Magnetohydrodynamic waves in the solar corona

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1. Introduction

The solar atmosphere, from the photosphere to the corona, is strongly structured by magnetic field and stratified by gravity. In such a medium the propagation of magnetohydrodynamic (MHD) waves is extremely complicated, with the well known results of a uniform medium providing only a limited guidance as well as to the behaviour of the modes in the inhomogeneous atmosphere of the Sun. As an illustration of solar stratification, it is curious to observe that in a region extending upwards from the photosphere to the low chromosphere, over which the temperature decreases from a photospheric value of about 6000 K to about 4000 K in the chromosphere, the plasma density declines by two orders of magnitude.

Magnetic fields introduce structuring into the atmosphere. At the photospheric level, the field is found to occur in magnetic clumps or flux tubes, which are isolated from their neighbours. the field strengths are high, generating a hierarchy ranging from 1.5 kG in intense flux tubes to 3 kG in sunspots. Intense flux tubes are the 'building blocks' of the photospheric field; they are found to reside preferentially in the down-droughts of supergranules and have a radius of about 10^2 km. Possessing such a small scale, they are below the resolution limit of ground based telescopes and are consequently to be the subject of several space missions.

Sunspots represents the largest spatial scale of photospheric structuring, possessing diameters typically in the 4×10^3 - 10^4 km range. Between the extremes of the intense tube and sunspot, knots and pores occur; field strengths are in the range of 1.5 - 2 kG and diameters of the order of 10^3 km.

Above the photospheric layers, in the low chromosphere, the isolated flux tubes rapidly expand to merge with their neighbours, completely filling the chromosphere and corona with magnetic field. The photosphere may be viewed as comprising of two media; a magnetic region where the plasma β is of the order of unity, and a weak field where β is very high. In the chromosphere, β is everywhere of order unity or less; in the corona, the magnetic field is generally dominant with $\beta << 1$. Magnetic structuring is also present in the corona. Whereas both structuring and stratification are significant in the photosphere, stratification is less important in the corona because many coronal structures (such as loops) have scales less than or at most comparable with the local pressure scale-height. Thus, the coronal plasma is in some ways easier to treat analytically than the flux tubes of the photosphere. Nonetheless, each region has its own set of problems. In the corona density and temperature inhomogeneities are common.

2. Waves in flux tubes

The evidence for the occurrence of magnetic flux tubes in astrophysical phenomena is increasing all the time. Flux tubes (or flux ropes) are believed to occur in extragalactic jets, in the magnetospheres of some planets, and in the Sun. Indeed in the case of the Sun almost all of the magnetic flux emerging through the photospheric surface is found to occur in concentrated forms, ranging in scale from the visible sunspot to the sub arcsec intense flux tubes. Thus, the source of coronal or solar wind magnetism is to be found in concentrated roots of magnetic field emerging from the deep interior. Firstly, we note that a tube is a wave guide. It permits waves to propagate without spatial attenuation. Thus a tube is likely to be a good communication channel perhaps providing a connection between an energy source and an energy sink. For example, photospheric flux tubes connect the convection zone - an ample source of energy, especially in granules - with the chromosphere and the corona. Secondly, we observe that a magnetic flux tubes is an elastic object (and so an elastic, not rigid, wave guide) and as such is likely to respond to sudden changes by guiding waves. Sunspots are known to support a wide variety of wave phenomena [1-8]. From an observational point of view the clearest evidence for flux tubes as magnetically distinct structures exists in the solar photosphere. An excellent review on the dynamics of flux tubes in the solar atmosphere is presented in [9]. In addition to this review, there have been other works on the observations of dynamics related to flux tubes [10-19] and from theoretical point of view [20-30].

2.1. Wave equations

The starting point for our discussion is the linear magnetohydrodynamic waves equations in a magnetically structured atmosphere. In this discussion we shall ignore a number of complications such as nonlinearity, non-adiabaticity, curvature, parametric wave coupling, gravitational stratification, and flows. All these effects are likely to prove important in various circumstances, but a full assessment of their significance is not yet available, though a number of aspects have been examined. For example, flows and nonlinearity have recently been discussed in a general form by Ballai and Erdelyi [31], Ballai et al [32], and parametric mode coupling has been discussed in Zaqarashvili [33] and Zaqarshvili and Roberts [34]. Our aim is to bring out the simplest form of the general features of wave propagation in structured medium as they apply directly to oscillations, and this is most conveniently done in terms of the linear equations of wave motion.

