2.5-dimensional solution of the advective accretion disk: a self-similar approach

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Abstract We provide a 2.5-dimensional solution to a complete set of viscous hydrodynamical equations describing accretion-induced outflows and plausible jets around black holes/compact objects. We prescribe a self-consistent advective disk-outflow coupling model, which explicitly includes the information of vertical flux. Inter-connecting dynamics of an inflow-outflow system essentially upholds the conservation laws. We provide a set of analytical family of solutions through a self-similar approach. The flow parameters of the disk-outflow system depend strongly on the viscosity parameter α and the cooling factor f.

Key words: accretion — accretion disk — black hole physics — hydrodynamics — galaxies: jets

1 INTRODUCTION

Most extragalactic radio sources are expected to form around spinning massive black holes (Meier et al. 2001; Meier 2002). The immense amount of matter, forming an accretion disk, is being accreted either from the interstellar medium or from its companion star. In these systems, the relativistic outflowing matter should only come from the inner regions of the accretion disk, unlike stellar outflows. This is particularly suggestive for quasars or micro-quasars which do not have an atmosphere of their own. Fender, Belloni & Gallo (2004) suggested a semi-quantitative model for the jet in black hole X-ray binaries where a correlation between the radio and the X-ray emission was estimated. Vadawale et al. (2001) established the X-ray and radio properties of micro-quasar GRS 1915+105. Time dependent interaction between the jet and the inner disk (e.g. Ueda et al. 2002) was evident from the observations of simultaneous X-ray/IR flares from a black hole/relativistic system. Rawlings & Saunders (1991) found a strong correlation between the narrow-line and radio luminosity in FRII type radio galaxies. This implies that the production of the optical line emission and large-scale radio emission are intrinsically linked. Therefore, the outflows or jets are expected to correlate with the disk controlling the accretion process, precisely the accretion dynamics around a central star. The jets or outflows extract matter, energy and angular momentum from the disk.

Thus, it is now clear that these two apparently dissimilar objects are related each other. In principle, one should study the disks and outflows leading to jets in a unified manner, which cannot be dealt with as separate flow dynamics. However, there are few models which simultaneously study the accretion-outflow dynamics on the same platform. Chakrabarti & Bhaskaran (1992) attempted to correlate the collimated bipolar outflows with the disk through a simplified model based on the ambipolar diffusion approximation and self-similarity in the radial direction. Blandford & Begelman (1999) modified the ADAF solution, originally proposed by Narayan & Yi (1994), where the accretion flow is well below the

Eddington limit, by including an outflow/wind which carries mass, angular momentum and energy away from the accretion disk. They later extended their work to two-dimensional adiabatic flow (Blandford & Begelman 2004). Although a new branch of wind solutions was discovered, it does not include the vertical fluxes in the hydrodynamical equations. The ADAF model (Narayan & Yi 1994, 1995) explained the under-luminous accreting sources. The interesting aspect of the ADAF model is that the Bernoulli's parameter at all radii (within the acceptable location of the validity of the self-similar approach) is positive, which leads to the concept that the outflows and jets might emanate from the advective disk. Later, stability of the solution under perturbation was studied with the inclusion of Coriolis force by Prasanna & Mukhopadhyay (2003). In recent times, a few simulations of disk-outflow coupling have been cultivated (Nishikawa et al. 2007; McKinney & Narayan 2007). However, the results are strongly dependent on the initial conditions (Ustyugova et al. 1999) and it is difficult to simultaneously simulate the disk and the outflow regions because the time scales of the accretion and outflow are in general very different. Moreover, in these simulations, how the matter gets deflected from the equatorial plane has been studied largely in the Keplerian regime.

In recent years, there has been a discovery of an unusual class of compact sources, the ultra-luminous X-ray sources (ULX), in the nearby star forming galaxies (Katz 1987, Fabbiano et al. 1989, Kaaret et al. 2001, Colbert & Ptak 2002, Miller et al. 2003, Begelman et al. 2006). These are optically thick, radiation pressure dominated systems with strong advection and the matter is strongly ejected out from the disk in the form of outflows/jets by strong radiation pressure. Using a slim disk model, Abramowicz et al. (1988) discovered a new branch of solutions at a super-critical rate which is stable and optically thick. A model for super-critical accretion with advection was attempted by Lipunova (1999). Ohsuga et al. (2005) have emphasized the importance of advective flows in the super-Eddington, radiation pressure dominated disk with photon trapping. Hence, these two opposite paradigms of black hole activities reveal a profound inter-connection between the inflow parameters and the outflows leading to jets, especially in the advective regime, which the standard optically thick Keplerian disk theory fails to explain.

