Asian Journal of Physics. Vol 13, Nos 3 & 4 (2004) 293 – 296

Black hole entropy: some skin deep subtleties sans strings

C Sivaram
Indian Institute of Astrophysics, Bangalore - 560034, India

The entropy increase by a huge factor when a star collapses into a black hole remains an enigma. In the recent past in attempt to understand the physics involved in the large number of microstates involved as well as other aspects like Hawking radiation it has become fashionable to invoke exotic objects like superstrings, D-branes, etc The black holes are assumed stuffed and stacked with these objects! Here an attempt has been made to understand several of these subtle features of black hole entropy and decay, linking them to 'skin deep' horizon properties We have also made connection with several remarkable recent experiments on coherent optical information storage Our braneless approach is mostly in the framework of familiar physical and astrophysical phenomena associated with general relativity

1. Introduction

The understanding of black hole entropy has been a long standing puzzle. What makes the entropy increase by such a huge factor [1] when a star collapses into a black hole is literally a mystery enshrouded in enigmas!

The difficulties in accounting for the vast number of states associated with the large increase in entropy has led in the recent past to several currently popular suggestions like packing the black hole [2] with exotic objects like superstrings, superbranes, D-branes etc (a black hole may not have hair but it could be very braney!) Despite some measure of success with these ideas it nevertheless appears rather strange that a massive star or cluster of stars made up of completely familiar matter (atoms, radiation, nuclei, etc ,) should suddenly transform into some thing made up of totally different exotic entities (like strings or branes) after it enters its Schwarzschild radius! In this context it may be pointed out that a 10^9 solar mass black hole while entering the horizon has an average density less than air One would not usually associate enigmatic states of matter at that density! Having said all this, the one thing which is definite, is that that black hole entropy is proportional to the area of the horizon, thus scaling as the square of the black hole mass This implies among other tings that a star around a solar mass increases its entropy by a factor of $\approx 10^{19}$ when it forms a black hole. This association of entropy only with the area (surface) of the horizon is embodied in the so called holographic principle [3, 4] This paper deals with the topic of black hole entropy with subtle connections with information theory and thermodynamics together quantum effects

2. Black hole entropy and quantum effects on the horizon

Classically the black hole horizon is just characterized by its Schwarzschild radius r_s, the temporal component of the metric tensor being given by $g_{00} = (1 - r_s/r)$ Thus at $r_s = r$, we have an infinite red shift (with no signal reaching the distant observer) However if one considers possible quantum effects [5], the horizon need not be exactly at $r = r_s$ but could differ by a Planck length, i.e , $r - r_s = L_{pl}$, where $L_{pl} \approx \sqrt{\hbar G/c^3} = 1\,6 \times 10^{-33} cm$. Since $r_s >> L_{pl}$, we now have instead of a vanishing g_{00} (as in the classical case),

$$g_{00} = 1 - \frac{r_s}{r} = \frac{(r - r_s)}{r} = \frac{L_{pl}}{r_s} \tag{1}$$

© Anita Publications All rights reserved

So that now a distant observer would instead of an infinite red shift factor of $g_{00}^{-1/2} \approx (r_s/L_{pl})^{1/2} \approx 10^{19}$ for a solar mass black hole (for the classical case, instead of a (finite) L_{pl} we would have $L_{pl} \to 0$, and recover the infinite red shift). **'Pinching the Horizon skin'** (membrane), by a **skin depth** L_{pl} has made this difference! The temperature as measured by a distant observer (T_∞) would also be red shifted by the same factor of 10^{19}, as in general relativity, we have

$$T(g_{00})^{\frac{1}{2}} = Constant \tag{2}$$

Thus

$$T_\infty \approx T_h(g_{00})_h^{\frac{1}{2}} = T_h\left(\frac{L_{pl}}{r_s}\right)^{\frac{1}{2}}, \tag{3}$$