Accordingly, we consider the linear equations of motion $\mathbf{v} = (v_r, v_\theta, v_z)$, written in the form (see for example Roberts [35]),

$$\rho_0\left(\frac{\partial^2}{\partial t^2} - c_A^2\frac{\partial^2}{\partial z^2}\right)\mathbf{v}_\perp = -\nabla_\perp\left(\frac{\partial p_T}{\partial t}\right), \tag{1}$$

$$\rho_0\left(\frac{\partial^2}{\partial t^2} - c_T^2\frac{\partial^2}{\partial^2}\right)v_z = -\left(\frac{c_s^2}{c_s^2+c_A^2}\right)\frac{\partial}{\partial z}\left(\frac{\partial p_T}{\partial t}\right), \tag{2}$$

where the total pressure variations p_T , given by

$$p_T = p + \frac{1}{\mu}\mathbf{B}_0\cdot\mathbf{B}, \tag{3}$$

are related to the flow $\mathbf{v}$ through

$$\frac{\partial p_T}{\partial t} = \rho_0 c_A^2\frac{\partial v_z}{\partial z} - \rho_0(c_s^2+c_A^2)div\mathbf{v}. \tag{4}$$

Here we have taken a non-uniform equilibrium magnetic field $\mathbf{B}_0 = B_0\hat{z}$ which is aligned with the z-axis of our coordinate system. The field strength B_0 is such as to balance plasma pressure p_0, through

$$\nabla\left(p_0 + \frac{B_0^2}{2\mu}\right) = 0. \tag{5}$$

The flow is $\mathbf{v} = v_z\hat{z} + \mathbf{v}_\perp$, for component v_z along the applied magnetic field and flow $\mathbf{v}_\perp$ in the plane perpendicular to the applied magnetic field. Written in this way, Eqs (1)-(5) may be applied both to flux tubes, described by a cylindrical coordinate system (r, θ, z), and magnetic slabs, described by Cartesian coordinates (x, y, z). Three basic speeds immediately arise in Eqs (1)-(4): the sound speed $c_s = (\gamma p_0/\rho_0)^{1/2}$, the Alfvén speed $c_A = (B0^2/\mu\rho_0)^{1/2}$, and the slow magnetoacoustic speed c_T defined through

$$c_T = \frac{c_s c_A}{(c_s^2 + c_A^2)^{1/2}}. \tag{6}$$

Each of these speeds is in general a function of the coordinate perpendicular to the applied magnetic field, be it the radius r in cylindrical coordinates or x in the Cartesian coordinates. Eqs (1)-(4) are the basic equations describing MHD wave propagation in a magnetically structured medium, with radial structuring in cylindrical geometry and structuring in x in a slab geometry (Roberts [35]). There is a rich structure in the equations, and this structure is important for oscillations. The corona in particular, is generally considered to be a low β plasma, so that $c_s << c_A$; consequently, the ordering of speeds is typically $c_T < c_s << c_A$. When applied to a discrete flux tube with Alfvén speed c_A inside the tube and an environment with Alfvén speed c_{Ae}, a further speed becomes important; it is the mean Alfvén, or kink speed, c_K, defined through

$$c_K = \left(\frac{\rho_0 c_A^2 + \rho_e c_{Ae}^2}{\rho_0 + \rho_e}\right), \tag{7}$$

where ρ_0 denotes the plasma density within the tube and ρ_e is the density in the tube's magnetic environment.

The speed c_K is intermediate between c_A and c_{Ae}. In the extreme of a magnetic field B_0 being everywhere uniform with the tube defined simply by virtue of a strong density enhancement ($\rho_0 >> \rho_e$), then $c_K = \sqrt{2} c_A$.

The speeds c_T and c_K have been recognized as important in a number of early studies in connection with photospheric flux tubes (Defouw [36], Roberts and Webb [37], Spruit and Roberts [38]) and coronal tubes (Wilson [39], Spruit [3], Edwin and Roberts [22]). The speed c_K also arises in the description of waves and instabilities on magnetic interfaces (Miles and Roberts [40]).

