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Kink Oscillations in the Solar Corona

A. Satya Narayanan, R. Ramesh, C. Kathiravan, and E. Ebenezer

Indian Institute of Astrophysics, Koramangala, Bangalore 560 034, India

Abstract. The solar coronal plasma is highly structured in the magnetic field, density and temperature. This plays a crucial role in the theory of MHD oscillations. In this study, we model a coronal loop to be made up of a cylindrical tube of constant cross-section. However, the magnetic field, the pressure and density are assumed to be different both inside and outside the tube. The tube admits modes such as sausage (symmetric), kink (asymmetric), surface and body modes. The dispersion relation of the modes for a cylindrical tube which is compressible, infinitely conducting with uniform flows inside the tube is derived. Limiting cases are discussed briefly. The phase speed of the kink mode can be used as a diagnostic for determining the magnetic field of the corona. For different values of the coronal parameters, the magnetic field varies from a few Gauss to 25 Gauss.

1. Introduction

There have been several studies on MHD waves in the corona in the context of coronal heating and acceleration of the solar wind, both theoretically as well as from an observational point of view. These waves also play an important role in the solar-terrestrial connections. Observations of MHD oscillations and waves have been done in almost all possible bands in the past few decades, in particular radio pulsations (Aschwanden 1987; Aschwanden et al. 1999).

Significant progress in MHD wave theory has been made in the recent past (see reviews of Roberts 2000; Nakariakov 2003). The discussion of other aspects of coronal oscillations and waves can be found, in particular, in wave theory (Goossens 1991), observations (Aschwanden et al. 1999) and prominence oscillations (Oliver 1999).

2. The Model

The coronal loop is assumed to be a straight cylindrical tube, as a first approximation (Roberts, Edwin, & Benz 1984; Nakariakov & Ofman 2001). The plasmas inside and outside the tube are assumed to have different densities, magnetic field (though uniform), which are compressible, infinitely conducting. We assume a uniform flow 'U_0' of the plasma inside the tube of radius 'a' as shown in Fig. 1. The effect of flows on the nature of kink oscillations is one of the main aim of this study.

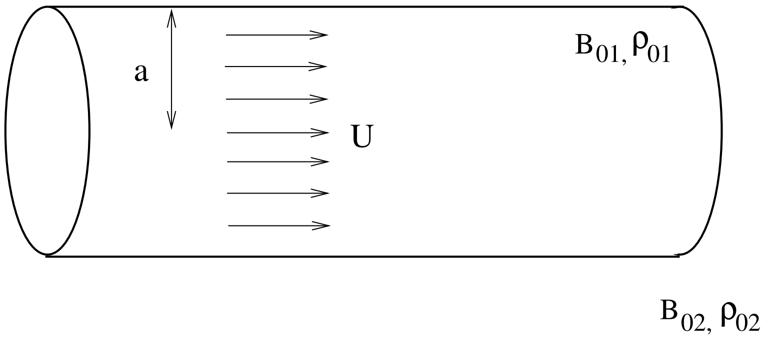

Figure 1. The Model.

3. Dispersion Relation

The equations of motion governing the electromagnetic and hydrodynamic properties of a compressible, infinitely conducting and moving plasma inside the cylinder of radius 'a' is linearized using the normal mode approach to derive the dispersion relation. The wave equation for the total pressure (gas pressure + magnetic pressure) is derived by algebraic simplifications.

For the modes that vary as

$$f(r, \phi, z, t) = \hat{f}(r) \exp[i(kz + l\phi - \omega t)] \tag{1}$$

where k is the axial wavenumber, l is the azimuthal wavenumber and ω is the angular frequency, it can be shown that the radial dependence of the flow variables satisfies the Bessel differential equation. By simple algebraic simplifications, the dispersion relation can be shown to be (Satya Narayanan 1990)

$$\rho_0[\Omega^2 - k^2 C_{\mathrm{A0}}^2] m_{\mathrm{e}} + \rho_{\mathrm{e}}[\omega^2 - k^2 C_{\mathrm{Ae}}^2] m_0 F_m(m_0, m_{\mathrm{e}}, a) = 0 \tag{2}$$

