

A NEW NORMAL INCIDENCE CONCAVE HOLOGRAPHIC GRATING MOUNTING

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In this paper we present a new spectrograph for extreme ultraviolet region. It is based on unconventional focussing properties of a concave holographic grating so far unexplored. The theoretical results shows that it will be very efficient as compared to the existing spectrograph in this region.

1. Introduction

The extreme ultraviolet region is one of the important spectral regions for the study of comets and the sun. In comets, West, Bradfield, Kohoutek and Halley observations have been taken with sounding rocket and international ultraviolet explorer (IUE) satellite at a resolution 12.5 Å. This spectral region includes the important spectral lines CI λ 1657, 1561 Å; CII λ 1355; CO bands at λ 1478 and 1510 Å; HI λ 1216 and OI λ 1304, 1356, observed in these comets. The solar ultraviolet spectra taken at a resolution ~ 0.05 Å, shows FeIII weak emission lines in three groups between λ 1575 and 1598 Å, 1503 and 1518 Å, and between λ 1431 and 1452 Å [1-4].

In the present paper we describe a new spectrograph for extreme ultraviolet region. This spectrograph is based on our studies on holographic concave grating. Our analysis shows that it will be more efficient in terms of resolution, speed and spectral sharpness.

2. Theoretical analysis

Let us take the centre of the grating rulings, O, as the origin of the cartesian coordinate system (fig. 1), the z -axis being parallel to the rulings, the x -axis along the grating normal and the y -axis as shown (fig. 1). Let A(x, y, z), B(x', y', z') and P(u, w, l) be points on the source slit, spectrum line and grating ruling respectively. We further assume that the recording sources are represented by C($x_c, y_c, 0$) and D($x_D, y_D, 0$) and that the difference of the distances of these recording sources from O is an integer multiple of λ_0 the wavelength of the recording laser light, and that the zeroth groove passes through O. Now let us take the cylindrical polar coordinates of the points A(x, y, z), B(x', y', z'), C($x_c, y_c, 0$) and D($x_D, y_D, 0$) as (r, α, z), (r', β, z'), ($r_c, \gamma, 0$) and ($r_D, \delta, 0$) respectively. All these angles are measured in the xy plane. The angles α and β are the

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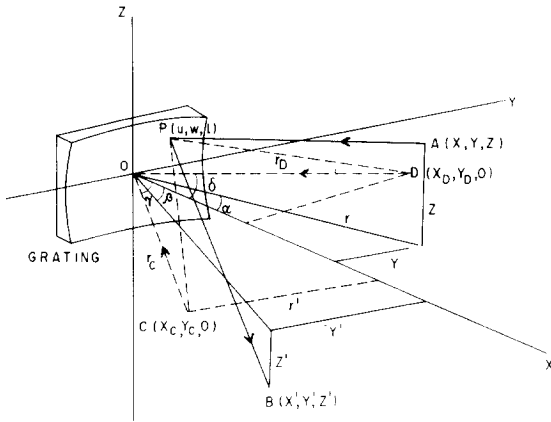


Fig. 1. Schematic diagram of the optical system.

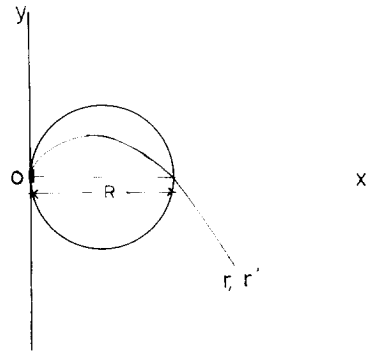


Fig. 2. Source curve and focal curve.

angles of incidence and diffraction respectively. The signs of α and β are opposite if points A and B lie on different sides of the xz plane. The same kind of sign condition holds good for γ and δ . The signs of α and β should be consistent with the signs of γ and δ .

Now following Noda et al. [5], the horizontal focal condition of the grating will be given by the relation

$$\frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} + \frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R} - \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left[\left(\frac{\cos^2 \delta}{r'_D} - \frac{\cos \delta}{R} \right) - \left(\frac{\cos^2 \gamma}{r_C} - \frac{\cos \gamma}{R} \right) \right] = 0, \quad (1)$$

where $R =$ radius of curvature of the grating surface.

