COMMUNICATIONS IN
THEORETICAL PHYSICS
VOL. 1, 1991, 121–137

TORSION, TIME AND TEMPERATURE (THE T^3 THEOREM)

Venzo De Sabbata

World Laboratory, Lausanne, Switzerland
Dipartimento di Fisica Università di Ferrara, Italy
Istituto di Fisica Nucleare, Sezione di Ferrara

C. Sivaram

Indian Institute of Astrophysics, Bangalore, India

(Received 12 June 1991)

Introduction of torsion in general relativity, that is physically considering the effect of the quantum of spin $\hbar$ and linking the resulting torsion to defects in space-time topology, is shown to give rise to a new uncertainty relation (a relation between time and temperature) where a minimal time is present. Some consequences of the minimal time for field theory, evaporation of black holes, information theory, particle decay and for cosmology are outlined.

1. Introduction

The concept of time has played a crucial role in discussions involving any aspect of a physical theory especially in considering the evolution of a physical system consisting of one or many objects including fields and particles. In Newtonian mechanics, time and space are distinct entities (we have the notion of absolute time) since fields propagate at infinite speeds. In special relativity, the finite speed of light or electromagnetic signals through space interlinks space and time. This gives a geometrical role for the velocity of light as it is now connected to the topologically invariant signature and dimensionality of space, i.e. we now have an invariant space-time interval rather than a purely spatial interval between neighbouring events. This interlinking of time with light velocity leads to the well known dilatation of time (or in alternate terms to

Doppler shifting of frequencies), both well tested experimentally. The topologically invariant interval also carries over into general relativity (i.e. when gravitational forces are present). This also has the consequences of time dilatation and redshifting of frequencies in the presence of gravitational fields showing that time is affected by presence of gravity.

Although this interconnection of space and time (with or without curvature!) represents considerable conceptual progress, the Newtonian problems with the singular behaviour of vanishing spatial and temporal co-ordinates of point particles (manifested for example by divergences in self energy) still persist. Again thermodynamically time has been linked to entropy especially in considering the notion of a time arrow. In cosmology the notion of zero time associated in big bang models with the instantaneous creation of matter is taken as an indication of the occurrence of an inevitable singularity suggesting a breakdown of the concepts involved.

In quantum mechanics, time is linked to energy via the uncertainty principle. Here the Planck's constant $\hbar$ plays a very fundamental role. In fact $\hbar$ has the units of energy × time! This suggests that $\hbar$ might have a basic role to play in the concept of time at microscopical scales. Similar to a geometrical role for 'c' through special and general relativity as defining a topologically invariant interval we can think of a geometrical role of '$\hbar$' in the structure of space-time at very small scales. In this context we note that '$\hbar$' enters into quantum mechanics by virtue of being the basic unit of intrinsic spin and therefore its interaction with the underlying geometry must necessarily give rise to torsion. Since '$\hbar$' is energy × time, and energy gives rise to space curvature, it is suggestive that time may be linked to torsion as '$\hbar$' is also the unit of intrinsic spin which is the source for torsion. So '$\hbar$' has a dual aspect: it is the source for both curvature and torsion geometrically, and physically it is the product of energy and time. So we have:

$$\hbar \Leftrightarrow \text{energy} \times \text{time}$$

$$\Leftrightarrow \qquad \Leftarrow$$

spin curvature torsion

and

zero point energy

2. Torsion, Space-Time Defects and Minimal Time

We can directly link all this with our recent work[1,2,3], wherein it was suggested that torsion gives rise to defects in space-time topology. We know that in the geometrical description of crystal dislocations and defects, torsion plays the role of defect density (in this context we consider space-time as an elastic deformable medium in the sense of Sakharov).

If we consider a small closed circuit and write :

$$l^{\alpha}=\oint Q^{\alpha}_{\beta\gamma}\, dA^{\beta\gamma},$$

where $dA^{\beta\gamma}=dx^{\beta}\wedge dx^{\gamma}$ is the area element enclosed by the loop and $Q^{\alpha}_{\beta\gamma}=\Gamma^{\alpha}_{[\beta\gamma]}$ is as usual the torsion associated with the connection $\Gamma^{\alpha}_{\beta\gamma}$, then l^{α} represents the closure failure, i.e. torsion has the intrinsic geometric meaning of the failure of the loop to close, analogous to the crystal case, l^{α} having the dimensions of length. In the above equation torsion can be related to the fundamental unit of intrinsic spin $\hbar$, by postulating that defects in space-time topology at the quantum level should occur in multiples of the Planck length $\left(\hbar G/c^{3}\right)^{1/2}$; i.e. we have

$$\oint Q dA \cong n\left(\hbar G/c^{3}\right)^{1/2} \tag{1}$$

Time would thus be defined in the quantum geometric level through torsion as

$$t=(1/c)\oint Q dA=n\left(\hbar G/c^{5}\right)^{1/2} \tag{2}$$

so torsion is essential to have a minimum unit of time $\neq 0$!

