THE FINAL STATE OF AN EVAPORATING BLACK HOLE AND THE DIMENSIONALITY OF THE SPACE-TIME

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1. Introduction

One of the main paradoxes of the contemporary physics of elementary particles is the apparent incompatibility of two main theoretical foundations: on the one hand the theory of general relativity which connects the force of gravity to the structure of space-time, on the other hand the theory of quantum mechanics. General relativity has been developed mainly to understand phenomena on a cosmic scale and the evolution of universe, while quantum mechanics regards mainly the atomic, subatomic and subnuclear world; this latter theory has been formulated for three of the four forces of nature, namely the strong, weak and electromagnetic interactions. In recent times unification of electromagnetic and weak interactions has been reached through gauge theories and one can try to include also strong interactions. Gravitation appears to be the more elusive to include in a true unification with all the other interactions and this may be due to the fact the energy at which gravity and quantum effects become of comparable strength, that is the energy at which one may hope to have unification of gravitational interaction with the other interactions is given by the so-called Planck energy

$$E_{Pl} = \left(\frac{\hbar c^5}{G}\right)^{1/2} \approx 10^{19}\,\mathrm{Gev}$$

But perhaps the main difficulty for unifying gravitation with other interactions lies in more deep property of this phenomenon known as 'force of gravity'. In fact, according to Einstein, gravity is not a force at all but is an intrinsic property of the space and time: this follows from the chief, very peculiar fact, that is the famous "equivalence principle" for which inertial and gravitational masses are equivalent.

So we are faced with the situation for which while 3/4 of modern physics (the physics of strong, weak and electromagnetic interactions acting at a microscopical

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level) are successfully described at present in the framework of a flat and rigid space-time structure, the remaining 1/4 (the macroscopic physics of gravity) needs the introduction of a curved, dynamic geometrical background.

In order to overcome this dichotomy, first of all we will try to extend the geometrical principles of general relativity also to microphysics, with the aim to establish a direct comparison, and possibly a connection, between gravity and the other interactions. Now we know that in general relativity matter is represented by the energy-momentum tensor which provides a description of the mass density distribution in space time, so that the mass-energy concept is sufficient to define the properties of the classical, macroscopic bodies; but if we go down to a microscopical level, we find that matter is formed by elementary particles which are characterized not only by mass but also by a spin. In that case, therefore, the energy-momentum tensor alone is no longer sufficient to characterize dynamically the matter sources, but also the spin density tensor is needed and the simplest and more natural way to take account of the spin in the Einstein theory, is the introduction of torsion, that is the antisymmetric part of the affine connection.

Of course this does not exhaust the problem to reconcile quantum mechanics with gravity and the aim of this course is to understand better the influence of quantum effects on gravity, or, viceversa the influence of gravitational fields on the quantum mechanics. This route seems to constitute a further step toward unification of all forces, including gravity and we believe also that the argument of quantum mechanics in curved space constitutes a necessary step to go toward the more difficult subject of quantum gravity. We like to understand better the influence of external gravitational fields on quantum matter and for that reason we have chosen to inquire what happens in the final state of an evaporating black hole. It is a problem that is strictly connected with the study of quantum fields in the presence of strong gravitational fields. We can say that the massive modes of closed superstring theories may play a crucial role in the last stages of black hole evaporation, and if the Bekenstein-Hawking entropy describes the true statistical entropy (the true degeneracy) of an evaporating black hole, it becomes favorable (entropically) that the black hole makes a transition to an excited state of massive string which, in turn, can decay to massless radiation, avoiding the naked singularity and also preserving quantum coherence as we will see.

2. Black hole evaporation

Are well known the consideration of Hawking [1] about the evolution of quantum field in background metric of classical black hole. He find that particle creation takes place and his semiclassical treatment of evaporation is valid if Schwarzschild radius >> Compton wave length i.e. $GM/c^2 \gg \hbar/Mc$, or $GM^2/(\hbar c) \gg 1$.

This quantity is also proportional to the entropy or area of the black hole. Emitted particles build a thermal spectrum and are uncorrelated among themselves. As black hole loses masses, its temperature rises, and evaporation accelerates (the rate is proportional to M^{-2}) as mass approaches the Planck mass = $(\hbar c^5/G)^{1/2}$ $\approx$ 10^{19}Gev). However at this stage the Compton length is comparable to the Schwarschild radius, i.e. GM/c^2 $\approx$ $\hbar/Mc$ for $M = M_{Pl}$, i.e. $GM^2_{Pl}/\hbar c \approx 1$, so that a quantum treatment of the gravitational field is required. The semiclassical approach would suggest explosive decay of black hole of mass M_{Pl} (temperature $\approx$ $(1/k)(\hbar c^5/G)^{1/2}$ $\approx$ 10^{32}) in a burst of duration $(\hbar G/c^5)^{1/2}$ $\approx$ 10^{-43}sec. In the absence of any conservation law prohibiting the decay one might expect decay into various elementary field quanta.