In the present work, without assuming a geometrically thin disk structure, we prescribe a new model for the accretion-induced outflow leading to a jet. We construct the inflow-outflow correlation model in a more self-consistent manner. The contribution of the magnetic field is neglected at the first instant. The magnetic field is more important for explaining the collimation and acceleration of the jet (apart from ultra luminous sources)[1]. The present model can, not only extend our model from quasars to micro-quasars, but also to neutron star X-ray binaries and in general to many sources with outflows from the disk. However, to describe the flow dynamics and consequently outflows in protostellar objects, the standard Keplerian disk model itself is enough. Our unification scheme is based on the fact that the astrophysical outflow and jet, its underlying disk and its interrelated dynamics at all scales, obey the same physical laws.

We arrange our paper in the following manner. In the next section, we formulate our model equations for the accretion-induced outflow. In Section 3, we present a complete analytical, but self-similar, solution of our model. Next, we study the properties of the class of solutions in Section 4. In Section 5, we end with a discussion and summary.

2 DISK-OUTFLOW CORRELATION AND MODEL EQUATIONS

We assume the disk to be steady and axisymmetric. For a generalized geometrically thick advective disk, we consider the $r\phi-$, $\phi z-$ and $rz-$ components of the shearing stress. The remaining stresses are believed to be negligible, which do not significantly contribute to controlling the disk-outflow dynamics. The flow parameters $v_r, \lambda, v_z, c_s, \rho$ and P are considered to be functions of both radial and vertical coordinates, which are radial velocity, specific angular momentum, vertical velocity, adiabatic sound speed, mass density and pressure, respectively. Here, throughout our calculations, we express radial and vertical coordinates in units of $2GM/c^2$, where M is mass of the central star, G is the gravitational

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[1] The conservation equations should remain valid amidst the nature of the model.

constant and c is speed of light. We also express velocities in units of the speed of light and specific angular momentum in $2GM/c$. The mass of the disk is assumed to be much less than that of the central object, hence the disk is not self-gravitating. Therefore, the general disk-outflow coupled equations are given below.

(a) Mass transfer:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{\partial}{\partial z}(\rho v_z) = 0. \tag{1}$$

(b) Radial momentum balance:

$$v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} - \frac{\lambda^2}{r^3} + F_{Gr} + \frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{1}{\rho}\frac{\partial W_{rz}}{\partial z} = 0, \tag{2}$$

where W_{rz} is the rz^{th} component of the stress tensor, when we consider that the shear stress tensor is symmetric (Landau & Lifshitz 1989) and F_{Gr} is the radial component of the gravitational force. To understand the importance of the term $\partial W_{rz}/\partial z$ in the above equation, we compare it with $\partial P/\partial r$ as

$$\left|\frac{\partial W_{rz}}{\partial z}\right| \Big/ \left|\frac{\partial P}{\partial r}\right| \sim \frac{r}{h}\frac{\nu_t}{c_s^2}\left|\left(\frac{v_r}{h} + \frac{v_z}{r}\right)\right|, \tag{3}$$

where we use a generic order of magnitude relation $\partial A/\partial x_j \approx O(A/x_j)$; A denotes any independent quantity as a function of an arbitrary coordinate variable x_j, $h(r)$ is the disk half-thickness. Note that we do not identify h here as the hydrostatic scale height, but rather as the photospheric height where the disk is coupled to the corona, ν_t is the turbulent kinematic viscosity. With $c_s^2 \sim P/\rho$ and from Equation (1) we obtain

$$\left|\frac{v_z}{v_r}\right| \sim \frac{h}{r}. \tag{4}$$

However, we can write from Equation (9) (as described below)

$$v_r \sim \frac{\nu_t}{r}. \tag{5}$$

Using Equations (4) and (5), and assuming an isotropic distribution of turbulence [2] such that $\nu_t \sim \alpha c_s h$, where α ($\alpha \leq 1$) is the Shakura-Sunyaev viscosity parameter (Shakura & Sunyaev 1973), Equation (3) reduces to

$$\left|\frac{\partial W_{rz}}{\partial z}\right| \Big/ \left|\frac{\partial P}{\partial r}\right| \sim \alpha^2 + \alpha^2\left(\frac{h}{r}\right)^2. \tag{6}$$

For a reasonable value of $h \sim r/2$ and α, the second term on the right hand side of Equation (6) can be neglected. Thus we retain $(\partial W_{rz}/\partial z)\Big/\frac{\partial P}{\partial r} \sim \alpha^2$; $\partial W_{rz}/\partial z$ cannot be neglected. In order to determine W_{rz}, we derive a simplified relation of W_{rz} with $W_{r\phi}$, which is the $r\phi^{th}$ component of the stress tensor, from the order of magnitude analysis and obtain $W_{rz} \sim \alpha W_{r\phi} h/r$.

