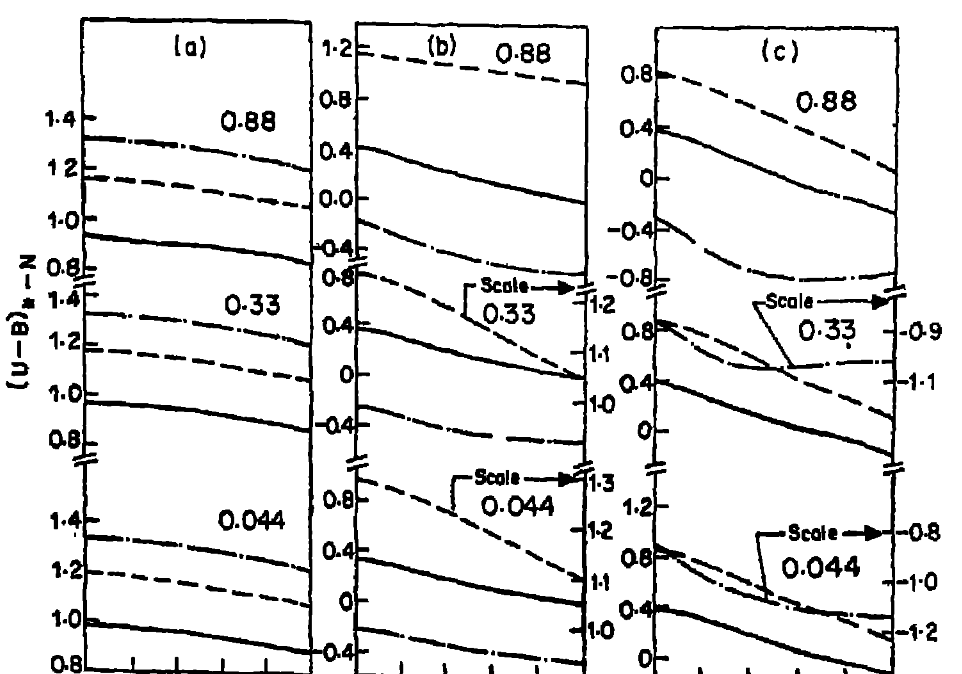

Figure 2 a,b,c. Same as figure 1 a,b,c but ordinate represents the colour difference $(U-B)_{*-N}$.

in the case of graphite and SiC cores with $r_1 = 0{\cdot}12$ μm. The composite grains having graphite core and ice mantle with $r_1 = 0{\cdot}12$ μm and $V_m/V_c \gtrsim 0{\cdot}3$ produce $(B-V)_{*-N}$ colour differences which are somewhat close to the observations. This

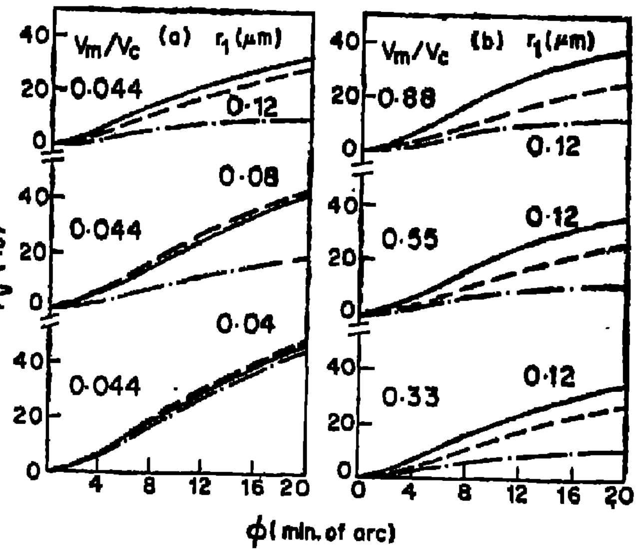

Figure 3 a,b. The polarization in V band for the case SBN containing core-mantle grains. The mantle is composed of ice. The core material is graphite (——), silicate (— —) and silicon carbide (—.—.). The numbers below r_1 and V_m/V_c represent core radii and mantle-to-core volume ratio, respectively, for each set.