This explosive disappearance when M reaches M_{Pl} would leave behind residue of thermal radiation which is a mixed state,in quantum mechanics term, i.e. the initial pure state has evolved into a mixed state of thermal radiation as seen by an observer at infinity. Such conversion would violate the fundamental tenets of quantum mechanics in flat space such as loss of quantum coherence; it appears also inconsistent with quantum unitary postulate: "time evolution is governed by unitary operator in Hilbert space of states" which will be maintained in a pratically stable remnant. The emitted quanta are an uncorrelated ensemble and cannot be described by a single wave function; some information about emitted radiation is contained inside event horizon in the form of correlations between photons absorbed by black holes and those emitted to infinity; i.e. unitary postulate is maintained by correlating each state ψ_n of radiation with a corresponding state Φ_n of black hole so that the joint system has a well defined overall wave function $|\psi_{tot}\rangle = \Sigma A_n |\psi_n^{rad}\rangle \times |\phi_n^{bh}\rangle$ that is we have a quantum state inside a black hole combined with thermal state of radiation to construct a pure state $|\psi_{tot}\rangle$ so that the unitarity is maintained.

Hawking process consists of pair creation in strong gravitational field, that is one member is emitted to infinity and the other is staying near horizon. When the degrees of freedom of the black hole are integrated out, external radiation is described by a density matrix $\bar{\rho}_{rad}$ = $\Sigma|A_n|^2|\psi_n^{rad}\rangle\langle\psi_n^{rad}|$ (only stationary states found for a black hole are consistent with the statistical nature of thermal radiation suggesting continous random emission of thermal radiation). If the residual mini black hole disappears completely by decaying into ordinary particles we will be deprived of the large reservoir of states in the black hole which is required for the construction of the overall wave function; the whole system will be described by the density matrix and unitary postulated is violated. Again as the initial and final states do not contain black holes we have to abandon even the weak assumption that an S-matrix exists between arbitrary initial and final asymptotic states, that is as the black hole evaporates, the entropy $\propto (GM^2/\hbar c)\, k$, decreases continously.

In order to preserve unitarity and coherence, residual mini black hole should not decay completely. Stable remnant should survive. Final state is not known but it is reasonable to assume from time reversal invariance that final state consists of particles not more exotic than the ones that made the black hole in the beginning.

There are essentially three possibilities:

a) the final state of evaporation may leave behind a naked singularity. This entails violation of cosmic censorship at quantum level [2]

b) black hole may evaporate completely leaving no residue giving rise to serious problem with quantum consistency described above; moreover by CPT theorem if there is no singularity initially, system must return to state with no singularity

c) stable remnant or residue of around Planck mass might remain; semiclassical back reaction and surface correction terms suggest that emission process might stop, but still better, and surely more physical, one may consider the possibility of a M_{Pl} black hole with spin ($\hbar$).

For a black hole with mass and spin of $a = s/Mc$, s being total spin, the black hole temperature is given by

$$T_{bh} = \frac{\hbar^2\ (M^2 - a^2)^{1/2}}{32\ \pi\ K_B M\ [M + (M^2 - a^2)^{1/2}]} \tag{1}$$

and can be zero for $M = a$.

For $M = M_{Pl}$, indeed, we have $GM_{Pl}/c^2 \simeq \hbar/M_{Pl}c$, i.e. $M = a$ that is $T_{bh} = 0$ for Planck mass black holes provided they have a spin of $\sim \hbar$. So in that case black holes may indeed be stable! With the attainment of zero temperature such black holes would stop radiating and may be stable.

A natural way of understanding spin effects in gravitation is through torsion. The modification of the metric by inclusion of torsion can be expressed as:

$$g_{oo} = 1 - \frac{2GM}{Rc^2} + \frac{3G^2s^2}{2R^4c^4} \tag{2}$$

The surface gravity of a black hole with torsion can be written as [3]:

$$\chi = \left(-\frac{GM}{R^2} + \frac{3G^2s^2}{2c^4R^5}\right)\cdot \text{const.} \tag{3}$$

For a mass $M = (\hbar c/G)^{1/2}$ and radius $R = (\hbar G/c^3)^{1/2}$, the Planck length, zero surface gravity would correspond to

$$s^2 = (2/3)(c^4/G)(\hbar c/G)^{1/2}(\hbar G/c^3)^{3/2} = (2/3)\hbar^2 \tag{4}$$

implying that for a spin $s \simeq \hbar$, for such a black hole, the surface gravity and hence the temperature vanishes. Thus in this case the torsion effects which enter with opposite sign, cancel those of gravity. This gives a minimum radius of $\sim (3/4)^{1/3} R_{Pl}$. Writing $s = \sigma R^3$, σ being the spin density, the torsion term in eq.(2) would correspond to that of an effective cosmological term of type $\Lambda_{eff} R^2$, the corresponding temperature being of the form $\left(\frac{\hbar c^3}{8\pi G K_B M} - \frac{\Lambda_{eff} M G \hbar}{K_B c} \right)$, vanishing for particular Λ_{eff}, i.e.for Schwarzschild-de Sitter metric. The entropy of this residual black hole of Planck mass and spin $\hbar$, would be $\sim K_B$, so that the entropy of these black holes is quantized in units of K_B, just as their spin is quantized in units of $\hbar$.