This in fact would give us the smallest definable unit of time as $\left(\hbar G/c^{5}\right)^{1/2}\cong 10^{-43}$s. In the limit of $\hbar \Rightarrow 0$ (classical geometry of general relativity) or $c \Rightarrow \infty$ (Newtonian case), we would recover the unphysical

$t \Rightarrow 0$ of classical cosmology or physics. So both $\hbar$ and c must be finite to give a geometric unit for time (i.e. $\hbar \Rightarrow 0$ and $c \Rightarrow \infty$ are equivalent). The fact that $\hbar$ is related to a quantized timelike vector, discretize time. This quantum of time or minimal unit of time also correspondingly implies a limiting frequency of $f_{max} \approx \left(c^5 / \hbar G\right)^{1/2}$. This would have consequences even for perturbative QED, in estimating self energies of electrons and other particles, i.e. the self energy integral (in momentum space) taken over the momenta of all virtual photons. To make the integral *converge* Feynman in his paper on QED[4], multiplied the photon propagator, k^{-2}, by the *ad hoc* factor : $-f^2/(k^2 - f^2)$, where k is the frequency (momentum) of the virtual photon. This convergence factor, although it preserves relativistic invariance, is objectionable because of its ad hoc character without any theoretical justification. Feynman considers f to be arbitrarily large without definite theoretical basis. Here the presence of space-time defects associated with the torsion due to the intrinsic spin would give a natural basis for the maximal value for f^2_{max} as (from eq. (2)) $\approx c^5 / G\hbar \approx 10^{96}$ (and extremely large as required by Feynman), giving finite result (instead of ∞) for the self energy. This makes f_{max} another fundamental constant for particle physics serving as a high frequency cut off which is not arbitrary.

Again in a recent paper[5] we had pictured particles with rest mass m as vortices, which would give them a life time related through torsion as (i.e. analogous to charge being connected to $\int BdA$ we had mass related to $\int QdA$ exploiting analogy between torsion and magnetism):

$$t \cong (1/c)\int QdA \cong \hbar / mc^2 \tag{3}$$

which is just the lifetime of quantum particles.

How can we understand the connection between eqs. (2) and (3)? By having an energy-dependent G! This is clearly stated in refs.[7,8,9]. We have

$$G \approx \hbar c / m^2. \tag{4}$$

For $m = M_{pl}$, we have $G = G_N$. For strong gravity with $m = m_P$, we have $G = G_f$ and so on. Substituting (4) in (2), we arrive at eq. (3), i.e. we have for strong (hadronic) interactions, the basic time unit (or scale) :

$$t_h = (1/c)\int Q dA = \left(\hbar G_f / c^5\right)^{1/2} = \hbar / m_P c^2 \approx 10^{-23} \mathrm{s}. \tag{5}$$

Similarly for weak interactions, where we have $G = G_w = G_F (c/\hbar)^2$ (G_F the Fermi constant) and $m = m_w = 250\,Gev$, $t_w = \left(\hbar G_w / c^5\right)^{1/2}$ and so on.

So we have space-time defects with defect lengths l^α scaling with energy as E^{-1}. In this picture, the existence of different interactions with characteristic time scales (or lifetimes of interacting particles) is brought out by the purely geometric existence of topological defects at different energy scales induced by torsion owing to the intrinsic spin $\hbar$. As shown in refs.[5,6,7,8,9] whatever be the energy scales, $\hbar$ is invariant.

Thus the postulate of the existence of defects with different length scales in space-time due to torsion gives rise to the different fundamental interactions, the strength being fixed by the value of the $G_{\rm eff}$ related to l^α through eqs. (2), (3) and (5). In this picture the existence of the different defect lengths l^α is primary (related to $\hbar$ and torsion) and the interactions, masses and lifetimes (and also charge as we have seen in previous works[5,6,8,10]) are secondary, i.e. derived concepts.