where $\Omega = \omega - k U_0$ is the Doppler-shifted frequency and

$$F_m(m_0, m_{\mathrm{e}}, a) = \frac{K_m(m_{\mathrm{e}} a) I'_m(m_0 a)}{K'_m(m_{\mathrm{e}} a) I_m(m_0 a)}, \tag{3}$$

$$m_0^2 = \frac{(k^2 C_{\mathrm{s0}}^2 - \Omega^2)(k^2 C_{\mathrm{A0}}^2 - \Omega^2)}{(C_{\mathrm{s0}}^2 + C_{\mathrm{A0}}^2)(k^2 C_{\mathrm{T0}}^2 - \Omega^2)}, \tag{4}$$

$$m_{\mathrm{e}}^2 = \frac{(k^2 C_{\mathrm{se}}^2 - \omega^2)(k^2 C_{\mathrm{Ae}}^2 - \omega^2)}{(C_{\mathrm{se}}^2 + C_{\mathrm{Ae}}^2)(k^2 C_{\mathrm{Te}}^2 - \omega^2)}. \tag{5}$$

Here C_{s0} and C_{se} are the sound speeds inside and outside the tube, respectively. C_{A0} and C_{Ae} are the Alfvén velocities inside and outside the tube while C_{T0} and C_{Te} are the tube speeds, respectively.

3.1. Limiting Cases

In the absence of a basic flow $U_0 = 0$, which implies $\Omega = \omega$, the dispersion relation reduces to (Roberts et al. 1984; Nakariakov & Ofman 2001)

$$\rho_0[\omega^2 - k^2 C_{\mathrm{A0}}^2] m_{\mathrm{e}} + \rho_{\mathrm{e}}[\omega^2 - k^2 C_{\mathrm{Ae}}^2] m_0 F_m(m_0, m_{\mathrm{e}}, a) = 0. \tag{6}$$

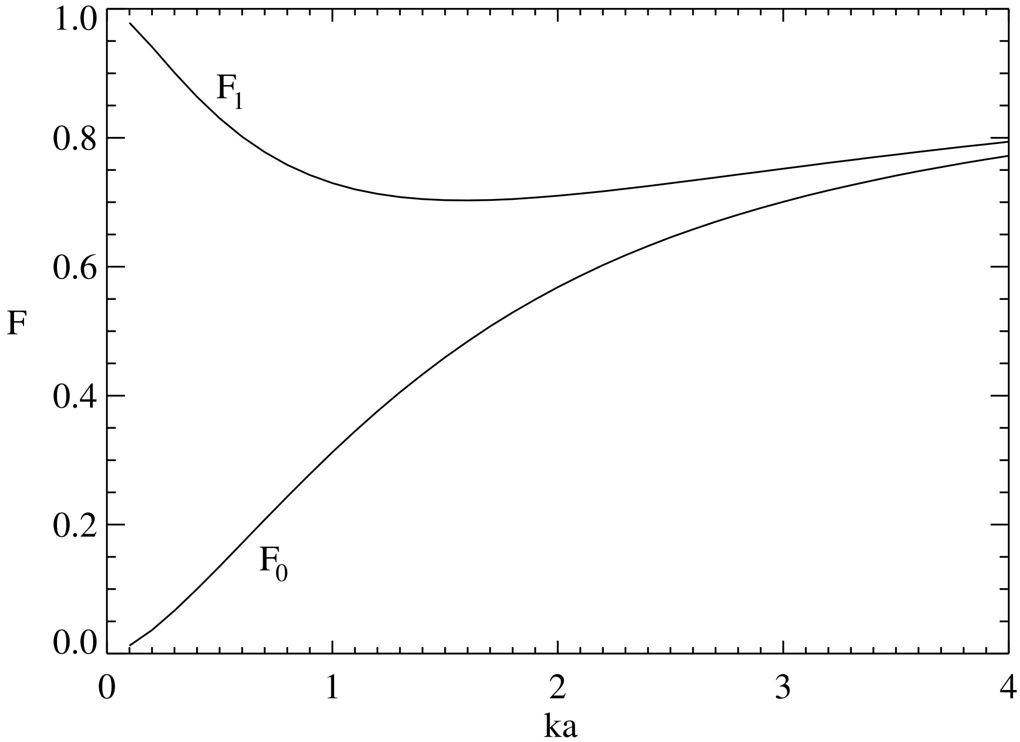

Figure 2. The behaviour of $F_m(ka)$ for different values of ka.