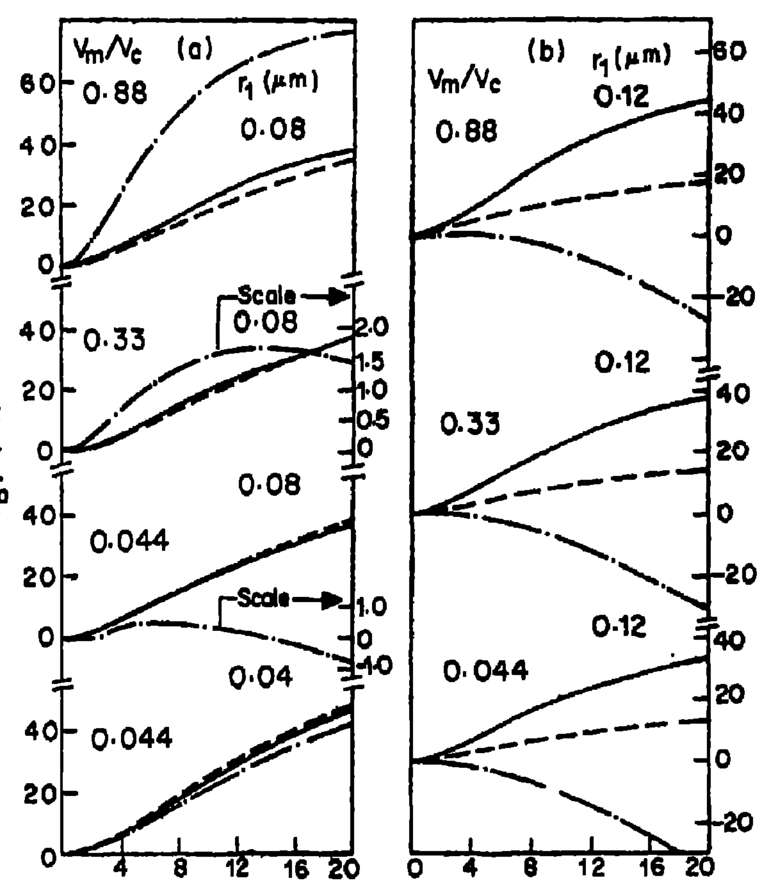

Figure 4 a,b. Same as figure 3 a, b except that polarization is in B band.

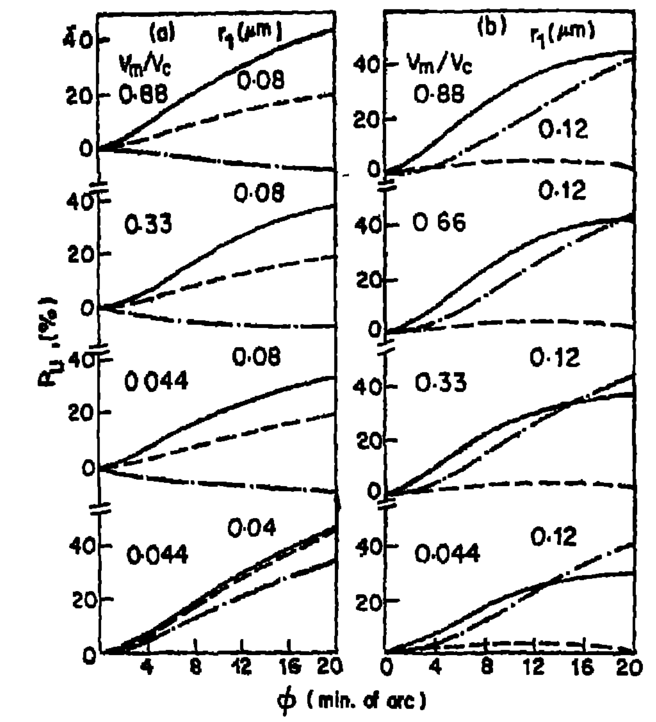

Figure 5 a,b. Same as figure 3a, b except for polarization in U band.