$((g_{00})_\infty) = 1$

T_h is the temperature at the horizon of the co-moving observer collapsing with the matter For a solar mass star the matter heats locally to a temperature of $T_h \approx 10^{12} Degree$ as it collapses This can be seen as follows. An object of mass M collapsing on a time scale of $\approx GM/c^3$, releases energy at the rate of (i.e., has a luminosity of:) $\approx Mc^2/GM/c^3 \approx c^5/G$ Dividing by the area of the horizon ($\approx (GM/c^2)^2$) this would imply a co-moving temperature for the collapsing matter estimated from

$$\sigma.\left(\frac{GM}{c^2}\right)^2 T_h^4 \approx \frac{c^5}{G}, \tag{4}$$

or

$$T_h \approx \frac{B}{M^{\frac{1}{2}}}, \tag{5}$$

(where B is a constant made up of G,c,σ, etc) This for $M \approx M_\odot$, implies $T_h \approx 10^{12} Degree$. Combining eqns (3) and (5), we have

$$T_\infty = \frac{B}{M^{\frac{1}{2}}}\left(\frac{L_{pl}}{r_s}\right)^{1/2} = \frac{D}{M}, \tag{6}$$

(as r_s proportional to M), D being another constant In eq (6), D turns out to be $\approx \hbar c^3/GkB$, thus T_{infty} being just the Hawking Temperature T_H if the black hole [6], which for a solar mass is $\approx 10^{-7} Degree$ ($T_{infty} \propto 1/M$, scaling inversely as the black hole mass).

We now understand why the entropy has increased by a factor of 10^{19} The temperature measured by the distant observer T_{infty} (orT_h) is 10^{19} times lower and as entropy is $\approx E/T$, this implies an increase in entropy by a factor of $\approx 10^{19}$. To give a familiar example the sun emits photons whose average energy kT corresponds to $T \approx 6000K$ The earth absorbs this radiation, and radiates it back after a time in space in the **infrared** with average energy $T_E \approx 300K$ So for every photon absorbed by the earth, $6000/300 \approx 20$ photons are emitted back into space The Earth loses 20 units of entropy, (for every unit absorbed from the sun) so that the net entropy of radiation has gone up by a factor of 20! Surely, we do not associate anything exotic with this! The energy emitted from the black hole (Hawking radiation) can then be deduced from (6) One can also see this from Liuoville's Theorem, (i e , conservation of phase space) Thus if the red shift (or Doppler) factor is f, then the source emits as if it were at a temperature of T_0/f (where T_0 is the temperature in the source frame) and the energy flux scales as f^4 So eq (6) would imply that Hawking flux scales as $1/M^2$ (as $f \approx \sqrt{r_s/L_{pl}}$ which for a solar mass black hole implies an energy emission rate of $\approx 10^{-17} ergs/sec$. i.e , $c^5/G(L_{pl}/r_s)^2 \approx c^5/G(M_{pl}/M)^2$, M_{pl} being the Planck mass). We can also link entropy and information via Shannon's theorem [7]. In the present case, the bandwidth of the radiation has been redshifted by a factor of $r_s/L_{pl}^{1/2}$, so that the information transformation is also **reduced** by the same factor as $I \propto \Delta\nu$, $\Delta\nu$ is the bandwidth This is tantamount to the entropy increasing by the same factor which for a solar mass black hole is $\approx 10^{19}$. We can consider the entropy of Hawking's radiance as entropy (uncertainty about the state) of the noise, which is adulterating the signal conveying the information The radiance power E is the sum of the noise and signal powers and the maximum rate at which information can be recovered from radiation by a suitable detector is,

$$I_{max} \approx \frac{dS}{dt} - \frac{dE}{dt},$$

dS/dt is the maximum entropy rate possible for given E

Again the red shift factor increases (the wavelength) and **decreases** the energy of individual photons (bits) by such a huge amount ($\approx 10^{19}$) that they would not register on the detectors and hence lead to an information black out The probability distribution for the black hole to spontaneously emit n quanta in mode $[i, w]$ when substituted in Shannon's formula can be shown to give the right amount of information lost in the given mode [8]