One solution of the above system of Eqs (1)-(4) is $p_T = 0$ with $v_r = v_z = 0$ and $\partial/\partial\theta = 0$; this describes a torsional Alfvén wave $\mathbf{v} = (0, v_\theta, 0)$ which satisfies the wave equation with Alfvén speed $c_A(r)$. More generally, when $p_T \neq 0$, it is usual to describe pressure variations (and the associated motion) in terms of a Fourier representation, writing

$$p_T(r,\theta,z,t) = p_T(r) exp[i(\omega t + n\theta - k_z z)], \tag{8}$$

for frequency ω, azimuthal number n = 0, 1, 2, ..., and longitudinal wavenumber k_z. The resulting equations may then be simplified to yield the ordinary differential equation (Edwin and Roberts [22])

$$\rho_0(r)(k_z^2 c_A^2(r) - \omega^2)\frac{1}{r}\frac{d}{dr}\left\{\frac{1}{\rho_0(r)(k_z^2 c_A^2(r) - \omega^2)} r\frac{dp_T}{dr}\right\} = \left(m^2(r) + \frac{n^2}{r^2}\right)p_T, \tag{9}$$

with m^2 given by,

$$m^2(r) = \frac{(k_z^2 c_s^2 - \omega^2)(k_z^2 c_A^2 - \omega^2)}{(c_s^2 + c_A^2)(k_z^2 c_T^2 - \omega^2)}.$$

Equation (9) possesses singularities at $\omega^2 = k_z^2 c_A^2$ and $\omega^2 = k_z^2 c_T^2$; these singularities generate the Alfvén and slow continua, respectively. The presence of these singularities is an indication of a number of interesting effects connected with the phenomenon of resonant absorption (Goossens [41]), of particular interest in coronal heating.

We consider the specific case of a flux tube of radius a (Figure 1), field strength B_0 and plasma density ρ_0 embedded in a magnetic environment with field strength B_e and plasma density ρ_e :

$$B_0(r) = B_{01}, \quad r < a, \tag{10}$$

$$B_0(r) = B_{02}, \quad r > a, \tag{11}$$

and a similar expression for the density. The Alfvén, sound and tube speeds with the tube are c_{A1}, c_{s1} and c_{t1}, with corresponding values in the external medium. The equation is solved in terms of Bessel function for p_T as $J_n(n_0 r)$ in $r < a$ and $K_n(m_e r)$ in r ¿ a, with the result,

$$\frac{n_0}{\rho_{01}(k_z^2 c_{A1}^2 - \omega^2)} \frac{J_n'(n_0 a)}{J_n(n_0 a)} = \frac{m_e}{\rho_{02}(k_z^2 c_{A2}^2 - \omega^2)} \frac{K_n'(m_e a)}{K_n(m_e a)}, \tag{12}$$

where

$$n_0^2 = \frac{(\omega^2 - k_z^2 c_{A1}^2)(\omega^2 - k_z^2 c_{s1}^2)}{(c_{s1}^2 + c_{A1}^2)(\omega^2 - k_z^2 c_T^2)},$$

$$m_e^2 = \frac{(k_z^2 c_{s2}^2 - \omega^2)(k_z^2 c_{A2}^2 - \omega^2)}{(c_{s2}^2 + c_{A2}^2)(k_z^2 c_{t1}^2 - \omega^2)}.$$

Equation (12) is the dispersion relation describing waves in a magnetic flux tube that is embedded in a magnetic environment; it applies when $m_e > 0$, corresponding to waves that are confined to the tube. The integer n that arises in the description of tube waves defines the geometry of the vibrating tube, the case $n = 0$ corresponds to the sausage wave (a symmetric pulsation of the tube, with the central axis of the tube remaining undisturbed). The case $n = 1$ describes a kink mode (involving lateral displacements of the tube, maintaining a circular cross-section, with the axis of the tube resembling a wriggling snake). A sketch of the above modes is given in Figure 2. Finally, there are fluting modes ($n \geq 2$), which ripple the boundary of the tube. Only the kink mode displaces the central axis of the vibrating mode. The restriction $m_e > 0$ imposed on the flux tube dispersion relation means that the amplitude of a wave declines with radius r ($> a$) so that far from the tube there is no appreciable disturbance. Inside the tube (for $r < a$), the disturbance is oscillatory if $n_0^2 > 0$, or non-oscillatory (evanescent) if $n_0^2 < 0$. Modes that inside the tube are oscillatory in r are called body waves. In the strongly magnetized coronal plasma, the modes are body waves. The waves are dispersive, the phase speed c ($= \omega/k_z$) of a tube wave depending upon its wavelength $2\pi/k_z$, through the combination $k_z a$. The dispersion relation (12) possesses two sets of modes, namely fast and slow body waves. There are no surface ($n_0^2 < 0$) waves. The fast waves are strongly dispersive, and arise only if $c_{A2} > c_{A1}$; if $c_{A2} < c_{A1}$, then the fast waves are leaky and propagate energy away from the region of high Alfvén speed. Fast body waves, then, are trapped in regions of low Alfvén speed, typically corresponding to regions of high plasma density. Regions of low Alfvén speed in a strongly magnetized plasma provide wave guides for fast magnetoacoustic waves (Uchida [42], Roberts et al [43]).