As the disk has a significant radial flow, we include ram pressure along with gas pressure (Mukhopadhyay & Ghosh 2003) in the equations and write $W_{r\phi} = -\alpha(P + \rho v_r^2)$. The radial momentum equation of the disk-induced outflow/jet thus reduces to

$$v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} - \frac{\lambda^2}{r^3} + F_{Gr} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\alpha^2}{\rho}\frac{\partial}{\partial z}[\frac{z}{r}(P + \rho v_r^2)] = 0, \tag{7}$$

where we have used the fact that for a thick disk, in general, $h \sim z$.

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[2] In reality turbulence is generally anisotropic for a thick disk.

(c) Azimuthal momentum balance:

$$v_r \frac{\partial \lambda}{\partial r} + v_z \frac{\partial \lambda}{\partial z} = \frac{1}{\rho r} \frac{\partial}{\partial r}(r^2 W_{r\phi}) + \frac{r}{\rho} \frac{\partial W_{\phi z}}{\partial z}. \tag{8}$$

The first term on the right hand side signifies the outward transport of angular momentum in the radial direction and the second term in the vertical direction due to the turbulent stress.

If $W_{r\phi}$ dominates the angular momentum transport, with the use of mass conservation Equation (1), we obtain

$$|v_r| \sim \frac{|W_{r\phi}|}{\rho v_\phi}. \tag{9}$$

If, on the other hand, $W_{\phi z}$ dominates the angular momentum transport, then we obtain

$$|v_z| \sim \frac{|W_{\phi z}|}{\rho v_\phi}. \tag{10}$$

Now, comparing Equations (9) and (10) and with the use of mass conservation Equation (1), we can write

$$W_{\phi z} \sim \frac{h}{r} W_{r\phi}. \tag{11}$$

Therefore, the azimuthal equation reduces to

$$v_r \frac{\partial \lambda}{\partial r} + v_z \frac{\partial \lambda}{\partial z} + \frac{\alpha}{r\rho} \left[\frac{\partial}{\partial r}[r^2 (P + \rho v_r^2)] + r^2 \frac{\partial}{\partial z}[\frac{z}{r}(P + \rho v_r^2)] \right] = 0. \tag{12}$$

(d) Vertical momentum balance:

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} + F_{Gz} + \frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{1}{r\rho} \frac{\partial}{\partial r}(rW_{rz}) = 0, \tag{13}$$

where F_{Gz} is the vertical component of the gravitational force. As before, we estimate $\left| \frac{1}{r} \frac{\partial}{\partial r}(rW_{rz}) \right| \Big/ \left| \frac{\partial P}{\partial z} \right| \sim \alpha^2 (\frac{h^2}{r^2} + \frac{h^4}{r^4})$. For $h \sim r/2$ and reasonable α, the quantity is negligible. The Equation (13) thus reduces to

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} + F_{Gz} + \frac{1}{\rho} \frac{\partial P}{\partial z} = 0. \tag{14}$$

In absence of the first term, the equation leads to a mean vertical outflow from the disk. On the other hand, if there is no outflow and jet: $v_z = 0$, then Equation (14) reduces to the well known hydrostatic equilibrium condition in the disk, from where one can calculate the hydrostatic disk-scale height.

(e) Energy conservation:

For an accretion-induced outflow, the energy budget can be computed by

$$\frac{1}{r} \frac{\partial}{\partial r}(r\mathcal{F}_r) + \frac{\partial \mathcal{F}_z}{\partial z} = 0, \tag{15}$$

where $\mathcal{F}_r$ and $\mathcal{F}_z$ are the radial and vertical components of the total energy flux $\mathcal{F}_i$ given by

$$\mathcal{F}_i = \rho v_i \left(\frac{v^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P}{\rho} + \phi_G \right) - v_j W_{ij} + F_i, \tag{16}$$

where we neglect the molecular heat conduction as this is insignificant in accretion disks and the nuclear heat generation/absorption (Mukhopadhyay & Chakrabarti 2000) for mathematical simplicity. Here

$v^2 = v_r^2 + \lambda^2/r^2 + v_z^2$, ϕ_G is the gravitational potential, F_i the radiative flux from the disk surface, W_{ij} the generalized stress tensor and γ the gas constant: $4/3 < \gamma < 5/3$.