This relation is in fact the basis of all mountings of the HRDG (holographic recorded diffraction grating). If in this equation we put some definite values of r and α we shall get a relation, involving r' and β , which will be the polar equation of the focal curve for the particular position (r, α) of the source. Now on an examination of eq. (1), we see that if we choose r and α in such a way that

$$\frac{R}{\sin \delta - \sin \gamma} \left[\left(\frac{\cos^2 \delta}{r'_D} - \frac{\cos \delta}{R} \right) - \left(\frac{\cos^2 \gamma}{r_C} - \frac{\cos \gamma}{R} \right) \right] = 1, \quad (2)$$

then eq. (1) reduces to

$$\frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} + \frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R} - \frac{\sin \alpha + \sin \beta}{R} = 0.$$

Now it is possible to choose a solution as

$$\frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} = \frac{\sin \alpha}{R} \quad \text{or} \quad r = \frac{R \cos^2 \alpha}{\cos \alpha + \sin \alpha}, \quad (3)$$

$$\frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R} = \frac{\sin \beta}{R} \quad \text{or} \quad r' = \frac{R \cos^2 \beta}{\cos \beta + \sin \beta}. \quad (4)$$

Relation (3) and (4) in fact represent the same curve. These equations represent the source curve and focal curve respectively. In fig. 2, the curve represented by these equations is given. This is an open curve and it cuts the normal at a distance R from the vertex of the grating. Based on these relation and curve, we have

presented the properties of HRDG in this work. The curves given by eqs. (3) and (4) were studied by Jobin Yvon [6] also.

3. Estimation of the aberrations and calculations of design parameters

By substituting eqs. (3) and (4) into the relation for optical path function F and applying Fermat's principle $\partial F/\partial l=0$ and $\partial F/\partial w=0$, we get the following expressions for the length of the spectral images (astigmatism) and the spread of the spectral images (due to coma, and other aberrations) for a point source, respectively as,

$$\Delta z' = l \left\{ 1 + \frac{\cos^2 \beta}{\cos \beta + \sin \beta} \left[\frac{\cos \alpha + \sin \alpha}{\cos^2 \alpha} - (\cos \alpha + \cos \beta) - \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left(\frac{R}{r_D} - \frac{R}{r_C} + \cos \gamma - \cos \delta \right) \right] \right. \\ \left. + \frac{w}{R} \frac{\cos^2 \beta}{\cos \beta + \sin \beta} \left[\frac{\sin \alpha (\cos \alpha + \sin \alpha) (\cos \alpha \sin^2 \alpha + \sin \alpha)}{\cos^4 \alpha} + \frac{\sin \beta (\cos \beta + \sin \beta) (\cos \beta \sin^2 \beta + \sin \beta)}{\cos^4 \beta} \right. \right. \\ \left. \left. + \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left(\frac{R^2}{r_C^2} \sin \gamma - \frac{R^2}{r_D^2} \sin \delta - \frac{R}{r_C} \sin \gamma \cos \gamma + \frac{R}{r_D} \sin \delta \cos \delta \right) \right] \right\} \left(1 + \frac{w}{R} \frac{\sin \beta (\cos \beta + \sin \beta)}{\cos^2 \beta} \right)^{-1}, \quad (5)$$