2.2. Thin flux tube waves

The region of the solar atmosphere lying between the photosphere and the corona is highly complicated. It contains an obvious energy source for wave motions in the granules and supergranules that reside in the convection zone, and it is a region of the Sun where stratification effects are most pronounced. Added to this is the complex architecture of the magnetic field, which tends to occur in

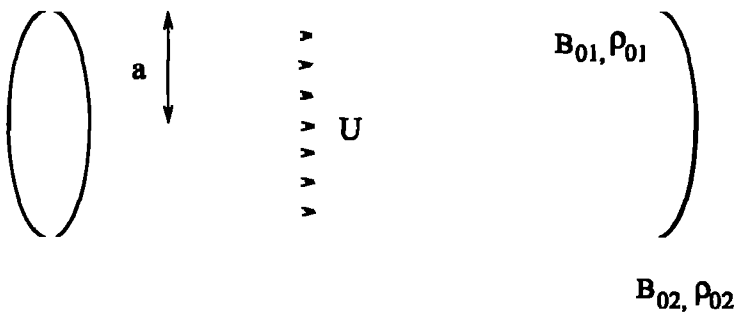

Figure 1: A simple model of a flux rube

concentrated structures (ranging from small-scale intense flux tubes through to pores and sunspots) in the photosphere but has spread out to fill the available plasma in the upper chromosphere and the corona. This complex magnetic architecture, forming three dimensional structures overlying regions that are largely field-free, is difficult to model and the description of MHD waves especially complex. One area where detailed theoretical studies are available is that of thin flux tubes. By assuming that magnetism is confined to thin tubes which expand outwards with height, it is possible to describe in detail the various modes of oscillation of the tube. The applicability of such theories to oscillations high above the photosphere, where the field has expanded out, is difficult to assess, but it seems reasonable to suppose that the features predicted for thin tubes are approximate guides to what might actually occur in this complex zone. The occurrence of Klein- Gordon equation, describing waves in flux tubes as well as sound waves offers some support that these descriptions are useful. The Klein-Gordon equation is given by

$$\frac{\partial^2 Q}{\partial t^2} - c^2\frac{\partial^2 Q}{\partial z^2} + \Omega^2 Q = 0. \tag{13}$$

Here $Q = Q(z,t)$ is given by,

$$Q(z,t) = \left[\frac{\rho_0(z)A_0(z)c^2(z)}{\rho_0(z)A_0(z)c^2(0)}\right]^{1/2}$$

$A(z,t)$ the tube area is related to the pressure $p(z,t)$. Ω^2, the cut-off frequency is a complicated expression involving the density, tube area, sound speed and the gravity.

The Klein-Gordon equation holds for a wide variety of thin elastic tubes. In addition to the modes described above, we have other types of MHD waves such as shear Alfvén Waves, Magneto Acoustic Waves, Internal Gravity Waves, Inertial Waves. These modes arise depending on the effects of compressibility, gravity, rotation taken into account. More details on these waves can be had from Priest [44].