Using Equations (1), (2), (8), (13) and (16), we obtain the disk-energy equation from (15) as

$$\rho v \cdot \nabla s = \frac{v_r}{\Gamma_3 - 1}\left[\frac{\partial P}{\partial r} - \Gamma_1 \frac{P}{\rho}\frac{\partial \rho}{\partial r}\right] + \frac{v_z}{\Gamma_3 - 1}\left[\frac{\partial P}{\partial z} - \Gamma_1 \frac{P}{\rho}\frac{\partial \rho}{\partial z}\right] = Q^+ - Q^- = fQ^+, \quad (17)$$

where s is entropy density. Here we assume the energy released Q^- due to radiative loss from the disk is proportional to the viscous heat generated, Q^+, where f is the cooling factor incorporating any kind of outflow and jet which is close to 0 and 1 for the flow with efficient and inefficient cooling respectively. The first and the second terms on the left hand side of the above equation represent the radial and vertical advection of the flow respectively, where we define (Cox & Giuli 1968, Mukhopadhyay & Ghosh 2003)

$$\Gamma_3 = 1 + \frac{\Gamma_1 - \beta}{4 - 3\beta},$$

$$\Gamma_1 = \beta + \frac{(4 - 3\beta)^2(\gamma - 1)}{\beta + 12(\gamma - 1)(1 - \beta)},$$

$$\beta = \frac{\rho k_B T/\mu m_p}{\bar{a}T^4/3 + \rho k_B T/\mu m_p}, \quad (18)$$

with $\beta = \frac{6\gamma - 8}{3\gamma - 3}$, the ratio of gas pressure to total pressure which is close to 0 for extreme radiation dominated flow ($\gamma = 4/3$) and to 1 for extreme gas dominated flow ($\gamma = 5/3$), $\bar{a}$ is the Stefan constant, m_p is mass of the proton, T is proton temperature, k_B is the Boltzmann constant, μ is average molecular weight.

In Equation (17), $Q^+ = W_{ij}^2/\eta_t$, η_t is the coefficient of turbulent viscosity. Thus for our case

$$Q^+ = \frac{1}{\eta_t}(W_{r\phi}^2 + W_{\phi z}^2 + W_{rz}^2). \quad (19)$$

As before, it can easily be shown that the contribution of W_{rz} is much less than that due to $W_{r\phi}$ $\left(W_{rz}^2/W_{r\phi}^2 \sim \alpha^2\left[\frac{h^2}{r^2} + \frac{h^4}{r^4}\right]\right)$. $W_{\phi z}$ contributes to the additional viscous heating in a geometrically thick advective disk with vertical outflow. Using mixed shear stress formalism (Chakrabarti 1996) and approximating $W_{\phi z}$ in terms of $W_{r\phi}$ as given by Equation (11), Equation (17) reduces to

$$\frac{v_r}{\Gamma_3 - 1}\left[\frac{\partial P}{\partial r} - \Gamma_1 \frac{P}{\rho}\frac{\partial \rho}{\partial r}\right] + \frac{v_z}{\Gamma_3 - 1}\left[\frac{\partial P}{\partial z} - \Gamma_1 \frac{P}{\rho}\frac{\partial \rho}{\partial z}\right] = -f\alpha(P + \rho v_r^2)\frac{1}{r}\left(\frac{\partial \lambda}{\partial r} - 2\frac{\lambda}{r} + \frac{z}{r}\frac{\partial \lambda}{\partial z}\right). \quad (20)$$

3 SOLUTION AND SELF-SIMILARITY

We follow the self-similar approach to solve the equations to obtain the class of solutions. For the present purpose we seek a generalized self-similar solution, unlike the previous case (Narayan & Yi 1994), where variation of the flow parameters as functions of the vertical coordinate along with the radial coordinate has been invoked for a coupled set of disk-outflow equations of the form

$$v_r(r, z) = v_{r0}r^u z^a, \ \lambda(r, z) = \lambda_0 r^v z^b, \ v_z(r, z) = v_{z0}r^w z^d, \ c_s(r, z) = c_{s0}r^g z^s, \quad (21)$$

where v_{r0}, λ_0, v_{z0} and c_{s0} are the dimensionless coefficients which will be evaluated from the conservation equations. We determine the exponents u, a, v, b, w, d, g and s by self comparison of various terms in the equations. Assuming the flow to be polytropic, as most likely the disk is, we consider the adiabatic equation of state as $P = k\rho^\gamma$, where $\gamma = 1 + 1/n$, n is the polytropic index of the flow, while the adiabatic sound speed $c_s = \sqrt{\gamma P/\rho}$.