It is to be noted that only black holes with spin can transform into a massive string (also an object of negative specific heat) as massive strings inevitably have angular momentum proportional to M^2, the same as for black holes with maximum spins. Otherwise we would have violation of angular momentum conservation. In all such transition, entropy would change in discrete units of K_B. Now, if this were the case then all primordial black holes formed with mass $< (\hbar c^4 / G^2 H_o)^{1/3}$ ($\sim 5 \cdot 10^{14}$g., H_o being the Hubble's constant, quite insensitive to H_o going as $H_o^{1/3}$) would now be Planck mass remnants! For the scale invariant perturbations in the early universe the number density of black holes in the initial mass range (M, M + dM) (with $P = \gamma \rho$ equation of state) is $dn = (\beta - 2)\, \Omega_{bh} \rho_c M_{H_o}^{\beta - 2} M^{-\beta}\, dM$ where ρ_c is the critical density and $\beta = (3\gamma - 1)/(2 + \gamma)$, $M_{H_o} = c^3 t$ the horizon mass, $\gamma = 1/3$ $\beta = 5/2$ in early universe. That means that the number density of Planck mass relicts left by evaporation of primordial holes is [4]:

$$n_{Pl} = (1/3)\Omega_{bh} \rho_c M_{H_o}^{1/2} M_{in}^{-3/2} \qquad (5)$$

where M_{in} is the mass of initially formed black holes.

$$n_P = \Omega_{Pl} \rho_c / M_{Pl} \simeq 1\ 10^{-25} \Omega_{Pl} (H_o/10^2)^2\ cm^{-3}. \qquad (6)$$

3. Black holes and strings

So we have seen that Planck mass black hole's may undergo a transition to a massive superstring. As string theory yields general relativity in low energy limit, this suggests that the Riemann geometry is embedded in a more general geometric structure. As a black hole is an excited state of gravitational field in general relativity, it can make a quantum transition to a new geometry. Again string theories unify all interactions supposedly at about Planck energies and that too in ten dimensions. So our understanding of the final state must incorporate these aspects. Fundamental object in string theories is one-dimension extended structure characterized by a string tension $\alpha = 1/M^2_{st}$ and interaction strength g. Their excitation spectrum includes an infinite tower of massive states with exponentially rising level starting at M_{Pl}!

Now as regards the string tension α we have some interesting implications in the context of strong gravity theory when considering also the dual models of hadrons [5]. In fact in the zero slope limit dual resonance models reduce to lagrangian field theories of the Yang-Mills or gravitational type.

In particular, the quantum theory of gravity (for e.g. the Gupta-Feynman theory) can be obtained as a zero-slope limit of the generalized Virasaro-shapiro model where the Regge intercept is fixed at two. One way to demonstrate such a correspondence involves computing the amplitude for graviton Compton scattering both in the Virasaro model as well as in linerized general relativity through Gupta-Feynman quantization [6]. The two expressions agree provided the identification

$$g^2 \alpha_R \cong G \qquad (7)$$

is made. Here g^2 is the strong interaction coupling (given as $g^2/\hbar c \simeq 1$), α_R is the universal Regge slope (related to the supestring tension μ) and G is the gravitational constant. So in general the corrispondence between dual resonance model and Yang-Mills-Einstein theory leads to such a relation between the gauge (strong) coupling constant and the gravitational constant through the Regge slope α_R. As $g^2 \approx \hbar c$, we see that this gives:

$$\alpha_R = G/\hbar c \qquad (8)$$

and in order to agree with the observed slope of Regge trajectory (i.e. $\alpha_R = 1\ (\mathrm{Gev})^{-2}$), it is necessary that G be the strong gravity coupling constant $G = G_f \simeq 6.7\ 10^{30}$ c.g.s.units.