So the absence of torsion implies absence of defects in space-time and consequently absence of masses, charges and of the very existence of time!

Defects have to be localized in space-time. In earlier papers[1,6,8] we had understood charge and mass of particles as arising from fluxes defined by Gauss's theorem over a closed surface characterizing the defect. Here we have extended the concept by defining time through eqs. (2) and (3).

However massless particles like neutrinos or photons (which always move at light velocity) are *not localized* in space-time. Defects are to be localized in space-time! So since neutrinos and photons are not defects, we can understand in this picture why they have no electric charge or mass, i.e. one cannot define a Gauss's theorem over a closed surface characterizing the defect so that there is no mass and charge. By eq. (3) this also implies that for $m \Rightarrow 0$ (i.e. no closed surface) the time scales are infinite, i.e. a massless particle cannot decay. So time defined through torsion has also these interesting implications. Eqs. (2), (3) and (4) would suggest that mass is generated by interactions and also time : time has no meaning at quantum level without interactions, i.e. when $G_{\text{eff}} \Rightarrow 0$ also $t \Rightarrow 0$.

3. Time-Temperature Uncertainty Relation

$\hbar$ is linked to energy through time as :

$$\text{time} \Rightarrow \hbar/\text{energy, i.e. we have } \hbar\nu = \text{energy} = E. \tag{6}$$

We have also the thermodynamic definition

$$E = k_B T \tag{7}$$

where k_B is the Boltzmann constant (the unit of entropy : see [9]) and t is the temperature.

Eqs. (6) and (7) imply the relation :

$$\text{time} \times \text{temperature} = \hbar / k_B = \text{constant}, \tag{8}$$

i.e. we have

$$\Delta t\, \Delta T = \hbar / k_B = 10^{-27} / \left(1.3 \times 10^{-16}\right) \approx 10^{-11}. \tag{9}$$

Eq. (9) is universally valid as can be seen by several examples. In the early universe, for $\Delta t = 10^{-43}$s, we have $\Delta T \approx 10^{32} K$ (from eq. (9)). This is just the temperature $T \approx \left(\hbar c^5 / G_N\right)^{1/2} \left(1 / k_B\right) \approx 10^{32} K$, at the Planck epoch. At the hadron era $\Delta t \approx 10^{-23}$s, $\Delta T \approx 10^{12} K$, the hadronic temperature $\left(m_p c^2 / k_B \approx 10^{12} K\right)$, etc. $\approx (\hbar c^5 / G_f)^{1/2} (1 / k_B)$. Also for a typical electromagnetic interaction time scale $\approx 10^{-16}$s, we have the corresponding temperature from eq. (9) as $T = 10^5\,K$, which is just that corresponding to the Rydberg theory (≈ 13 ev) for ionized atoms.

So we can say that the existence of these times and temperatures in the early universe (as implied by eq. (9)) fixes the strength of the dominating interactions at the different epochs, G_N at t_{pl}, G_f at t_{hadron} etc., i.e. G is automatically determined, once eq. (9) is assumed.

The time-temperature uncertainty relation eq. (9) is valid in also curved space-time. In general in curved space-time, the temperature T is modified as :

$$T(g_{oo})^{1/2} = \text{const.} \tag{10}$$

But we also know that frequencies are redshifted in a gravitational field, i.e.

$$\upsilon(g_{oo})^{1/2} = \text{const.}$$

This is dilated in a gravitational field as $t(g_{oo})^{-1/2}$ (in general $\Delta\upsilon\Delta t = 1$!). So we have :

$$\Delta T(g_{oo})^{1/2} \cdot \Delta t(g_{oo})^{-1/2} = \Delta T\Delta t = \hbar / k_B = \text{const.}$$

So the time-temperature uncertainty relation also holds in curved space and $\hbar$ and k_B are universal constants even in the presence of gravitation. In a recent paper[11] we had related temperature to curvature K (via acceleration) as:

$$a = c^2(K)^{1/2}, \quad T = \hbar c K^{1/2} / k_B\,, \tag{11}$$

i.e. temperature scales as square root of curvature.

Eq. (9) would then imply that time would scale inversely as square root of curvature, i.e.

$$t \propto 1 / K^{1/2}. \tag{12}$$

For the maximal curvature, $K_{max} = c^3 / \hbar G \approx 10^{66} cm^{-2}$, eq. (11) would imply a temperature $\approx 10^{32} K$ at the Planck epoch which corresponds (from eqs. (10) and (12)) to the time $\approx 10^{-43} s = t_{pl}$.