We propose a generalized gravitational potential $\phi_G(r,z) = -(r^{-1} - \frac{1}{k+2}r^{-3}z^2)z^k$, where the index k induces the variation along the z-axis which we determine self-consistently. When the disk does not have strong outflow and jet, $k = 0$ and ϕ_G reduces to conventional Newtonian potential up to the second order in (z/r).

Substituting the solutions from Equation (21) in Equations (1) and (7) and comparing the exponents of r and z we obtain $w = u - 1, a = s, d = a + 1, u = -1/2, v = 1/2, g = -1/2, b = a, k = 2a$. Equations (1), (7), (12) and (20) can now be written, with the use of Equation (21), respectively

$$v_{r0} + 2\left(\frac{2an + a + 1}{1 - 2n}\right)v_{zo} = 0\,, \tag{22}$$

$$\left[\frac{1}{2} - \gamma\alpha^2[2a(n+1)+1]\right]v_{r0}^2 + \left[n - \alpha^2[2a(n+1)+1]\right]c_{s0}^2 + \lambda_0^2 - av_{r0}v_{z0} - 1 = 0\,, \tag{23}$$

$$\left(\frac{1}{2}v_{r0} + av_{z0}\right)\lambda_0 + \alpha\left[(n+1)(2a-1)+3\right]\left(v_{r0}^2 + \frac{n}{n+1}c_{s0}^2\right) = 0 \tag{24}$$

and

$$\frac{n(\Gamma_1 - 1) - 1}{\Gamma_3 - 1}\left[\frac{1}{2}v_{r0} - av_{z0}\right]c_{s0}^2 + \frac{1}{2}f\alpha\left(a - \frac{3}{2}\right)\left[\frac{n+1}{n}v_{r0}^2 + c_{s0}^2\right]\lambda_0 = 0\,. \tag{25}$$

Solving Equations (22)–(25), we compute the coefficients of Equation (21)

$$v_{r0} = \frac{\mathcal{D}}{\left[\mathcal{B}\mathcal{X} + \mathcal{D}(\mathcal{A}\mathcal{D} - a) + \mathcal{K}^2\frac{\mathcal{X}^2}{\mathcal{G}^2}\right]^{1/2}}\,, \tag{26}$$

$$\lambda_0 = \frac{\mathcal{H}\mathcal{X}}{\mathcal{G}\left[\mathcal{B}\mathcal{X} + \mathcal{D}(\mathcal{A}\mathcal{D} - a) + \mathcal{K}^2\frac{\mathcal{X}^2}{\mathcal{G}^2}\right]^{1/2}}\,, \tag{27}$$

$$v_{z0} = \frac{1}{\left[\mathcal{B}\mathcal{X} + \mathcal{D}(\mathcal{A}\mathcal{D} - a) + \mathcal{K}^2\frac{\mathcal{X}^2}{\mathcal{G}^2}\right]^{1/2}}\,, \tag{28}$$

and

$$c_{s0} = \frac{\mathcal{X}^{1/2}}{\left[\mathcal{B}\mathcal{X} + \mathcal{D}(\mathcal{A}\mathcal{D} - a) + \mathcal{K}^2\frac{\mathcal{X}^2}{\mathcal{G}^2}\right]^{1/2}}\,, \tag{29}$$

where $\mathcal{G} = \mathcal{X} + \gamma\mathcal{D}^2$, $\mathcal{X}$ is given by

$$\mathcal{X} = -\frac{\gamma}{4\mathcal{E}}\left[4\mathcal{D}^2\mathcal{E} + (2a + \mathcal{D})\mathcal{K}\left[1 + \left(1 + \frac{8\mathcal{D}^2\mathcal{E}}{(2a + \mathcal{D})\mathcal{K}}\right)^{1/2}\right]\right] \tag{30}$$

and $\mathcal{A}$, $\mathcal{B}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{H}$ and $\mathcal{K}$ are represented as

$$\mathcal{A} = \left[\frac{1}{2} - \gamma\alpha^2[2a(n+1)+1]\right], \ \mathcal{B} = \left[n - \alpha^2[2a(n+1)+1]\right],$$

$$\mathcal{D} = 2(2\,a\,n + a + 1)/(2n-1), \ \mathcal{E} = \alpha\left[(n+1)(2a-1)+3\right],$$

$$\mathcal{H} = \left[\frac{n(\Gamma_1 - 1)-1}{\Gamma_3 - 1}\right](a - \frac{1}{2}\mathcal{D})/\frac{1}{2}f\alpha(a - \frac{3}{2}), \ \mathcal{K} = (a - \mathcal{D}/2). \tag{31}$$

The generalized Bernoulli equation is then

$$\left[\frac{1}{2}\left(v_{r0}^2 + \lambda_0^2 r^{-2} + v_{z0}^2 r^{-2}z^2\right) + nc_{s0}^2 - \left(1 - \frac{1}{2(1+a)}r^{-2}z^2\right)\right]r^{-1}z^{2a} = B_E, \tag{32}$$

where B_E is the Bernoulli constant.