The Regge trajectories of hadrons given by a relation of type $J(M) = \alpha_R M^2$, is consistent with the observed mass spectroscopy of a large number of hadronic resonance states lying on these trajectories. The above spectrum can also be interpreted as the spectrum of the rotational states of a string with the string tension μ related to the strong gravity constant by [7],[8]:

$$\alpha_R = c(2\pi\mu)^{-1}, \qquad \alpha_R \simeq G_f/\hbar c \qquad (9)$$

We can also observe that dual models give an upper limiting temperature as related to α_R as: $T \propto (1/\alpha_R)^{1/2}$, the constant of proportionality being the Boltzmann constant K_B. The degenerary of the N^{th} energy level of the string is given for large N by asymptotic formula $P(N) =$ constant$A(N)$ $\exp(M_N/T_H)$ where T_H is the Hagedorn limiting temperature; if we use for α_g the expression given by strong gravity we get [9]

$$T \simeq (1/K_B)(\hbar c^5/G)^{1/2} \simeq 10^{13}\ ^oK \qquad (10)$$

which agrees with the Hagedorn temperature given by the dual model.

Entropy of a black hole of energy E is $S_{bh} \approx 4\pi E^2$ and entropy of a massive string mode of energy E is: $S_{string} = - a \ln E + bE$ with $a \simeq 10$ and $b \simeq \pi(2 + \sqrt{2}\,)\alpha^{1/2}$ for heterotic string. All units are in Planck units.

In order that there is no information loss, true degeneracy of a black hole which has evaporated down to Planck mass is $\sim \exp(4\pi M^2)$ >> degeneracy of massive string modes of energy E given by $E^{-10}\exp(bE)$ where $E \simeq E_{Pl}$ so it is unlikely for a black hole to transform into massive string modewithout also radiation, but when its mass is less than γM_p, $S_b(E) < S_s(E)$, then the transition becomes highly probable.

Specific heat of massive string $C = - (1/T^2)(E^2/a)$ is negative! Specific heat of a black hole $dM/dT \propto 1/T^2$ is also negative. Gas consisting of massive superstring excitations behaves in many respects like a black hole, i.e. black hole and massive string excitations can never be in thermal equilibrium with an infinite heat reservoir. A body with negative specific heat can be in equilibrium with a finite heat bath. For a black hole the condition for a stable equilibrium is that energy of heat bath (i.e.radiation) be less than 1/4 mass of the black hole. The corresponding condition for equilibrium of massive excitations of heterotic string with massless modes of a radiation can be obtained.

In ten dimension space-time energy of massless gas of bosons and fermions is

$E = \sigma V T^{10}$; $\sigma = (8\pi^5/3465)\left[n_b + (1 - 1/2^9)n_f\right]$ where n_b is the number of bosons and n_f is the number of fermions. The most probable values of E_{st}, E_γ maximize $S_{st} + S_{rad}$ and this happens when the second derivative of total S is negative.

(for $n_b = n_f = 4032$, $\sigma = 6\ 10^4$ and S is very high)

We can study the various phases of a three component system (blach holes, string and radiation) [10]. We have three distinct phases: (1) black hole and radiation, (2) string and radiation (3) pure radiation.

Total energy E in a volume V is:

Phase (1) $E_{bh+rad} = \sigma V T^4 + 1/(8\pi T)$

$$S_{bh+rad} = 4/3(\sigma V T^3) + 1/(16\pi T^2) \qquad (11)$$

Phase (2) $E_{st+rad} = \sigma V T^4 + aT/(bT-1)$

$$S_{st+rad} = 4/3(\sigma V T^3) + baT/(bT-1) - (1/T)(bT-1) \qquad (12)$$

Phase (3) $E_{rad} = \sigma V T^4$

$$S_{rad} = 4/3(\sigma V T^3) \qquad (13)$$

We have S_{bh+rad}, S_{st+rad} and S_{rad} as function of E and V. Given E and V the system with higher S would be preferred. There is a critical volume above which only the radiation can be in thermal equilibrium for both systems: black hole + radiation and massive string + radiation. For black holes + radiation, as is known, $V_c = 9\ 2^{20}\sigma E^5/125$ and for strings + radiation $\sigma V_c = (E + 3a/2b - D)b^4(D - 5a/2b)^4/(D - 3a/2b)^4$ with $D = [4Ea/b + (3a/2b)^2]^{1/2}$. The two critical volume curves intersect at $E \approx 7\ E_{Pl}$ and $V \simeq 10^5$ in Planck units. Below this value of E, V_c for string plus radiation is higher than that for black hole + radiation. Thus a black hole bathed in radiation with total $E \approx 5\ M_{Pl}$ can, if the volume increases slowly, undergo a phase transition to enter a string and radiation phase. For further increase in volume, the massive strings will evaporate to pure radiation.
Thus in this thermodynamic process black hole has evaporated through its transition to a string phase and if the string theory is free of singularity (but this is not yet known also if conceivable) no singularity will be left. If superstring unification of gravity with other interactions takes place at $\approx M_{Pl}$, then it is necessary to include other degrees of freedom associated with massive string excitation to understand final stages of black hole evaporation.