Now we had shown in other recent works[12,13] that the entropy of the universe would scale inversely as the square root of curvature (with total energy a fixed value) i.e.

$$S \propto 1/K^{1/2}. \tag{13}$$

Thus eqs. (12) and (13) seem to indicate that time and entropy have the same direction of increase with decreasing curvature as the universe expands.

Now entropy in general is defined as :

$$S = k_B ln\ w \tag{14}$$

w being the total number of microstates characterizing the system, k_B would correspond to the minimal unit of entropy, i.e. the smallest value which is not zero[14,15,16]. The analogy between entropy and time would suggest that we should have a similar relation to eq. (14) to describe time statistically in terms of a large number of discretized temporal events. This can come about naturally if we consider for example inflationary expansion in the early universe, for which we can write :

$$t = H_{pl}^{-1}\ ln\ R \tag{15}$$

This follows from the relation for the scale factor

$$R = R_{pl}\ \exp\left(H_{pl} t\right), \tag{16}$$

i.e. exponential expansion of the scale factor with t. Here H_{pl} is the "Hubble constant" for the inflationary expansion, i.e.

$$H_{pl} \approx \left(G\rho_{pl}\right)^{1/2} \tag{17}$$

$\left(\rho_{pl} \approx c^5/G_N \hbar\right)$. H_{pl} is just the inverse of the Planck time t_{pl}. We can consider the expansion to increase in a number of consecutive stages which would imply eq. (15) $R = R_1 \ldots R_n$.

Corresponding to k_B (minimal entropy) we have H_{pl}^{-1} as minimal time unit (compare eqs. (14) and (15)).

4. Black Hole Evaporation and Minimal Time

We can explore the consequences of the existence of a minimal unit of time i.e. $\approx t_{pl}$, for black hole evaporation. We can understand black hole evaporation in a different manner as a process of quantum diffusion. For an observer falling into the black hole towards the centre there is no longer the singular behaviour of classical general relativity. The minimal length and time are l_{pl} and t_{pl} (and not zero). However for an outside observer all events within the horizon are inaccessible. So his smallest interval is $t_{hor} \approx 2GM/c^3 \approx 10^{-5}$s for a solar mass black hole. The redshift factor is no longer ∞ but $GM_{\odot}/c^3\left(\hbar G/c^5\right)^{1/2} \approx 10^{38}$ owing to the existence of a minimal time.

As far as the behaviour of quanta within the horizon is concerned, we can consider them to diffuse *inside* the horizon with a mean free path $\approx l_{PL}$ (i.e. matter does not collapse to ∞ density). Similar to the case of radiation diffusion within an ordinary star, we can consider the radiation inside the event horizon to random walk. So by the analogy to the case of the ordinary star, the diffusion length is estimated as follows: the total number of scatterings is given by :

$$\sqrt{N} = R_s/l_{pl} \approx \left(GM/c^2\right)/\left(\hbar G/c^3\right)^{1/2} \tag{18}$$

$$N \approx \left(GM/c^2\right)^2 . c^3/\hbar G = \left(G^2M^2/c^4\right)\left(c^3/\hbar G\right) = GM^2/\hbar c. \tag{19}$$

Total distance travelled by a quantum of radiation is

$$N.l_{pl} = \left(GM^2/\hbar c\right).\left(\hbar G/c^3\right)^{1/2} \tag{20}$$

As measured by an observer inside the horizon, the time taken for the radiation to travel a distance R_s is given by

$$t_s = \left(GM^2/\hbar c\right)\left(\hbar G/c^5\right)^{1/2} \tag{21}$$

(the quantum of time enters the formula).

For an observer outside the horizon, this would be multiplied by the redshift factor found earlier, i.e. $GM_{\odot}/c^3\left(\hbar G/c^5\right)^{1/2}$, so that we have the time scale for the radiation to leak out of the black hole as

$$t_h = G^2M^3/\hbar c^4 \tag{22}$$

which is the same as Hawking's formula! We can write

$$t_h = \left(GM^2/\hbar c\right)\left(GM/c^3\right) = \left(GM^2/\hbar c\right)\, t_{h(\text{smallest})}. \tag{23}$$

For $M = M_{pl}$, this just becomes : $t = \left(\hbar G/c^5\right)^{1/2}$, the quantum unit of time. So we have a scaling law for evaporation time :

$$t_h = \left(\hbar G/c^5\right)^{1/2}\left(M/M_{pl}\right)^3 \propto M^3 \tag{24}$$

to be compared with the scaling for entropy $\propto \left(M/M_{pl}\right)^2$, spin $\propto \left(M/M_{pl}\right)^2$, charge $\propto \left(M/M_{pl}\right)$ etc.[9,15,16].