The above solutions can explain both super-critical and sub-critical accretion flows, where the flow is more likely to be strongly advective with the strong possibility of the outflow and jet. Super-critical accretion, of the order of $\dot{M} \gtrsim (10^{-3} - 10^{-6})\ M_\odot/\mathrm{yr}$, corresponds to high luminosity sources with mass of the central star $M \sim 10\ M_\odot$. In this case, the flow is expected to be radiation pressure dominated with a maximum physically plausible γ of 1.444, corresponding to $P_r \sim P_g$.

To determine the exponent a, we vertically integrate Equation (14) from $-h$ to $+h$ after substituting the solutions given by Equations (21), (26)-(29). As the outflow is not likely to emanate from the equatorial plane, the solution loses its relevance there because the torque due to $W_{\phi z}$ exerted on the matter is zero. However, from a certain finite height h_0, they are relevant, describing a disk-outflow system. We consider $h/r \sim t \le 1$ and $h_0/r \sim t_0 \ll 1$, where t and t_0 are kept constant throughout our analysis. The realistic flow, when $v_r < 0$ and $v_z > 0$, demands that a cannot be positive. This helps us to fix the boundary condition of the outflow in the vertical direction. We demand a situation for which t_0 is the minimum to yield a physically realistic a. For super-critical flows exhibiting high luminosity sources, using Equation (22), we obtain the most physically acceptable solution for a given by $a \sim -(2/11+\epsilon)$ for an appropriate $t_0 \sim 0.02$ corresponding to a reasonable $t \sim 0.5$, when ϵ is a very small number $\lesssim 10^{-3}$. For sub-critical flows exhibiting under-luminous sources, on the other hand, the highly sub-critical mass accretion rate $\dot{M} \le (10^{-10} - 10^{-12})\,M_\odot\ \mathrm{yr}^{-1}$ or $\dot{M} \le (10^{-5} - 10^{-7})\,M_\odot\ \mathrm{yr}^{-1}$ corresponds to black holes of mass $M \sim 10\,M_\odot$ or $M \sim 10^6\,M_\odot$ respectively, for which $P_g \gg P_r$ and $\gamma \lesssim 5/3$. With a similar argument as above, we obtain here $a \sim -(1/4 + \epsilon)$ for an appropriate $t_0 \sim 0.07$ corresponding to $t \sim 0.5$.

4 PROPERTIES OF SELF-SIMILAR SOLUTIONS

Here we study the properties of our solution for cases of under-luminous and highly luminous sources at sub-critical and super-critical accretion rates respectively. The standard model of Shakura & Sunyaev (1973) is ineffective for describing these two cases of the geometrically thick advective disks having a substantial outflow. We describe a typical set of solutions for the accretion-induced outflow using the flow parameters obtained in the last section in these two opposite paradigms. A detailed family of solutions and their observational implications will be discussed elsewhere (Ghosh et al. in prep.).