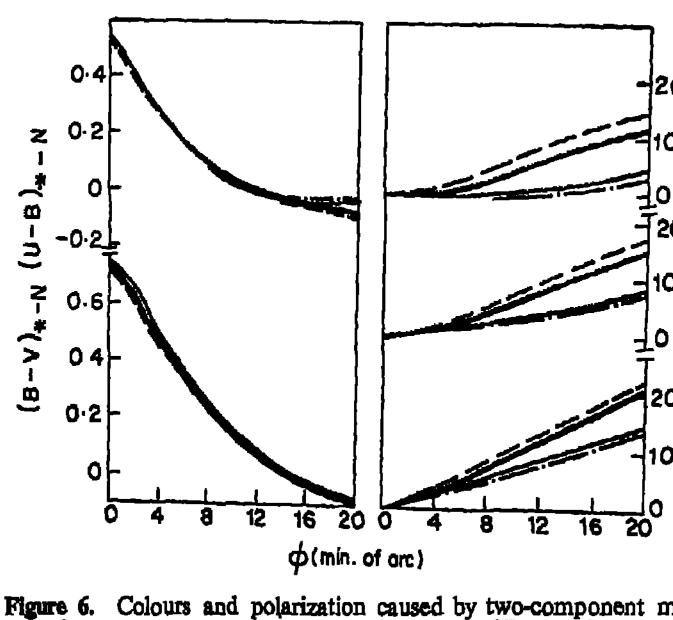

Figure 6. Colours and polarization caused by two-component mixture of ice and graphite grains in proportion $n_1 : n_2$ for the case SBN. The ratio $n_1 : n_2$ is 1 : 2 for ——, 2 : 1 for ——, 1 : 3 for — — 3 : 1 for — . —.

also holds for $(U-B)_{*-N}$ colours shown in figure 2. However, proper gradient of the curves and the range of colours are not adequate.

The general trends in the variation of polarization in the V, B and U bands are shown in figures 3, 4 and 5, respectively. For $r_1 = 0{\cdot}04$ μm and $V_m/V_c = 0{\cdot}044$, the polarization for a fixed offset angle ϕ is almost the same for graphite and silicate core materials in all the three bands. The deviation from SiC core increases as one goes from V to U band. . As r_1 increases up to $r_1 = 0{\cdot}12$ μm graphite and silicate

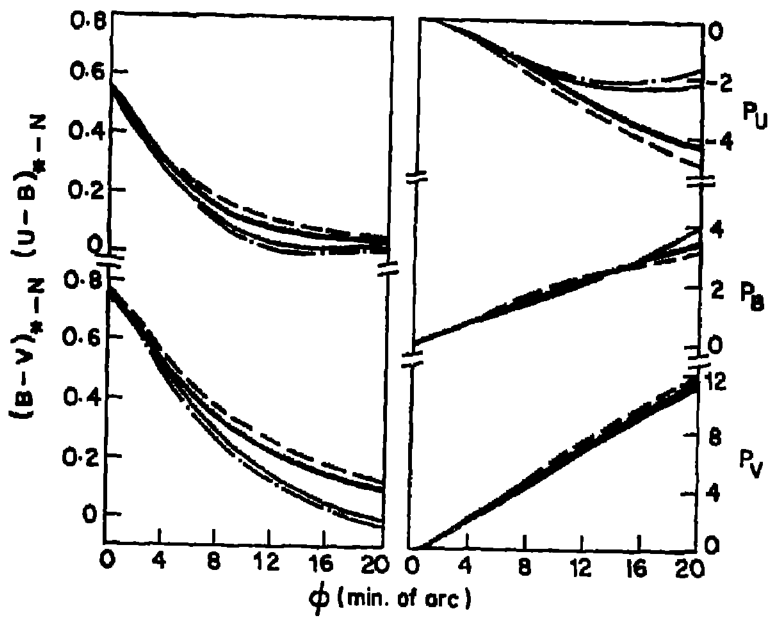

Figure 7. Same as figure 6 but with mixture of ice (n_1) and silicate (n_3) grains.