The expression for m_0^2 is modified accordingly, with Ω being replaced by ω.

In the absence of a magnetic field ($B_{01} = B_{02} = 0$), the dispersion relation with $l = 0$ reduces to

$$\rho_0\Omega^2 m_{\rm e} + \rho_{\rm e}\omega^2 m_0 F_m(m_0, m_{\rm e}, a) = 0. \tag{7}$$

For an incompressible flow, $C_{\rm s0}$ and $C_{\rm se} \to \infty$ so that m_0^2 and $m_{\rm e}^2 \to k^2$. In this case, the dispersion relation can be solved analytically (Somasundaram & Satya Narayanan 1987) with

$$F_m(m_0, m_{\rm e}, a) = F_m(ka). \tag{8}$$

Here

$$F_m(ka) = \frac{K_m(ka)I'_m(ka)}{K'_m(ka)I_m(ka)}. \tag{9}$$

The behaviour of $F_m(ka), m = 0, \pm 1$ for $ka \to 0$ and ∞ is shown in Fig. 2.

$$\frac{\omega}{kV_{\rm A0}} = \frac{V \pm [(1+\eta F_0(ka))(1+\alpha^2 F_0(ka)) - V^2\eta F_0(ka)]^{(1/2)}}{[1+\eta F_0(ka)]} \tag{10}$$

where $\alpha = B_{02}/B_{01}$, $V = U_0/C_{\rm A0}$ and $\eta = \rho_{\rm e}/\rho_0$.

In the limit $ka \to 0$, the above equation reduces to

$$\frac{\omega}{kC_{\rm A0}} = V \pm 1. \tag{11}$$

It follows that for values $V \leq 1$, there exists only one positive value for ω/kC_{A0}. For $V > 1$, there are two branches. In the limit $ka \to \infty$, $F_0(ka) \to 1$ so that Eq. (10) reduces to

$$\frac{\omega}{kC_{A0}} = \frac{V \pm [(1+\eta)(1+\alpha^2) - V^2\eta]^{(1/2)}}{(1+\eta)}. \tag{12}$$

For $U_0 = 0$ and $a \to \infty$, the cylindrical geometry reduces to the case of an infinite fluid with a single interface. In this case the dispersion relation is given by

$$\psi_1(\omega, k)(m_e^2 + l^2)^{1/2} + \psi_2(\omega, k)(m_0^2 + l^2)^{1/2} = 0 \tag{13}$$

where

$$\psi_{1,2}(\omega, k) = \rho_{01,2}(k^2 C_{A1,2}^2 - \omega^2). \tag{14}$$

4. Kink Oscillations with Flows

Assume the plasma $\beta \ll 1$. The pressure balance condition is given by

$$p_0 + \frac{B_0^2}{2\mu} = p_e + \frac{B_e^2}{2\mu}. \tag{15}$$

Define : $\alpha = \rho_e/\rho_0$, $\epsilon = U_0/C_{A0}$, $x = \omega/kC_{A0}$. For low β plasma, it can be shown that

$$m_0 = k[1 - (x-\epsilon)^2]^{1/2} = m_0^* \tag{16}$$

$$m_e = k[1 - \alpha x^2]^{1/2} = m_e^*. \tag{17}$$

The dispersion relation for low beta plasma with flow can be written as

$$[(x-\epsilon)^2 - 1](1-\alpha x^2)^{1/2} + \alpha(x^2-1)[1-(x-\epsilon)^2]^{1/2} F(m_0^*, m_e^*, a) = 0 \tag{18}$$

$$F(m_0^*, m_e^*, a) = \frac{K_m(m_e^* a) I_m'(m_0^* a)}{K_m'(m_e^* a) I_m(m_0^* a)}. \tag{19}$$

The above relation is highly transcendental and will have to be solved numerically. However, for $ka \ll 1$, one can show that $F(m_0^*, m_e^*, a) \approx 1$ so that the dispersion relation would reduce to

$$[(x-\epsilon)^2 - 1](1-\alpha x^2)^{1/2} + \alpha(x^2-1)[1-(x-\epsilon)^2]^{1/2} = 0. \tag{20}$$

The above dispersion relation will be solved for certain parametric values pertaining to the corona in due course.