4. Model for string gas

The above energy levels and corresponding degeneracies are confined to a space volume taken as 9 dimensions. Energy and entropy are correlated to volume and temperature like photon gas except that the space is 9-dimensional and the 2 polarization states of photons replaced by 8064 fold degeneracy of massless string (1/2 of the states are fermionic).

$$E_{st(9)} = 9Q\ V\ T^{10}_{st(9)} \qquad (14)$$

$$S_{st(9)} = 10Q\ V\ T^{9}_{st(9)} \qquad Q = 8\ \pi^{5}/15$$

Gas temperature $\neq$ String temperature

For subsystem consisting of all massless strings and heaviest strings the energy is
Energy is

$$E = M_N + E_g \qquad M_N = \text{mass of}$$

heaviest string. The dependence of entropy is:

$S(N,V,M) = S_N + S_\sigma$ at fixed (V,M) or N or M_N/M

we have the cases:

a) $M > M_N$: system dominated by gas of masless strings

b) for increasing N, entropy increases: system dominated by one string

c) Entropy is not a monotonic function of N: in general both massive string and gas of massless strings in equilibrium are important. For $T_g < T_H$, massless strings dominate.

Even at $T_g = T_H$ massive strings contribute less than 0.3% to entropy. Gas must be in state for which

$$M_{tot} < 9\, Q\, V\, T_H^{10}.$$ The result depends on

$$\gamma = M_{PL}\sqrt{\alpha}.$$ We choose $\gamma = 10$

Using mean square radius for bosonic string in 9 dimension for high N, is natural to assume the volume (V) larger than volume of 9-dimensional sphere of radius R(N), otherwise the string would not fit the radiation box.

This gives

$$M_{st} < 0.2\, V^{2/9}$$

$$M_{st}/M_{tot} < 7\ 10^5\ V^{-7/9}$$

which is a negligible fraction of total energy of string gas. Possibility of the black hole going over directly into heavy string without massless strings has problems with conservation laws like angular momentum. Black hole nearing the end of is life usually is assumed to have zero angular momentum and heavy strings with $J = 0$ don't exist.

The most probable state of system of non-interacting heterotic strings in a box of volume V, may be a single string, or a heavy string in equilibrium with a gas of massless strings, or a gas of massless strings.

Each of these at low mass has an entropy which is higher than the entropy of a black hole having the same mass.

The d-dimensional Schwarzschild-de Sitter metrics:

$$ds^2 = -\left(1 - \frac{2\,G\,M\,F(d)}{\gamma^{d-3}} - \frac{\Lambda\, r^2}{d-1}\right) dt^2 + \left(1 - \frac{2\,G\,M\,F(d)}{\gamma^{d-3}} - \frac{\Lambda\, r^2}{d-1}\right)^{-1} dr^2 + r^2 d\Omega^2_{d-1} \tag{15}$$

generalize to higher dimensions the standard asymptotic analysis of solutions to Einstein equations in 4 dimensions. Higher dimensional de Sitter space behaves as though it has intrinsic temperature

$$T = (2\pi)^{-1}\,[\,(d-1)/\Lambda\,]^{-1/2}. \tag{16}$$

For given values of Λ, the number of space dimensions tends to decrease the de Sitter temperature.

For d-dimensional black holes:

entropy scales as $\propto M^{(d-2)/(d-3)}$ (that is as M^2 in 4-dimensions)

Schwarzschild radius scales as $\propto (2GM)^{1/(d-3)}$

and temperature scales as $\propto 1/M^{d-3}$

(2GM and 1/M respectively for d = 4).

Effect of shadow state particles in $E_8 \times E_8'$ superstrings is to accelerate the decay of black holes by a factor of about 2 [12]. Compactified manifold with radius of curvature $\sim R_{Pl}/\alpha^{1/2} > R_{Pl}$ imply black holes with $R > R_{Pl}$ or $M > M_{Pl}$

Considering higher dimensional black holes, the Schwarzschild solution for d-dimensions is given by (15). The temperature of higher dimensional black holes scales as

$$T \simeq \frac{1}{M^{d-3}} \quad \text{that for } d = 4 \text{ gives } T \simeq 1/M.$$

Entropy scales as $S \propto M^{(d-2)/(d-3)}$ that for $d = 4$ gives $S \propto M^2$. Evaporation rate is $\propto M^{(4d-d^2-2)}$ that for $d = 4$ gives M^{-2}.