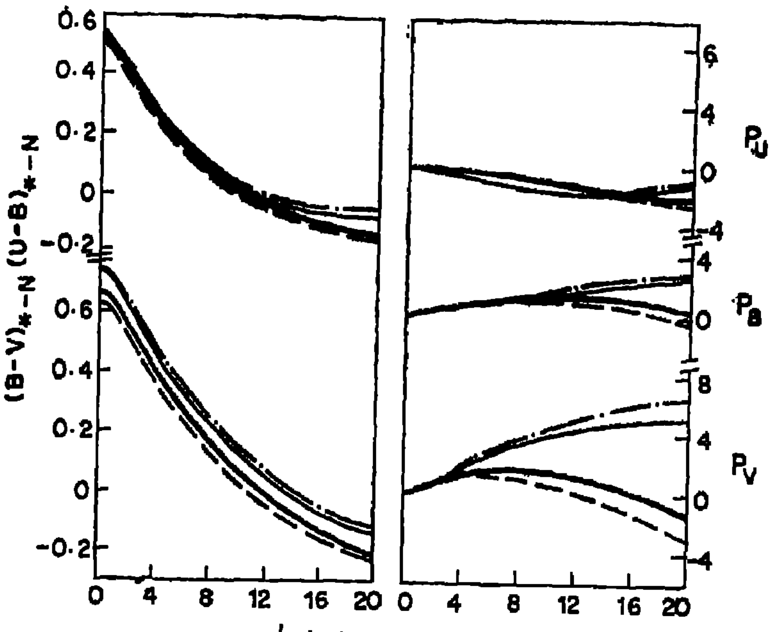

Figure 8. Same as figure 6 but with mixture of ice (n_1) and silicon carbide (n_4) grains.

cores still produce nearly the same polarization in B and V bands. The difference between graphite and silicate cores is brought out clearly in the case of U band especially when $V_m/V_c > 0{\cdot}044$ as shown in figure 5. SiC core with $r_1 = 0{\cdot}12$ μm in figure 4 produces undesirably high negative polarization in the B colour band. Note that the observations have positive polarization in both the bands, B and V.

4. Two-component mixture of grains

We have considered a two-component mixture of grains consisting of (i) ice and graphite, (ii) ice and silicate and (iii) ice and silicon carbide. The results for representative proportions $n_1:n_2 = 1:2$, 2:1, 1:3, and 3:1 for each of these mixtures are plotted in figures 6, 7 and 8 respectively. In the case of ice and graphite the quantities $(B-V)_{*-N}$ lie very close when $n_1:n_2 = 2:1$ and 3:1. Therefore, only one curve is plotted for these two cases. In the case of ice and silicate, polarization P_B or P_V for $n_1:n_2 = 3:1$ and 2:1 are almost the same. Polarization by mixture of ice and gaphite reaches upto 15 to 20% at $\phi = 20'$ for the three bands U, B and V. The mixture of ice and silicate produces quite a low P_B and a moderate P_V but the P_U is negative for all ϕ's and all $n_1:n_1$ considered here. The mixture of ice and SiC gives quite a low polarization and even negative values when $n_1:n_2 = 1:2$ and 1:3.

5. Three-component mixture of grains

Here we consider three of the four materials of the grains at a time. The proportions $n_1:n_2:n_3 = 1:1:2$, 1:2:1, 2:1:2, 2:2:1, 1:2:4 and 1:4:2 have been selected to illustrate the general trends. The resulting colours and polarization are set out in figures 9 10 and 11. Note that the colours in figure 9 for $n_1:n_2:n_3 = 1:2:1$ and 2:1:2 merge below $\phi \lesssim 10$. Also, P_U for $n_1:n_2:n_3 = 1:2:4$ is close to case of 1:1:2. From the nature of figures 9 to 11, it seems that polarization provides better criteria for sorting out mixtures and proportions. In particular, one can at once rule out ice + silicate + SiC mixtures (figure 11) on this basis. The polarization curves in figures 9 and 10 allow one to infer that, in general, higher proportion of ice, graphite or silicate in comparison with SiC is favoured to obtain the necessary P_V, P_B and probably P_U of at least = 10%.