3. Analogy with optical system

For closed systems in which the evolution of the quantum state is unitary between process and measurement, there is an intrinsic time symmetry. On the contrary time symmetry no longer holds in open systems into which information is irretrievably lost. The information loss rate in the black hole case is $\approx c^3/GM bits/sec$ One subtle aspect is that for **any black hole**, the Hawking Temperature implies at any given time **one photon** (of wavelength $\approx r_s$) in the entire volume $\approx r_s^3$ This can be **converted** into a **surface integral** by Gauss' Theorem, which implies an emission rate of $\approx 1/\Delta t \approx (c^3/GM) bits/sec$ through the horizon surface. One can quantify this with a master diffusion equation for quantum communication signals Why does the radiation (or information) appear to take so long to diffuse out of a black hole? This can be understood as a transport process on the horizon, each 'step' of random (Brownian) diffusion corresponds to L_{pl} So the time taken for the diffusion over the horizon is $t_d \approx (r_s^2/L_{pl})c$ Now the velocity of light in a gravitational field is less than its vacuum (or distant observer) value c_0 This is verified in the so called Shapiro time delay As is well known as far as the propagation of light [9] is concerned the gravitational field is like a refractive medium with a refractive index $\approx g_{00}$ So the velocity of light in the gravitational field is $c = c_0(1 - r_s/r)$ If $r_s = r$, the light is completely 'stopped' and c=0. However if as above, $r - r_s = L_{pl}$ then $c = c_0(L_{pl}/r_s)$, the external observer would see photons slowed down in an intense gravitational field Then the diffusion time t_d above is now given as [10]

$$t_d \approx r_s^2/(L_{pl}\, c_0 L_{pl}/r_s) \approx r_s^3/(c_0 L_{pl}^2) \approx G^2M^3/\hbar c_0^4, \tag{7}$$

(substituting for L_{pl}^2 and r_s^3) Eq (7) is precisely the Hawking evaporation time scale for the black hole For a solar mass black hole $t_d \approx 10^{71} sec$ and $c \approx 10^{-38} c_0$, so in this picture the horizon with its skin depth $\approx L_{pl}$, appears to an external observer to slow down light which now diffuses over a time t_d

In this context it is of much interest to note that in recent experiments [11], the group velocity of light in specially prepared media was brought to almost zero value leading to observation of coherent optical information storage [12] (in a very thin layer) In the L_{pl} skin deep horizon layer we had

$c \approx 10^{-38} c_0 \approx 10^{-28} cm/sec$. A similar formula to that (modified for the gravitational case) used in the above optical phenomena gives a value for the group velocity $\approx c_0 L_{pl}/r_s$ We can also evaluate the total amount of information or (degrees of freedom) bits stored using similar formulae [12] and [13] (i.e., for the number of modes N), which in this case for a cut off (skin depth) of wave number $k_c \approx L_{pl}^{-1}$ is:

$$N \approx \frac{V k_c^3}{48\pi^2} \tag{8}$$

Here V is volume of the thin layer given by $V \approx G r_s^2 L_{pl}$ With this eq (8), gives (substituting for r_s, L_{pl}, k_c, etc).

$$N \approx \frac{GM^2}{\hbar c}, \tag{9}$$

which is precisely the total number of microscopic states of the black hole

4. Concluding remarks

In the limit $L_{pl} \to 0$, $T_h \to 0$, $t_d \to \infty$, we have the classical black hole which is completely black. In principle we could have higher powers of L_{pl}/r_r in g_{00}, but their contributions would be much smaller as ($L_{pl} << r_s$). In conclusion, we have been able to have an understanding of several aspects of black hole entropy, decay, number of microstates etc , in the framework of well tested physical pictures and without invoking exotic (unknown) physical objects' (i.e , braneless and sans strings!)

References

1 Bekenstein J D, *Phys. Rev. D*, 7 (1973), 2333.
2 Klebanov T, *Nucl. Phys B*, 496 (1997), 231
3 Sivaram C, *Mod. Phys. Lett A*, 14 (1999), 2363
4 Susskind L, *J. Math. Phys.*, 36 (1995), 6377
5 Sivaram C, *Astrophys. J* , 520 (1997), 454
6 Sivaram C, *Phys. Rev. Lett.*, 84(2000), 3209
7 Shannon C E, *Bell Systems Journal*, (1949).
8 Sivaram C, (2004), In preparation
9 Landau L D, Lifshitz, *The classical Theory of Fields*, Pergamon Press, (1962).
10 Sivaram C, *Gen. Rel. Grav.*, 33 (2001), 1
11 Hau L et al , *Nature*, 397(1999), 594
12 Liu C et al., *Nature*, 409 (2001), 490
13 Phillips D et al , *Phys Rev. Lett*, 86 (2001), 783

Received : November 01, 2004 Revised December 03, 2004 Accepted December 07, 2004