Lifetime is $\propto \int M^{-(4d-d^2-1)} dM \propto M^{d^2-4d+3}$. For $d = 4$, i.e. the usual four-dimensional case, the lifetime is given by:

$$t = \frac{G^2 M^3}{\hbar c^4} \qquad (17)$$

For the d-dimensional case, this generalizes to:

$$t_d = \frac{\hbar M^{(d^2-4d+3)} G^{(d^2-4d+4)/2}}{c^2 \hbar^{(d^2-4d+4)/2} c^{(d^2-4d+4)/2}} \qquad (18)$$

$$= \frac{\hbar}{c^2} M^{(d^2-4d+3)} \left(\frac{G}{\hbar c}\right)^{(d^2-4d+4)/2}$$

For d = 4 , we set the usual formula. As $G/\hbar c \approx M_{Pl}^{-2}$, this can be written

$$t_d = \frac{\hbar M^{(d^2-4d+3)}}{c^2 M_{Pl}^{(d^2-4d+4)}} = \frac{\hbar}{M_{Pl} c^2}\left(M/M_{Pl}\right)^{(d^2-4d+3)} \qquad (19)$$

Then for $M \approx 10^2 M_{Pl}$ black hole, the lifetime in d = 10 dimension, for instance, is >> age of the universe, so it is practically stable unlike in 4-dimensions. This raises the possibility of a stable remnant of the order of a few Planck masses of the evaporating black hole in higher dimensions, solving problems of unitarity and coherence.

The temperature of a 4-dimensional black hole is:

$$T \approx \frac{\hbar c^3}{8\pi G K_B M} . \tag{20}$$

Correspondingly for a d-dimensional black hole it is:

$$T_d \approx \frac{\hbar^{d-3} c^{d-1}}{8\pi G^{d-3} K_B M^{d-3}} , \tag{21}$$

thus much reducing the temperature for a black hole with $M >> M_{Pl}$ as compared to four dimensional case.

For $d = 10$ entropy $\propto M^{8/7}$, lifetime $\propto M^{63}$ and temperature $T \propto M^{-7}$. Thus temperatures are much lowered for higher dimensional black holes and life times much enlarged. Similarly for d-dimensional de Sitter space

$T \approx (2\pi)^{-1} [(d - 1)/A]^{-1/2}$ i.e. temperature is much lower for increase in d.

For d-dimensional black hole solutions of these models, we have that metric tensor has the form $g_{oo} = 1 - 2GM/R^{(d-3)}$ and the Schwarzschild radius is $(2GM)^{1/(d-3)}$. Entropy of higher d black holes scales respect to M as $S \propto M^{(d-2)/(d-3)}$ that for $d = 4$ gives the usual $S \propto M^2$. Each Planck mass black hole has one unit of entropy. If we assume that in the earliest epoch the universe began with $\sim 10^{60}$ Planck mass black holes in d-dimensions which evaporated to give the observed 4-dimensional entropy, then the total entropy S of the universe of $\sim 10^{60} M_{Pl}$ black holes would scale in d-dimensions as $(M/M_{Pl})^{(d-2)/(d-3)} \sim (10^{60})^{(d-2)/(d-3)}$. This entropy when released by black hole evaporation cannot exceed $S_{microwave}$ in microwave background, i.e. $S_m \sim 10^{88}$. For $d = 6$ this gives $S \sim 10^{60 \cdot 4/3} \sim 10^{80}$; for $d = 5$ we have $\sim 10^{90}$ which is a few orders larger than that of the observed entropy S_m of the microwave background. For $d = 2$ no entropy release (an impossible situation) and for $d = 3$ $S \rightarrow$ infinite!; $d = 7$ gives too small an entropy. So as far as the entropy release in the d-dimensional black holes arising from the compactified dimensional space in the early universe is concerned, $d = 6$ seems to be the optimum dimension of the compactified space.

The scenario we have here is as follows: we already have an expanding four-dimensional space-time with several compactified objects of mass $\sim M_{Pl}$ (strings, membranes etc.), in d compact dimensions which collapse to form black holes;

as when excited, these objects produce an energy spectrum of states with energies several times Planck mass. So the compactified objects collapse to form higher dimensional (d-dimensional) black holes which then evaporate to produce entropy, particles etc. This total entropy released on their evaporation must be comparable to that in the microwave background observed now. This would require $d = 6$ for the dimension of the compact space as other values of d, as seen above, are inconsistent with the observed entropy. Again as entropy of black hole is connected with area, this might have to do with geometrical property of area in d-dimensions which as we shall see below is maximized for $d = 6$. So the two pictures can be connected.

5. Ten dimension ?

We have in fact another argument [13] that also leads to d = 6: it is an 'a priori' argument that can privilege 10 space-time dimensions. 10 dimensions in all means 9 + 1 that is 9 spatial dimensions. As we live in a three dimensional space, six dimensions must be hidden from view, thereby leaving only the four familiar dimensions of space-time to be observed. The six extra spatial dimensions must be curled up to form a structure so small that it cannot directly be seen.