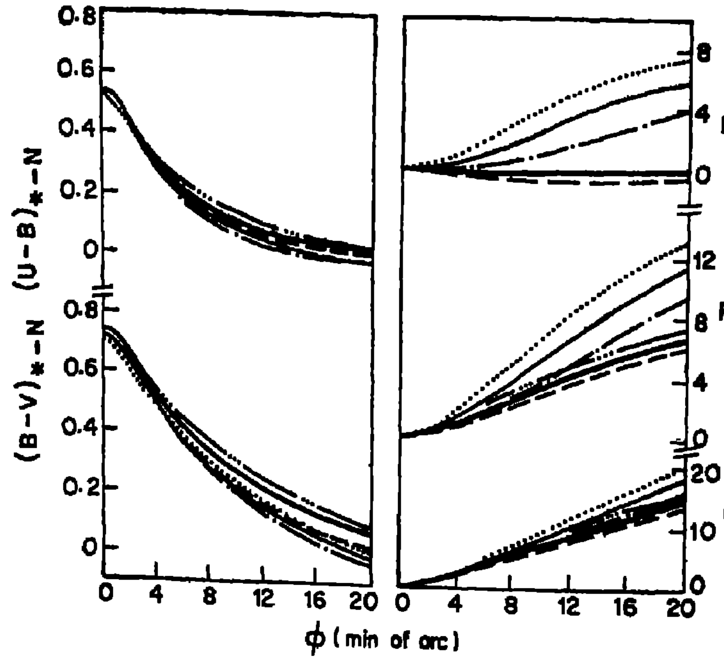

Figure 9. Colours and polarization produced by a three-component mixture of ice (n_1), graphite (n_2) and enstatite silicate (n_3) grains for the case SBN. The mixture proportions are $n_1:n_2:n_3 = 1:1:2$ for —— 1 : 2 : 1 for —— 2 : 1 : 2 for — — 2 : 2 : 1 for —.— 1 : 2 : 4 for —···— and 1 : 4 : 2 for....

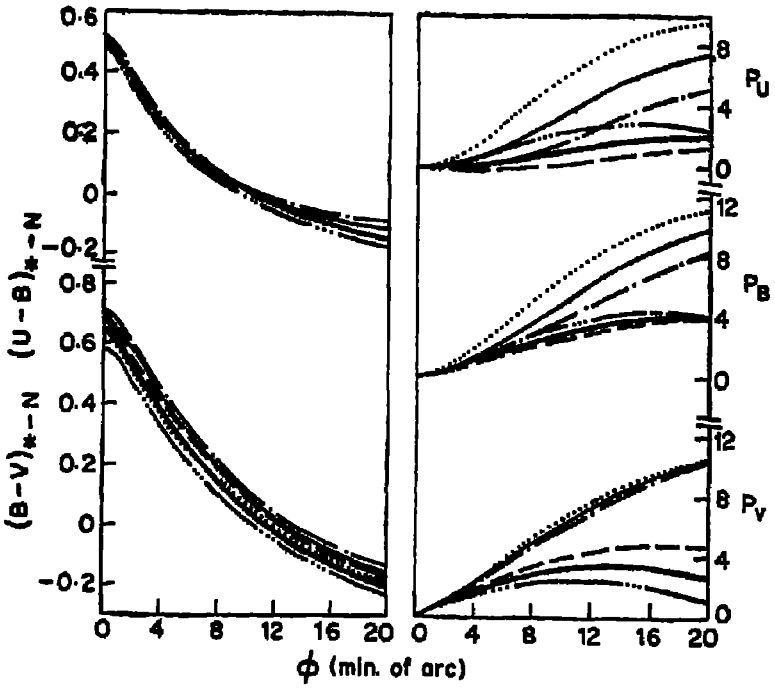

Figure 10. Same as figure 9 but the mixture consists of ice (n_1), graphite (n_2) and SiC(n_3).