4.1 Super-critical Accretion Regime

Let us consider a case where radiation pressure P_r dominates over gas pressure P_g for the flow with a high Eddington-accretion rate. We choose $\gamma \sim 1.4$ corresponding to $\beta \sim 1/3$, appropriate for the above class of flow. The flow is radiation trapped, optically thick and hot. Figure 1 describes the variations of flow parameters as functions of radial and vertical coordinates for two values of f at a typical α. We see that v_r, v_z, λ and c_s fall off rapidly with the increase of f at a fixed z. With the decrease of f,
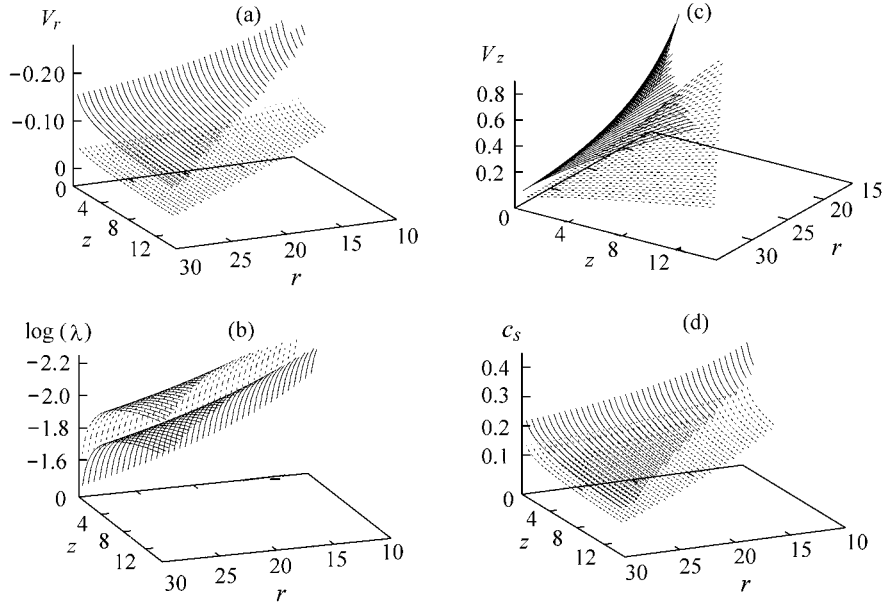
Fig. 1 Variation of (a) velocity, (b) specific angular momentum, (c) vertical velocity, (d) sound speed, as functions of radial and vertical coordinates for super-Eddington accretion flows. Solid and dashed sheets are for $f = 0.4, 0.7$ respectively. Other parameters are $\alpha = 0.05$, $\gamma \sim 1.4$ and corresponding $\beta \sim 0.3$.

the outflow velocity v_z increases rapidly due to strong radiation pressure which blows up the matter as shown in Figure 1c. The disk gets possibly truncated due to strong outflow having both radiation and gas at a region around $r \sim 17$ for $f \sim 0.4$, and $r \sim 10$ for $f \sim 0.7$.

In general, an increase of f leads to inefficient cooling. This renders the disk puffed-up and more quasi-spherical. As a result, the disk angular momentum decreases due to its extraction by the outflow/jet (see Figs. 1b,c). However, at higher f (> 0.5), the system becomes radiatively very inefficient, which may result in the decrease of possible outflux with an increase of f, rendering an increase of the flow angular momentum. At low α (~ 0.01), when the residence time of the infalling matter in the disk is high, angular momentum of the system is such that the disk becomes centrifugally dominated. At this stage, with the decrease of f, angular momentum may increase resulting in the radial and vertical velocity of the flow to be enhanced significantly in order to overcome the strong centrifugal barrier. The outflow is then centrifugally dominated.

The Bernoulli's number in Figure 2 is similar to the velocity profiles in Figure 1. B_E is always positive and high at low f, which indicates the plausibility of the outflow to be very strong, and falls off rapidly with r. The probability of the outflow and jet is low at high f. With an increase of z, B_E initially decreases. This is due to the fact that the first term of the potential ϕ_G, which dominates at small z, is attractive in nature. Then B_E gets a kick as the repulsive part of ϕ_G dominates with the increase in z.