Now why six compact extra dimensions appear to be preferred? We can observe that when we calculate the area of an hypersurface, we find that (referring to a unitary radius) it is maximum when it is an esasurface [14]. We know that there is a general principle in physics for which every physical system tends to put itself in the state of minimum energy. More generally one uses the superlative in order to express in concise form a general principle which covers a great variety of phenomena. In this sense we say that a straight line is the "shortest" distance between to points (or in a non-euclidean, curved space-time we speake of the "geodesic"), on the sphere such a path is the great-circle route between two points: in this sense the statement that a physical system so acts that some function of its behaviour is least (or greatest) is the starting point for theoretical investigations.

The mathematical formulation of the superlatives is usually that the integral of some function has a smaller (or larger) value for the actual performance of the system. This is the case of the action integral and we are led to the variational method: certain integral has to be minimized or maximized; in other words we search for an 'extreme' value of this integral so that it has either a minimum or a maximum (or a point of inflexion). Usually we can tell from the physical situation which of these cases are true.

Now if we investigate the area of an hypersurface of a unitary radius we find out that it has a maximum value when it has six dimensions. To be more precise: let E_n be an n-dimensional space and S_{n-1} an hypersurface whose volume is V_n and surface area A_n. If we indicate V_2 as the area of the circle, V_3 the volume of the sphere and V_n the n-hypervolume we find:

TABLE 1

n	A_n	$A_n(1)$
2	$2\pi r$	6.28
3	$4\pi r^2$	12.56
4	$2\pi^2 r^3$	19.73
5	$8\pi^2 r^4/3$	26.31
6	$\pi^3 r^5$	31.00
7	$16\pi^3 r^6/15$	33.07
8	$2\pi^4 r^7/6$	32.46
9	$32\pi^4 r^8/105$	29.68
10	$2\pi^5 r^9/24$	25.50
.		
40	$2\pi^{20} r^{39}/19$	$1.44\ 10^{-7}$

$$V_{2k} = \frac{\pi^k r^{2k}}{k!} \quad ; \qquad V_{2k+1} = \frac{(2\pi)^{k+1} r^{2k+1}}{\pi(2k+1)!!} \qquad (22)$$

$(n = 2k$ or $n = 2k+1)$

where $k! = 1\ 2\ 3.....k$ and $(2k+1)!! = 1\ 3\ 5\ 7....(2k+1)$, while if we indicate A_2 as the circumference of a circle, A_3 the area of a sphere surface and A_n the area of the S_{n-1} hypersurface in a n-space we find:

$$A_{2k} = \frac{2\pi^k r^{2k-1}}{(k-1)!} \quad ; \qquad A_{2k+1} = \frac{(2\pi)^{2k+1} r^{2k}}{\pi\ (2k-1)!!} \qquad (23)$$

$(n = 2k$ or $n = 2k+1)$.
For unitary radius, (see Tab.1), the maximum area is for hypersurface S_6 (embedded in a 7-dimensional space).
In this context the natural unit of measure is the Planck length.

6. Impossibility of deflation

As regards the vacuum dominance in a collapsing universe, for a Lorentz invariant and general covariant vacuum $T_{\mu\nu}$ we require at all temperatures $(\varepsilon + p)_{vac} = 0$, so vacuum domination is characterized by $p < 0$, that is by negative pressure. In early universe the vacuum energy dominates radiation energy being $\varepsilon + 3p < 0$, whenever temperature T_U falls below T_v, that is $\varepsilon_U(T_U) \le \varepsilon_v(T_v)$. The period of the exponential expansion is driven by negative pressure, negative energy becoming larger with increasing volume, resulting in creation of positive energy paticles. Entropy multiplication took place (to present $\sim 10^{88}$) at reheating epoch when latent heat was released.
Vacuum dominated de Sitter phase is only a supercooled state of metastable equilibrium. Supercooled state becomes vacuum dominated as soon as radiation temperature falls below T during the expansion of early universe.

For contracting universe we would have the corresponding metastable superheated phase, which always has $p > 0$, which must be always radiation dominated as only for $p < 0$, vacuum can dominate. Thus in any contraction of the universe there can no deflation that is vacuum domination superheating leads to instability aiding collapse.

The mass of black holes accreting black body radiation in a collapsing universe, would diverge in a finite time; the rate of accretion is [15]:

$$dM(t)/dt \simeq 4\pi R_s^2(t)\rho(t)/c \ ; \quad R_s(t) = 2GM(t)/c^2 \ ;$$
$$\rho(t) = aT^4(t) \ ; \quad R(t) \propto 1/T(t) \propto (t_* - t)^{1/2}. \qquad (24)$$

Substituting, we see that M(t) diverges so does entropy;

black hole accretion causes instability in a collapsing universe.