So the existence of a minimal time is crucial in understanding quantum evaporation. For zero minimal time we have ∞ redshift factor and ∞ time for evaporation. If N be the number of scattering, the probability that a distance 'd' is travelled, is given by the Fokker-Planck equation for $P(d, N)$ [17]:

$$\partial P/\partial N = \left(\Delta^2/d^2 6\right)\nabla^2 P \text{ (where } \Delta \text{ is mean free path per scattering)}$$

which has the solution :

$$P(d,N) = \left(N\Delta^2/d^2 6\right)^{-3/2} \exp\left(-d^2/N\Delta^2\right). \tag{25}$$

The probability is ≈ 1 if the number of scattering is $N \simeq d^2/\Delta^2$, i.e. $\sqrt{N} = d/\Delta \left(\text{here } \Delta = \left(\hbar G/c^3\right)^{1/2},\ d = GM/c^2\right)$. This was the relation we had used above.

The above diffusion equation also holds in general relativity as it is generally covariant. So our result is exact. It only depends on there being a *minimal* time!

As far as the outside observer is concerned, the temperature associated with the horizon is $T_h = \hbar c^3/Gk_B M$, and the minimal time scale is $t_h = GM/c^3$, so that the Temperature-time relation, i.e. eq. (9) $T_h \cdot t_h = \hbar/k_B$ is satisfied. For the outside observer the temperature is redshifted by a factor of $\approx 10^{38}$ (as compared to Planck temperature) i.e. being $T_{pl} = 10^{32}K$, is $T_h \approx 10^{-6}$ (for $\sim M_\odot$ black hole).

The minimal time scales are also redshifted correspondingly, i.e. 10^{-43}s becomes 10^{-5}s. But the product remains constant i.e. $\Delta T . \Delta t =$

$\hbar / k_B = 10^{-11}$ (that is 10^{-5}s.$10^{-6}k = 10^{-11}$).

For the case of infinite redshift, the horizon temperature would be zero, just as in classical general relativity. So the existence of a finite temporal and spatial unit gives finite value for the horizon temperature and evaporation time.

5. Minimum Temperature Operationally Definable

As regards the minimal temperature it can be related to minimal possible energy allowed in the cosmological context. Also from time-temperature uncertainty relation $\Delta t \Delta T \approx \hbar / k_B$ we have that the maximal time is related to the Hubble H_0 constant being

$$\Delta t_{\max} \approx 1/H_0 \tag{26}$$

is

$$\Delta T_{\min} \approx \hbar H_0 / k_B \approx 10^{-27}.10^{-18}/10^{-16}. \tag{27}$$

So the smallest possible operationally definable temperature is

$$\Delta T_{\min} \approx 10^{-29} K. \tag{28}$$

Here $\hbar H_0$ can have the interpretation as the minimum amount of energy that can be operationally defined in a closed universe[18].

In general one can not reach absolute zero. One can come arbitrarily close (at present we have reached $\approx 10^{-6}$ K). It is remarkable that this minimal temperature can also be arrived at by considering a black hole of maximum possible mass, i.e. mass of the universe $\approx 10^{55}\,g \approx 10^{60}\,M_{pl}$. Since temperature of a black hole scales inversely of mass: $T \approx \hbar c^3 / G K_B M$, maximal possible $M_{\max}$ will give minimum possible operationally definable temperature which is consistent with (28)!

Also from entropy considerations we have the maximal possible entropy of $\approx 10^{120} k_B$ [13]. This implies minimum temperature of $\approx 10^{-29} K$.

6. Other Consequences of Existence of a Minimal Unit of Time

Another implication of minimal time is connected with the information theory. In fact the minimal time of $\approx 10^{-43}$s implies a maximal possible information processing rate of $\approx 10^{43}$bits/s !

So no computer can ever process information faster than 10^{43}bits/s.