3. Coronal waves

The high resolution observations of the solar corona by the instruments on board SOHO and TRACE missions have brought us a breakthrough in the experimental study of coronal wave activity [45]. The majority of the coronal wave phenomena discovered by these missions is associated with variations of the EUV emissions produced by coronal plasmas. The characteristic speeds of the variations are

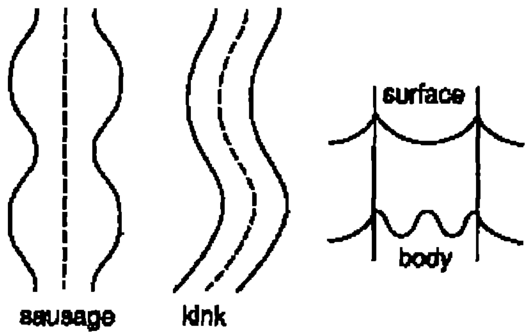

Figure 2: Different modes in a flux tube

found to be about several minutes. Observationally determined properties of various coronal waves and oscillations are quite different, allowing us to distinguish between several kinds of phenomena. In particular, kink and longitudinal modes are confidently interpreted in the data. The modes of other types, in particular sausage and torsional, predicted by theory, have not been identified in EUV data. However, the sausage modes are believed to be routinely observed in the radio band. Unresolved torsional waves can be responsible for the modulation of non-thermal broadening of coronal emission lines. The observational discovery of coronal waves reinforced wave-based theories of coronal heating and led to the creation of a new branch of coronal physics: MHD coronal seismology, first proposed as a theoretical possibility by Uchida [46] and Roberts et al [43]. Measuring the properties of MHD waves and oscillations (periods, wavelengths, amplitudes, temporal and spatial signatures, characteristic scenarios of the wave evolution), combined with theoretical modeling of the wave phenomena (dispersion relations, evolutionary equations etc.), we can determine values of the mean parameters of the corona, such as the magnetic field strength and transport coefficients. The first practical implementation of this method was made by Nakariakov et al [47] and Nakariakov and Ofman [48]

Theoretical aspects of MHD waves in the solar coronal plasma have been investigated for decades. There have been also several reports on the existence of MHD phenomenon in the solar corona [49-54]. With the spatial detection of oscillations by the Transition Region and Coronal Explorer (TRACE) spacecraft recently, theories on MHD waves take on a new vigour. It has now proved possible to systematically study coronal loop oscillations and their decay [55,56]. Oscillations in hot loops have also been very recently detected by SUMER (Solar Ultraviolet Measurements of Emitted Radiation) spectrometer on the Solar and Heliospheric Observatory (SoHO) [57,58]. The theory of coronal loop oscillations has recently been reviewed in [59-63]. However, it is evident that the subject is developing apace, led by the recent observational discoveries which have prompted a re-examination of theoretical aspects.

3.1. Acoustic waves in coronal loops

Slow magnetosonic waves (acoustic waves) are an abundant feature of the coronal wave activity, known from observations such as SOHO/EIT, UVCS, SUMER and TRACE. These modes are longitudinal in nature, perturbing the density of the plasma and the parallel component of the velocity. Both propagating and standing waves are observed.

3.2. Propagating longitudinal waves

With imaging telescopes, propagating longitudinal waves are observed in both open and closed coronal magnetic structures. The standard observational technique is the use of the stroboscopic method : the emission intensities along a chosen path, taken in different instants of time are laid side-by-side to form a time-distance map. Diagonal stripes of these maps exhibit disturbances which change their position in time and, consequently, propagate along the path. This method allows the determination of periods (or wavelengths), relative amplitudes and projected propagation speeds. The first observational detection of longitudinal waves came from analyzing the polarized brightness (density) fluctuations The fluctuations with periods of about 9 min were detected in coronal holes at a height of about 1.9 $R_\odot$ by Ofman et al [64,65] using the white light channel of the SOHO/UVCS. Theoretical models of the propagation of longitudinal waves in stratified coronal structures (Ofman et al [66], Ofman et al [67], Nakariakov et al [68]), describe the evolution of the wave shape and amplitude with the distance along the structure s in terms of the extended Burgers equation,

$$\frac{\partial A}{\partial s} - \alpha_1 A - \alpha_2 \frac{\partial^2 A}{\partial \xi^2} + \alpha_3 A \frac{\partial A}{\partial \xi} = 0, \tag{14}$$

where the coefficients α_1, α_2, α_3 are in general functions of s and describe α_1 - the effects of stratification, radiative losses and heating, α_2 - dissipation by thermal conductivity and viscosity and α_3 - nonlinearity; and the $\xi = s - C_s t$ is the running coordinate.