5. Coronal Magnetic Field

As an illustration, we use the results of Nakariakov & Ofman (2001) for determining the magnetic field. They had shown that in the limit $ka \ll 1$, there are

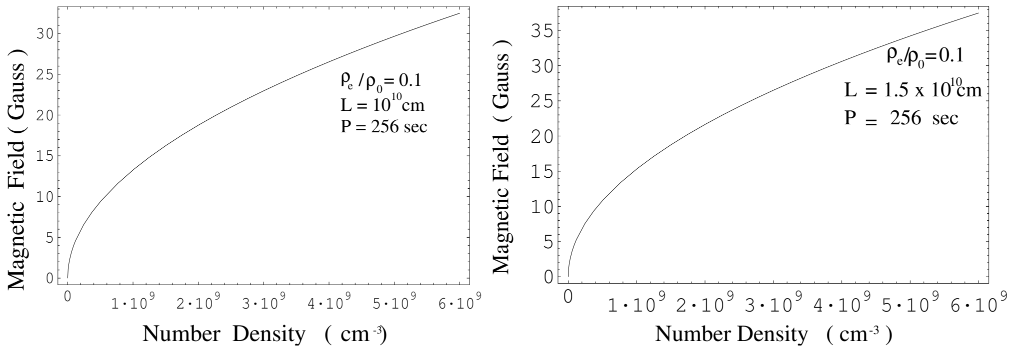

Figure 3. The magnetic field as a function of the density.

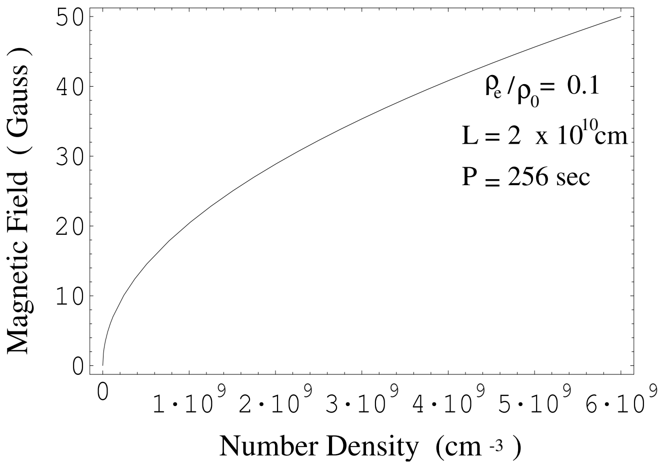

Figure 4. The magnetic field when L is changed.

two kink modes, the slow and the fast with their respective phase speeds. They argued that the magnetic field can be determined by the following formula :

$$B_0 = (4\pi\rho_0)^{1/2} C_{\mathrm{A}0} = \frac{\sqrt{2}\pi^{3/2} L}{P}\sqrt{\rho_0(1 + \rho_{\mathrm{e}}/\rho_0)}. \tag{21}$$

Figure 3 shows the dependence of the magnetic field on the density for different values of the kink speed. It is evident from Eq. (21) for determination of the magnetic field B_0 in the corona, that the strength of the magnetic field depends on L (length between the foot points of the loop), P (the period) and the ratio of the plasma densities. This puts a certain constraint on the determination of the field accurately. However, one can deduce the strength if one has good observations of L, P and the ratio of the densities. One can in principle assume a certain density model and work out. Figure 4 shows the magnetic field when L is increased, keeping the ratio of the densities and period to be the same. There is a marginal increase in the magnetic field.

Ramesh et al. (2003) have presented metric observations of transient, quasi periodic radio emission in association with a 'halo' CME and an 'EIT' wave event to determine the strength of the magnetic field in the corona. We hope to use the theory of kink oscillations to interpret some of our recent radio observations to determine the magnetic field of the corona. The data analysis is being carried out and will be reported later.

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