This is the *absolute maximum* possible. In general the theoretical limit given by Brillouin, is temperature dependent, i.e. at a temperature T the energy expended in ordering one bit of information (binary unit) is $\approx k_B T \ln 2$. So for an energy E or power supplied of P, the maximum thermodynamically allowed processing rate is

$$I_{max} = P / k_B T \ln 2 \quad \text{bits/s} \cdot \tag{29}$$

For $P = 1$ watt $= 10^7$erg/s at $T \approx 3 \cdot 10^2 K$ (room temperature) we have

$$I_{max} = 10^7 / 10^{-16}\, 3 \cdot 10^2 \text{bits/s} \approx 10^{20} \text{bits/s}. \tag{30}$$

The above absolute upper limit of 10^{43}bits/s is *independent* of *temperature* and of power expended, purely arising from the existence of a minimal time unit.

Now we like to examine some implications for beta decay : the decay time in weak interaction β–decay scales inversely as M^5, the mass of the particle. So we have :

$$M_\mu^5 / M_\tau^5 = t_\tau / t_\mu \tag{31}$$

where t_τ, t_μ are lifetimes for muon and τ-particle (leptons) respectively. So the existence of a minimal lifetime of 10^{-43}s would imply an upper limit to the mass of any lepton decaying by β–decay (lepton by definition can take part in only β–decay and not decay by strong or electromagnetic interactions to conserve lepton number). So if $t_\mu \approx 10^{-6}$s an $t_{min} \approx 10^{-43}$s, we can have an upper limit to lepton mass as :

$$M_{\max}/M_{\mu} \approx \left(t_{\mu}/t_{\min}\right)^{1/5} \approx 10^{7}. \tag{32}$$

This therefore gives an upper limit to mass of particle undergoing β-decay as (or maximum lepton mass) :

$$M_{\max} \approx 10^{7} M_{\mu} \approx 10^{6} Gev = 10^{3} Tev. \tag{33}$$

Since this would be within the energy range of the next generation of accelerators (currently ≈ 40 *Tev*), this can be tested experimentally.

Similar limit can exist for maximum mass of pseudoscalar meson undergoing decays like $\pi^0 \Rightarrow 2\gamma$ (π^0 is the lightest such particle). Here the scaling is like M^{-3}. π^0 has decay time of $\approx 10^{-16}$s. We have

$$M_{\max,\ \mathrm{scalar}} \approx 10^{9} m_{\pi^0} \approx 10^{8} Gev = 10^{5}\, Tev. \tag{34}$$

This in turn would give maximum energy of photons in decay.

The evaporation of black hole need not be in contradiction with the hypothesis that the total energy of a black hole is zero (i.e. no gravitational field outside). The black hole decay can be understood purely as a quantum mechanical phenomenon dictated by the uncertainty principle. The lifetime of evaporation of a solar mass black hole for instance is $\approx 10^{71} s$. So from the uncertainty principle this would imply an energy exchange of $\Delta E \approx \hbar / \Delta t$ (almost zero) for Δt very large. In any case the quantum effects dominate only for very small black holes $\approx 10^{15} g$ (Hawking holes). For the case of such black holes :

$$GM / c^{2} \leq \hbar / m_{\pi} c \approx 10^{-13}. \tag{35}$$

So even if the total internal energy of the hole is zero[16], by the fact that the size of the horizon is $< \hbar / m_{\pi} c$, particle pairs would still be created purely by quantum mechanical uncertainty principle effects.

If $GM/c^2 \le \hbar/m_e c$, (m_e is the electron mass) e^+e^- pairs would be created and so on. So by uncertainty principle, the rate of energy emission by creation of virtual pairs is

$$dE/dt \approx m_\pi c^2 / \left(\hbar / m_\pi c^2\right) = m_\pi^2 c^4 / \hbar \qquad (36)$$

and since

$$\hbar / m_\pi c \approx GM/c^2 \text{ (for significant evaporation effect)}, \qquad (37)$$

we have :

$$dE/dt \approx m_\pi c^2 / \left(GM/c^3\right) = m_\pi c^5 / GM = [\text{from } (37)]\, \hbar c^6 / (GM)^2 \qquad (38)$$

and for lifetime :

$$t \approx Mc^2 / \hbar c^6 / (GM)^2 = G^2 M^3 / \hbar c^4 \qquad (39)$$

which is exactly the same as Hawking's formula. So the Hawking process can be considered purely as a quantum mechanical effect arising from the horizon size of small black holes being of the order of particle Compton wavelength and therefore subject to uncertainty principle effect.