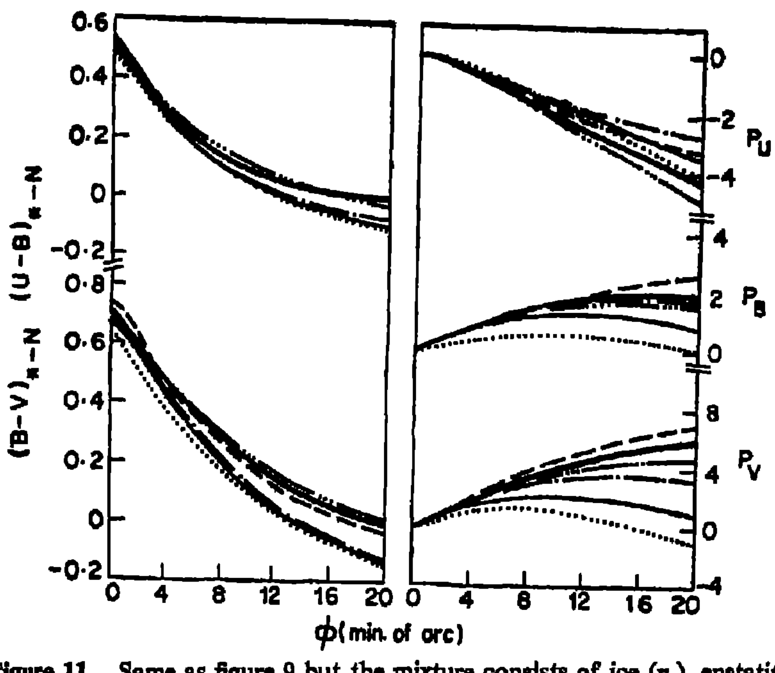

Figure 11. Same as figure 9 but the mixture consists of ice (n_1), enstatite silicate (n_2) and SiC(n_3).

6. Conclusion

In the present case of a star behind the reflection nebula containing core-mantle grains, the *UBV* colour differences show either large values and or shallower gradient compared to the observations and to the results on single grains considered in paper I. The observed polarization can be reproduced for certain core and mantle sizes and compositions except perhaps for SIC core with a thin mantle of ice. The results for a thick layer of ice on a SiC and other cores would appear similar to the dielectric case of single homogeneous spherical grains.

Regarding the mixture of grains, one obtains reasonable colours but it is difficult to cover the observed range and gradient. In general, the polarization provides better criteria for choosing the appropriate size parameters, proportions and compositions of the grains. SiC grains do not seem to be an important component of the mixture of grains particularly in the Merope type of reflection nebula.

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References

Aannestad P A and Purcell E M 1973 *Ann. Rev. Astron. Astrophys.* **11** 309
Allen C W 1973 *Astrophysical Quantities* (London: Athlone Press)
Boggess A and Borgman J 1964 *Astrophys. J.* **140** 1636
Elvius A and Hall J S 1966 *Lowell Obs. Bull.* **6** 257
Elvius A and Hall J S 1966 *Lowell Obs. Bull.* **7** 17
Greenberg J M 1968 *Nebulae and Interstellar Matter* eds B M Middlehurst and L H Aller (Univ. of Chicago Press) Ch. 6
Greenberg J M 1975 *The Dustry Universe* eds G B Field and A G W Cameron (New York: Neale Watson Acad. Publ.) p 131
Greenberg J M and Roark T P 1967 *Astrophys. J.* **147** 917
Güttler A 1952 *Ann. Phys.* **11** 65
Huffman D R and Stapp J L 1971 *Nature (London) Phys. Sci.* **229** 45
Mie G 1908 *Ann. Phys.* **25** 377
Philipp H R and Taft E A 1962 *Phys. Rev.* **127** 159
Shah G A 1974 *Pramāṇa* **3** 338 (Paper I)
Shah G A 1977 *Astron. Nachr.* **298** 319
Van de Hulst H C 1957 *Light Scattering by Small Particles* (New York: John Wiley & sons Inc., London: Chapman & Hall Ltd).
Zellner B H 1974 in *Planets, Stars and Nebulae Studied with Photopolarimetry* ed T Gehrels (Tucson: Univ. of Arizona Press)