Solutions of the above equation are in satisfactory agreement with the observed evolution of the wave amplitude. Also, full MHD 2D numerical modeling of these waves gives similar results. The energy carried and deposited by the observed waves is certainly insufficient for heating of the loop. However, Tsiklauri and Nakariakov [69] has shown that wide spectrum slow magnetoacoustic waves, consistent with currently available observations in the low frequency part of the spectrum, can provide the rate of heat dissipation sufficient to heat the loop.

3.3. Kink oscillations

Movies created with the use of coronal imaging data show fast-decaying quasi-periodic displacement of loops, often responding to an energy release nearby, in a form of a flare or eruption (flare generated oscillations). Analysis of 26 oscillating loops with lengths of 74-582 Mm, observed in EUV with TRACE [56] yielded periods 2.3 - 10.8 min, which is different for different loops. It is known that coronal loops are anchored in the dense plasma of the photosphere, so it is reasonable to assume that any motions in the corona are effectively zero at the base of a loop. The observed properties of these oscillations can be interpreted in terms of a kink fast magnetosonic mode [22]. The first observation of kink oscillations was after the flare on the 14th July 1998 at 12.55 UT. The oscillation was identified as a global mode, with the maximum displacement situated near the loop apex and the nodes near the foot points. Using the above theory Nakariakov and Ofman [48] estimated the magnetic field in an oscillating loop as 13 ± 9 G. The effect of uniform flows on kink oscillations was studied by Satya Narayanan et al [27]. We briefly describe their results : Assume the plasma $\beta << 1$. The pressure balance condition is given by,

$$p_0 + \frac{B_0^2}{2\mu} = p_e + \frac{B_e^2}{2\mu}. \tag{15}$$

For $\alpha = \rho_e/\rho_0$, $\epsilon = U_0/c_{A1}$, $x = \omega/kc_{A1}$ and low-β plasma, it can be shown that

$$m_0 = k[1 - (x - \epsilon)^2]^{1/2} = m_0^*, \tag{16}$$

$$m_e = k[1-\alpha x^2]^{1/2} = m_e^*. \tag{17}$$

The dispersion relation for low-β plasma with flow can be written as,

$$[(x-\epsilon)^2-1](1-\alpha x^2)^{1/2} + \alpha(x^2-1)[1-(x-\epsilon)^2]^{1/2} F(m_0^*, m_e^*, a) = 0, \tag{18}$$

$$F(m_0^*, m_e^*, a) = \frac{K_m(m_e^* a) I_m'(m_0^* a)}{K_m'(m_e^* a) I_m(m_0^* a)}. \tag{19}$$

The above relation (Eq. 18) is highly transcendental and will have to be solved numerically. However, for $ka << 1$, one can show that $F(m_0^*, m_e^*, a) \approx 1$, so that the dispersion relation would reduce to,

$$[(x-\epsilon)^2-1](1-\alpha x^2)^{1/2} + \alpha(x^2-1)[1-(x-\epsilon)^2]^{1/2} = 0. \tag{20}$$

For long wavelengths, the phase speed of the kink mode is about equal to the so-called kink speed c_K which, in the low-β plasma is,

$$c_K \approx \left[\frac{2}{1+n_e/n_0}\right]^{1/2} c_{A1}, \tag{21}$$

where n_0 and n_e are the plasma concentrations inside and outside the loop, respectively, and c_{A1} is the Alfvén speed inside the loop.

It was shown by Nakariakov and Ofman [48] that the formula for the kink speed can be utilized to determine the magnetic field as follows :

$$B_0 = (4\pi\rho_0)^{1/2} c_{A1} = \frac{\sqrt{2}\pi^{3/2} L}{P} \sqrt{\rho_0 (1+\rho_e/\rho_0)}. \tag{22}$$

3.4. Sausage oscillations

Modulated coronal radio emission which have periodicities in the range 0.5 - 5 s have been interpreted in terms of a fast magnetoacoustic mode, the sausage mode, associated with the perturbations of the loop cross-section and plasma concentration by Nakariakov et al [70]. Quasi periodic pulsations of shorter periods (0.5 - 10 s) may be associated with sausage modes of higher spatial harmonics [43,48]. There have been quasi periodic pulsations in the periods 14 - 17 s, which oscillate in phase at a loop apex and its foot points which have been observed at radio wavelengths. Those modes have a maximum magnetic field perturbation at loop apex and nodes and at the foot points.
The dispersion relation for magnetoacoustic waves in cylindrical magnetic flux tubes has many types of long wavelength solutions in the fast mode branch (n = 0, 1, 2, ...) with the lowest ones called the sausage ($n = 0$) and kink mode ($n = 1$). Kink mode solutions extends all the way to the long wavelength limit ($ka \to 0$) while the sausage mode has a cut-off at a phase speed,

$$v_{ph} = v_{A2}, \tag{23}$$

which has no solutions for wavenumbers $ka < k_c a$.