Now superstring theories are valid at very early expansion; in late stage of collapse as the universe contracts there would be phase transition to a system of massive string excitations and we will have a corresponding rise in entropy. The gas consisting of massive superstring excitations behaves like a black hole with negative specific heat proportional to $1/T^2$. In ten dimension space-time of superstrings entropy density of a gas of bosons and fermions is

$$S = 10\,\sigma\, T^9 \;;\quad \sigma = (8\pi^5/3465)[n_b + (1 - 1/2^9)n_f]$$

(that is $(1 - 1/8)n_f = (7/8)n_f$ for 4-dimension space-time).

$n_b \sim n_f = 4032$ massless modes of heterotic strings

$\sigma = 6\;10^4$; $T \approx M_{Pl} \approx 10^{32}K$ and S is very high.

Also in Klein Kaluza theories, the $(4 + d)$ dimensional scalar curvature R is the Lagrangian and the $(4 + d)$ $g_{\mu\nu}(x)$ describes general relativity as well as gauge field in a compactified manifold with radius of curvature given by $R_c^2 = \alpha_g R^2_{Pl}$ where α_g is the gauge coupling constant. The entropy for n-dimension is $S \sim const/\hbar^n$.

In the d-dimensional space, Planck spectral density is of the form:

$$E=\sigma_d\, A\, T^{d+1} \qquad (25)$$

$$\text{where}\quad \sigma_d = \frac{2\pi^{(3d+1)/2}}{\Gamma[(d+1)/2]}\,\Gamma(d+1)\zeta(d+1)\,\frac{K_B^{d+1}}{h^d\, c^{d-1}}$$

is the d-dimensional Stefan-Boltzmann constant and $\zeta(x)$ the Riemann zeta function.

σ_3 is usual Stefan-Boltzmann constant for three space dimensions that is:

$$\sigma_3 = 2\pi^5 K_B^4 /15\, h^3 c^2 = 5.67\; 10^{-5}\; erg\; cm^{-2}\; deg^{-4}$$

$\sigma_1 = \dfrac{\pi^2 K_B^2}{3h}$, $\left(E_1 = \dfrac{\pi^2 K_B{}^2 T^2}{6h}\right.$ is the well-known case of one-dimensional thermal radiation i.e. Johnson noise or Nyquist noise in electrical networks!)

$$\sigma_9 = \frac{32\,\pi^{14}\,K_B{}^{10}}{99\, h^9\, c^8},$$

$$\sigma_{25} = \frac{10779541504\, \pi^{38}\, K_B^{26}}{1403325\, h^{25}\, c^{24}},$$ appropriate for 26-dimensional bosonic strings! etc.

Again the Schwarzschild metric at $\approx R_{Pl}$ may be modified by quantum corrections. So horizon is different. For m-loop terms, corrections are:

$$ds^2 = -(1 - \sum_n \alpha_n R_{Sch}^n r^{-n} + \sum_n \sum_m \beta_{nm} R_{Sch}^n R_{Pl}^{2m} r^{-(n+m)})dt^2 - \text{etc.}$$

For $r >> R_{Pl}$ one recovers the usual solution. These corrections would make the horizon fluctuate [12].

Again low energy limit of string theories give gravity with higher order curvature terms with dilatons. Here the corresponding Schwarzschild solution has smaller horizon. G.'t Hooft [16] has pointed out that black hole theory is related to string theory provided the string constant equal $T = 1/(8\pi G)$, with negative sign however! We point out the correspondence between action leading to equation for oscillations of black hole horizon and the string action used in describing Veneziano amplitude, provided T is identified as $1/(8\pi G)$. Thus end point of a black hole evaporating may be tied up with unificationo of interactions!

It turns out that string corrections reduce the black hole temperature [17] with

$T = T_{bh}(1 - F_D\, \alpha'^3/M^6]$, with F_D constant and

α' the string slope expansion parameter. One sets a vanishing value for temperature at a particular value of mass of black hole. Gauss-Bonnett corrections (R^2 corrections) also have similar effects. Again back reaction effect of Hawking radiation on the Schwarzschild geometry in the presence of massless graviton, lowers the black hole temperature.

For a number N of weak boson fields, with a vacuum contribution due to polarization of $\rho = 41\cdot N/7680\ \pi^2 M^4$, the black hole temperature approaches zero as mass $M \rightarrow (41\ N/240\ \pi)^{1/2}$ which implies that $T \doteq 0$ for $M \simeq 5M_{Pl}$ in the case of superstring theory with N = 496. In short there are several ways by which the terminal stage of black hole evaporation may be indefinitely prolonged!

A mechanism for considerably prolonging the lifetime of the remnant state of mass $\sim M_{Pl}$ was suggested for instance by Carlitz [18] where the period of Hawking radiation is followed by much larger period during which remnant is radiated away, producing a pure state with unusual long range correlations.

There are also arguments against the decay: in fact if the remnant evaporates into N quanta, the average wave length of final N quanta is $\approx \lambda \approx (M_{Pl}/N)^{-1} \approx N\ R_{Pl}$ that is larger by a factor of N than the size of decaying