and

$$\Delta \lambda = \frac{\sigma}{m} \left\{ \frac{(\Delta z')^2 - 2l \Delta z' \frac{\sin \beta (\cos \beta + \sin \beta)^2}{\cos^4 \beta}}{2R^2} + \frac{l^2}{2R^2} \left[\frac{\sin \alpha (\cos \alpha + \sin \alpha) (\cos \alpha \sin^2 \alpha + \sin \alpha)}{\cos^4 \alpha} \right. \right. \\ \left. \left. + \frac{\sin \beta (\cos \beta + \sin \beta) (\cos \beta \sin^2 \beta + \sin \beta)}{\cos^4 \beta} \right. \right. \\ \left. \left. + \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left(\frac{R^2}{r_C^2} \sin \gamma - \frac{R^2}{r_D^2} \sin \delta - \frac{R}{r_C} \sin \gamma \cos \gamma + \frac{R}{r_D} \sin \delta \cos \delta \right) \right] \right\} \\ \left. + \frac{3w^2}{2R^2} \left[\frac{\sin^2 \alpha (\cos \alpha + \sin \alpha)}{\cos^2 \alpha} + \frac{\sin^2 \beta (\cos \beta + \sin \beta)}{\cos^2 \beta} + \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left(\frac{R^2}{r_C^2} \sin \gamma \cos^2 \gamma \right. \right. \right. \\ \left. \left. \left. - \frac{R^2}{r_D^2} \sin \delta \cos^2 \delta - \frac{R}{r_C} \sin \gamma \cos \gamma + \frac{R}{r_D} \sin \delta \cos \delta \right) \right] \right\} \\ \left. + \frac{w(l^2 + w^2)}{2R^3} \left[\frac{\cos \alpha \sin^2 \alpha + \sin \alpha}{\cos^2 \alpha} + \frac{\cos \beta \sin^2 \beta + \sin \beta}{\cos^2 \beta} \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left(\frac{R}{r_D} - \frac{R}{r_C} + \cos \gamma - \cos \delta \right) \right] \right\} \\ \left. + \frac{w^3}{R^3} \left[\frac{2 \sin^3 \alpha (\cos \alpha + \sin \alpha)^2}{\cos^4 \alpha} + \frac{2 \sin^3 \beta (\cos \beta + \sin \beta)^2}{\cos^4 \beta} - \frac{\sin^2 \alpha (\cos \alpha + \sin \alpha)}{2 \cos^2 \alpha} - \frac{\sin^2 \beta (\cos \beta + \sin \beta)}{2 \cos^2 \beta} \right. \right. \\ \left. \left. + \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left(\frac{2R^3}{r_C^3} \sin^2 \gamma \cos^2 \gamma - \frac{2R^3}{r_C^3} \sin^2 \gamma \cos \gamma - 2 \frac{R^3}{r_D^3} \sin^2 \delta \cos^2 \delta \right. \right. \right. \\ \left. \left. \left. + \frac{2R^2}{r_D^2} \sin^2 \delta \cos \delta - \frac{R}{2r_C} (\cos^2 \gamma R/r_C - \cos \gamma)^2 + \frac{R}{2r_D} (\cos^2 \delta R/r_D - \cos \delta)^2 \right) \right] \right\}. \quad (6)$$

From eq. (5), we get the following expression, for the length of the astigmatic images formed by a point source as,

$$[z']_{\text{ast}} = L \left\{ 1 + \frac{\cos^2 \beta}{\cos \beta + \sin \beta} \left[\frac{\cos \alpha + \sin \alpha}{\cos^2 \alpha} - (\cos \alpha + \cos \beta) - \frac{\sin \alpha + \sin \beta}{\sin \delta - \sin \gamma} \left(\frac{R}{r_D} - \frac{R}{r_C} + \cos \gamma - \cos \delta \right) \right] \right\}, \quad (7)$$

where L is the total length of the grooves. The condition for zero astigmatism is given by

$$\frac{R/r_D - R/r_C + \cos \gamma - \cos \delta}{\sin \delta - \sin \gamma} = \left(\frac{\cos \beta + \sin \beta}{\cos^2 \beta} - (\cos \alpha + \cos \beta) + \frac{\cos \alpha + \sin \alpha}{\cos^2 \alpha} \right) \frac{1}{\sin \alpha + \sin \beta} = f(\alpha, \beta). \quad (8)$$

For normal incidence ($\alpha = 0^\circ$) and $\beta = 10^\circ$, $f(\alpha, \beta) = 1.2075$, the value of $[z']_{ast}/L$ is zero at $\lambda(\text{\AA}) = 658.57$. Fig. 3 represents $[z']_{ast}/L$ at normal incidence at different wavelength for $f(\alpha, \beta) = 1.2075$ and $f(\alpha, \beta) = 1$.