The cut-off wavenumber k_c is given by,

$$k_c = \left[\frac{(c_s^2+v_{A1}^2)(v_{A2}^2-c_T^2)}{(v_{A2}^2-v_{A1}^2)(v_{A2}^2-c_s^2)}\right]^{1/2} \left(\frac{j_0}{a}\right). \tag{24}$$

Under coronal conditions the sound speed $c_s \approx 150 - 260$ km/s and Alfvén speed is $v_A \approx 1000$ km/s. Therefore,

$$c_s << v_A. \tag{25}$$

Here tube speed is similar to sound speed,

$$c_T \approx c_s. \tag{26}$$

The expression for the cut-off wavenumber reduces to,

$$k_c \approx \left(\frac{j_0}{a}\right) \frac{1}{[(v_{A2}/v_{A1})^2 - 1]^{1/2}} \tag{27}$$

For a typical density ratio in the solar corona (0.1 - 0.5), the cut-off wavenumber $k_c a$ falls in the range $0.8 \leq k_c a \leq 2.4$. Therefore, the long wavelength sausage mode oscillation is completely suppressed for the slender loops. The occurrence of global sausage modes therefore requires special conditions : (i) very high density contrast ρ_0/ρ_e, (ii) relative thick loops to satisfy $k > k_c$. The high density ratio $\rho_0/\rho_e >> 1$ or $v_{A2}/v_{A1} >> 1$ yields the following simple expression for the cut-off wavenumber k_c,

$$k_c a \approx j_0 (v_{A1}/v_{A2}) = j_0 (\rho_e/\rho_0)^{1/2}. \tag{28}$$

The cut-off wavenumber condition $k > k_c$ implies a constraint between the loop geometry ratio (2a/L) and the density contrast ratio (ρ_e/ρ_0) which turns out to be,

$$\frac{L}{2a} \approx 0.65\sqrt{\rho_0/\rho_e}. \tag{29}$$

Also it can be shown that the period of the sausage mode satisfies the condition,

$$P_{saus} < \frac{2\pi a}{j_0 v_{A1}} \approx \frac{2.62a}{v_{A1}}. \tag{30}$$

Observations of radio burst emission from the 'disturbed' Sun at meter wavelength seem to provide the bulk of available evidence for coronal oscillations [71]. One of the important reasons for this is the high temporal resolution with which data can be obtained. Also, according to Aschwanden [51], the favourable conditions for MHD oscillations occur mainly in the upper part of the corona. Since the radio emission observed in the meter wavelength range originate mainly at heights > 0.2 $R_\odot$ above the solar photosphere, they play an useful role in this connection. We recently analyzed the data obtained with the Gauribidanur radioheliograph [72] and Mauritius radio telescope [73] for quasi-periodic emission from the solar corona. The theory of MHD oscillations was used to determine the Alfvén speed and magnetic field. The estimated values are in the range 800-1200 km/s and 3-30 G, respectively [74-76].

4. Summary

In summary, the observational and theoretical investigations of solar coronal waves is a rapidly developing branch of solar physics. Kink and longitudinal MHD waves with periods 2 - 15 min are observed in the EUV. There is also strong evidence for the presence of shorter periodicities in radio and X-ray bands. High resolution observations of solar coronal wave activity provide us with a new powerful method for the investigation of the coronal plasma (coronal seismology). One of the most important missions to study the Sun is underway and is being developed in Japan. This is the

Solar-B spacecraft, which is scheduled to be launched in the year 2006 [77]. The detection of MHD oscillations is one of its main objectives. With the advancement of technology which will enable us to get more information of the Sun through observations with good resolution (both spatial and temporal), more theoretical improvements need to be carried out to interpret the observations in the years to come.

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