4.2 Sub-critical Accretion Regime

For highly sub-critical accretion flows, which are associated with very low density plasma, the possibility of transfer of viscous energy from ions to electrons due to the Coulomb collisions is very negligible. This results in a gas pressure dominated, geometrically thick accretion disk. The flows have strong advection due to inefficient cooling and are optically thin (Narayan & Yi 1994, 1995). To analyze our
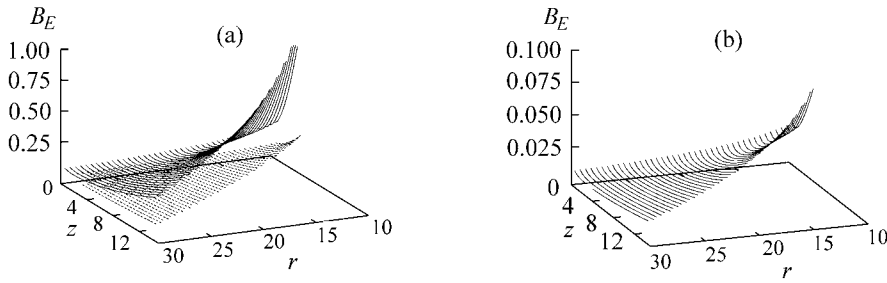
Fig. 2 (a) Variation of Bernoulli's constant for high mass accretion flows. Solid and dashed sheets are for $f = 0.4, 0.7$ respectively. Other parameters are $\alpha = 0.05$, $\gamma \sim 1.4$ and corresponding $\beta \sim 0.3$. (b) Same as in (a), but for low mass accretion flows. $\alpha = 0.05$, $f = 0.9$, $\gamma = 1.6$ and corresponding $\beta \sim 0.89$.
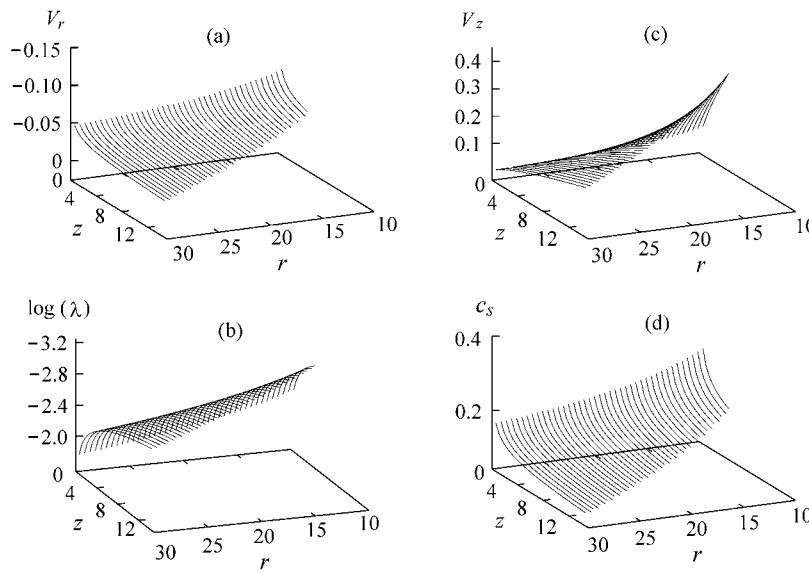
Fig. 3 Same as in Fig. 1, but for sub-Eddington accretion flows. Other parameters are $\alpha = 0.05$, $f = 0.9$, $\gamma \sim 1.6$ and corresponding $\beta \sim 0.89$.

result, we choose $\gamma = 1.6$, which corresponds to $\beta \sim 0.89$ and $f = 0.9$. Figure 3 shows the profiles of the flow parameters as functions of r and z, which are generally similar to those in the super-critical flows. The velocity profiles signify that the magnitudes of v_r and v_z are much less compared to that in the super-critical accretion flows. In low mass accretion flows, the disk may get truncated much nearer to the central star.

5 SUMMARY

We have presented a self-consistent model of accretion-induced outflow and resulting jet. We have established our model equations in a more general way than done earlier, without making any hypothesis,

and without restricting ourselves to the Keplerian geometry. Our equations uphold the conservation laws as the outflows and jets extract matter, energy and angular momentum from the inflowing matter. In its analytical self-similar form, it is easier to analyze and study the family of solutions (with variation of $\alpha \& f$) and to understand the significance of individual terms on the coupled dynamics of the flow. The only limitation we have kept is to ignore the importance of the magnetic field in the disk-outflow system. While not including the magnetic field is an assumption, the outflows and resulting jets in ULX are expected to emerge due to strong radiation pressure. Therefore, the collimation of the jet in ULX might not be magnetically linked (Jaroszyński & Abramowicz 1980; Fabrika 2004). Therefore, for ULX and highly luminous AGN, the assumption of neglecting magnetic effects could be quite appropriate. We also do not aspire to describe the mechanism for the formation of jets, for that the inclusion of magnetic terms might be mandatory, but rather, we try to understand the accretion flow dynamics with the inclusion of the vertical flow. Moreover, to include a magnetic field, solve the equations, and obtain the solution in its present form is beyond the scope.

The new insights that we have provided in the work are:

(1) We have analytically studied the complete set of axisymmetric Navier-Stokes equations for accretion-induced outflow.
(2) The vertical flow, which represents outflow, has been explicitly and self-consistently incorporated in our model, thus invoking a 2.5-dimensional accretion flow.
(3) We do not assume hydrostatic equilibrium.
(4) We have explicitly included $\phi z-$ and $rz-$ components of the stress tensor apart from the usual $r\phi$-component in order to include outflow dynamics into the disk.
(5) All the flow parameters are considered to be functions of both r and z coordinates. We explicitly have shown, by order of magnitude analysis, which terms are relevant and which others can be discarded.

Two extreme cases of the geometrically thick advective accretion disk consisting of super-critical and high sub-critical accretion flows have been studied. It shows that the dynamics of the system depend strongly on f. The model shows that the outflows and jets are less probable in the sub-critical flows compared to that of the super-critical flows. Although we have made a self-similar analytical study, it exhibits some reasonable features in understanding the dynamics of the accretion-induced outflow.

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