The equation for secondary focal curve in polar coordinates (r'_h, β) is given by

$$r'_h = R \left((\cos \alpha + \cos \beta) - \frac{\cos \alpha + \sin \alpha}{\cos^2 \alpha} + (\sin \alpha + \sin \beta) f(\alpha, \beta) \right)^{-1}. \quad (9)$$

This relation has been obtained using equations from ref. [5].

If we plot both the primary focal curve given by eq. (4) and the secondary focal curve given by eq. (9) on the same diagram, the points of intersection of these curves, give the position for zero astigmatism and the separation between these curves at different points an insight into the astigmatic properties of the spectral image. Fig. 4 represents these curves for $f = 1.2075$ and $f = 1$. For the first case ($f = 1.2075$) there are two points of minimum astigmatism and represents better condition.

By using eqs. (2) and (8) we get the relations for calculating the recording parameters r_C and r_D at some selected values of γ, δ and $f(\alpha, \beta)$. These recording parameters will give spectral lines without astigmatism at the wavelengths given by the values of (α, β) which have been used in eq. (8) i.e. $f(\alpha, \beta)$ for these calculations. These relations are given below,

$$\frac{R}{r_C} = \frac{\sin^2 \delta}{\cos \delta + \cos \gamma} + \frac{\sin \delta - \sin \gamma}{\cos^2 \delta - \cos^2 \gamma} [1 - f(\alpha, \beta) \cos^2 \delta], \quad (10)$$

$$\frac{R}{r_D} = \frac{\sin^2 \gamma}{\cos \delta + \cos \gamma} + \frac{\sin \delta - \sin \gamma}{\cos^2 \delta - \cos^2 \gamma} [1 - f(\alpha, \beta) \cos^2 \gamma], \quad (11)$$

$$\sigma = \frac{\lambda_0}{\sin \delta - \sin \gamma} = \text{grating constant at the centre [5]}. \quad (12)$$

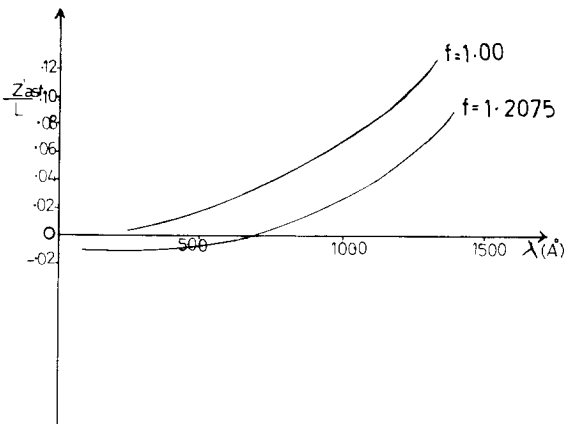


Fig. 3. Astigmatism per unit grooves length at different wavelengths.

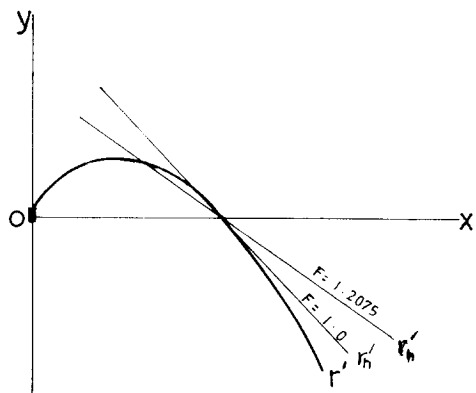


Fig. 4. Secondary focal curve and primary focal curve.

Eq. (12) limits the choice for δ and γ for a particular σ . We obtained the following recording parameters for $f(\alpha, \beta) = 1.2075$, $\sigma = 0.37936 \mu\text{m}$, $\lambda_0 = 0.45793 \mu\text{m}$, $R = 1000 \text{ mm}$ as,

$$\gamma = -45^\circ, \quad \delta = 30^\circ, \quad r_c = 1627.074 \text{ mm}, \quad r_D = 448.209 \text{ mm}.$$