7. Black Hole Decay as a Gravitational Analogue of Zeldovich-Popov Effect

Zeldovich and Popov pointed out[19] that when the atomic number of a nucleus exceeds a certain critical value, i.e. $Z > Z_{crit} \approx 170$, the electrostatic binding energy ($= Ze^2/r$) of the K-shell electrons becomes of the order of the negative of the electron rest mass energy ($-m_e c^2$), i.e. $Ze^2/r \approx m_e c^2$. This can also roughly be seen from the well known fact that when $Z\alpha > 1$ (α is the fine structure constant) the Dirac equation has negative energy solutions. This is for a point nucleus, but when corrections are made for a finite nuclear size, Z_{crit} turns out to be $\approx$ 170. This signals instability of the vacuum of electron-positron pairs surrounding the nucleus giving rise to spontaneous production of e^+e^- pairs out of the vacuum i.e. the virtual pairs now manifest as real pairs. Since the total energy, i.e. binding + rest energy, is zero we have conservation of energy. So one can argue that the field strength is sufficiently large for the pairs to be created.

In the black hole case, we have the gravitational analogue of the above effect. This happens because the gravitational binding energy at the Schwarzschild radius is of the order of the rest energy, i.e. $GMm/r \approx mc^2$, so that the total energy is again zero (confirming the view that gravitational field outside a black hole is zero[20]). We thus have again the instability of the vacuum giving rise to spontaneous production of particle pairs, at a rate solely dependent on the size of the hole. So uncertainty principle gives the rate of energy production as estimated above. This happens even if the total internal energy of the hole is zero, analogous to the total internal energy of the atomic system being zero in the Zeldovich-Popov case.

REFERENCES

1. V. De Sabbata and C. Sivaram, Torsion and quantum Effects in vol. devoted to 85th jubilee of D.D. Ivanenko, (World Sci. Publ., Singapore; 1991).

2. V. De Sabbata, C. Sivaram, H.-H. v. Borzeszkowski and H. -J. Treder, *Annalen der Physik* (volume of centenary), 1991.

3. V. De Sabbata and C. Sivaram, *Intern. Journal Theor. Phys.* **30**, 123 (1991).

4. P. A. M. Dirac, New Directions in Physics, (J. Whiley, N.Y.; 1978).

5. V. De Sabbata and C. Sivaram, Universality of charge arising from torsion *Foundation of Physics* (submitted) (1991).

6. V. De Sabbata and C. Sivaram, *Nuovo Cimento* **105**, 1181 (1990).

7. V. De Sabbata and C. Sivaram, *Astrophys. and Space Sci.* **158**, 347 (1989).

8. V. De Sabbata and C. Sivaram, Some consequences of the torsion model of the electron for particle physics and cosmology, *Internat. Journ. of Theoret. Physics* (submitted) (1991).

9. V. De Sabbata and C. Sivaram, Entropy and spin of black holes, *Ann. der Physik* (submitted) (1991).

10. V. De Sabbata and C. Sivaram, *Ann. der Physik* **47**, 511 (1990).

11. V. De Sabbata and C. Sivaram, *Astrophys. and Space Sci.* (1991).

12. V. De Sabbata, C. Sivaram and Dingxiong Wang, *Ann. der Physik* (submitted) (1991).

13. V. DE SABBATA AND C. SIVARAM, Torsion, quantum effects and the problem of cosmological constant, in vol. devoted to 75th jubilee of Peter Gabriel Bergmann, (Plenum Press N.Y.; 1991).

14. V. DE SABBATA, C. SIVARAM AND DINGXIONG WANG, *Ann. der Physik* 47, 508 (1990).

15. V. DE SABBATA AND C. SIVARAM, The final state of an evaporating black hole, in '*Quantum Mechanics in curved space-time*' ed. by J. AUDRETSCH AND V. DE SABBATA, (Plenum Press; 1991).

16. V. DE SABBATA AND C. SIVARAM, Charge and Magnetic field of black holes, *Ann. der Physik* (submitted) (1991).

17. L. D. LANDAU, *Sov. Phys. J. Exp. Theor. Phys.* **7**, 203 (1937).

18. C. SIVARAM, *Amer. J. Phys.* **50**, 279 (1982).

19. YA. B. ZELDOVICH AND V. POPOV, *Sov. Phys. Uspekhi* **14**, 673 (1972).

20. V. DE SABBATA AND K. TAHIR SHAH, *Lett. Nuovo Cimento* **23**, 409 (1978).