4. Ray tracing, spot diagrams and resolution

By using eqs. (5) and (6) the spot diagrams for a point source for the recording parameters given in the previous section are presented in fig. 5, at $\lambda(\text{\AA}) = 330.80, 658.57, 981.78, 1297.41$ and 1603.17 . In table 1, we have presented $\Delta z'$ (mm), $\Delta\lambda$ (mm) and resolution at different wavelengths at $f = 1.2075$, $R = 1000 \text{ mm}$, $R_c = 1627.07 \text{ mm}$, $R_D = 448.21 \text{ mm}$, $\sigma = 0.37936 \mu\text{m}$. The International Ultraviolet Explorer (IUE) observations were at 12.5 \AA . Feldman et al. [1], Woods et al. [2], McCoy et al. [3], and Jordan et al. [4], obtained observation of solar ultraviolet region at a resolution $\sim 0.05 \text{ \AA}$. In the present set up the resolution at $\lambda(\text{\AA}) = 330.80, 658.57, 981.78, 1297.41$ and 1603.17 is $0.0020 \text{ \AA}, 0.0037 \text{ \AA}, 0.0042 \text{ \AA}, 0.0029 \text{ \AA}$ and 0.0006 \AA , respectively. From these results it is evident that the proposed mounting will be much better than the existing mountings.

5. Suggested design

A suitable arrangement for the proposed mounting is shown in fig. 6, which GOG' represents the concave grating, the rulings of which are perpendicular to the plane of the diagram and OS is the normal to the grating at its middle point O. The source is placed at S. The spectrum is recorded from P to P'. Distances $OS = R$, $OP = 0.618R$, $OP' = 0.916R$, respectively, where R is the radius of curvature of the grating. In this set up the source position S is very convenient as compared to the source position in Egle mounting. The wavelength of the spectral lines for $\sigma = 0.37936 \mu\text{m}$ at P is 1603.17 \AA and at P' is 330.8 \AA .

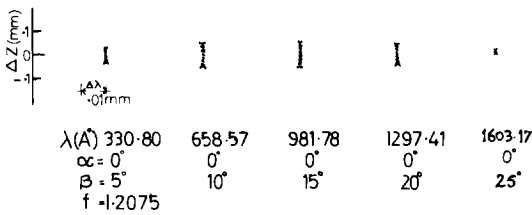


Fig. 5. Spot diagrams at different wavelengths.

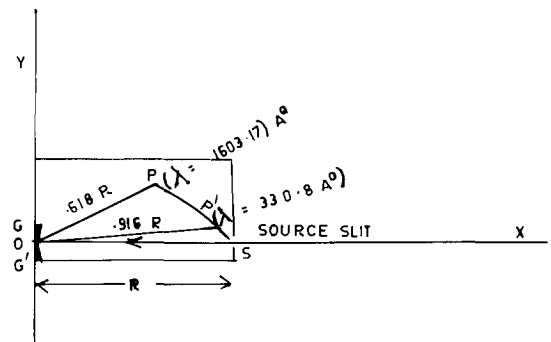


Fig. 6. Arrangement for the proposed mounting.

Table 1

β ($\alpha=0$)	$\lambda(\text{\AA})$	$\Delta z'$ (mm)	$\Delta\lambda$ (mm)	Resolution (\AA)
5°	330.80	0.0673	0.0007	0.0020
10°	658.57	0.1066	0.0012	0.0037
15°	981.78	0.1143	0.0013	0.0042
20°	1297.41	0.0858	0.0009	0.0029
25°	1603.17	0.0151	0.00017	0.0006

References

- [1] P.D. Feldman, M.F.A. Nearn, M.C. Festou, L.A. McFadden, H.A. Weaver and T.N. Woods, *Nature* 324 (1986) 433.
- [2] T.N. Woods, P.D. Feldman, K.F. Dymond and D.J. Sahnou, *Nature* 324 (1986) 436.
- [3] R.P. McCoy, C.B. Opal and G.R. Carruthers, *Nature* 324 (1986) 439.
- [4] C. Jordan, J.D.F. Bortoe, G.E. Brueckner and K.P. Dere, *Nature* 324 (1986) 444.
- [5] H. Noda, T. Namioka and M. Seya, *J. Opt. Soc. Am.* 64 (1974) 1031.
- [6] Jobin and Yvon Co., 1969, Arcueil, France, Technical Bulletin.