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Torsion Effects in Black Hole Evaporation

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Torsionseffekte bei der Verdampfung schwarzer Löcher

Recently several different approaches have been followed in understanding the final stage of black hole evaporation. This topic is of much current interest. The complete disappearance of an evaporating black hole, presumably in an explosive burst of duration $\sim 10^{-43}$ s, when it reaches a mass of the order of the Planck mass $[(\hbar c/G)^{1/2} \sim 10^{19}$ GeV], is inconsistent with fundamental tenets of quantum mechanics such as loss of quantum coherence and the unitary postulate (for instance if the initial and final state do not contain black hole we have to abandon the assumption that an S-matrix exists between arbitrary initial and final asymptotic states). Again the initial pure state would be converted into a mixed state of normal radiation as seen by a distant observer, the emitted quanta being an uncorrelated ensemble cannot be described by a single wave function. The unitarity postulate is preserved by correlating each state of emitted radiation with a corresponding quantum state inside a black hole so that the joint system has a well defined overall wave function. If the residual black hole disappears completely we would not have the large reservoir of states in the black hole required for constructing the overall wave function and the whole system would be described by the density matrix violating the unitarity postulate. There are essentially three possibilities:

(1) the final state of evaporation may leave behind a naked singularity violating cosmic censorship at quantum level.

(2) black hole may evaporate completely leaving no residue giving rise to the serious problems with quantum consistency described above. Moreover by CPT theorem if there is no singularity initially the system must return to state with no singularity.

(3) a stable remnant of residue of around Planck mass might remain. The emission process might stop owing to semiclassical backreaction or surface correction terms.

A possibility is to consider a Planck mass (M_{Pl}) black hole with spin $\hbar$. For an extremal Kerr black hole ($M = a$, $a = s/Mc$, s being total spin) the black hole temperature, given by [1]

$$T_{bh} = \frac{\hbar^2 (M^2 - a^2)^{1/2}}{32\pi K_B M[M + (M^2 - a^2)^{1/2}]}, \tag{1}$$

can be zero. For $M = M_{Pl}$, indeed $GM_{Pl}/c^2 \approx \hbar/(M_{Pl}c)$, i.e. $M = a$, i.e. $T_{bh} = 0$ for Planck mass black hole provided they have a spin of $\sim\hbar$. With the attainment of zero temperature such black holes would stop radiating and may indeed be stable [2].

A natural way of understanding spin effects in gravitation is through torsion. The modification of the metric by inclusion of torsion can be expressed as [3, 4]

$$g_{00} = 1 - \frac{2GM}{Rc^2} + \frac{3G^2s^2}{2R^4c^6}. \tag{2}$$

The surface gravity of a black hole with torsion can be written as

$$X = \left(-\frac{GM}{R^2} + \frac{3G^2s^2}{c^4R^5}\right) \cdot \text{const.} \tag{3}$$

For a mass $M = (\hbar c/G)^{1/2}$ and radius $R = (\hbar G/c^3)^{1/2}$, the Planck length, zero surface gravity would correspond to

$$s^2 = (3)\,(c^4/G)\,(\hbar c/G)^{1/2}\,(\hbar G/c^3)^{3/2} = (3)\,\hbar^2, \tag{4}$$

implying that for a spin $s \sim \hbar$, for such a black hole, the surface gravity and hence the temperature vanishes. Thus in this case the torsion effects which enter with opposite sign, cancel those of gravity. This gives a minimum radius of R_{Pl}. Writing $s = \sigma R^3$, σ being the spin density, the torsion term in eq. (2) would correspond to that of an effective cosmological term of type $\Lambda_{\text{eff}}R^2$, the corresponding temperature being of the form $\left(\frac{\hbar c^3}{8\pi GK_BM} - \frac{\Lambda_{\text{eff}}MG\hbar}{K_Bc}\right)$, vanishing for particular Λ_{eff}, i.e. for Schwarzschild-de Sitter metric [1]. The entropy of this residual black hole of Planck mass and spin $\hbar$, would be $\sim K_B$ so that the entropy of these black holes is quantized in units of K_B, just as their spin is quantized in units of $\hbar$ [5]. In fact it has been proved that there exists a minimum increase in the areas of black hole surface [6]

$$(\Delta A)_{\min} \geq 8\pi\hbar G/c^3. \tag{5}$$

Combining the well known Bekenstein-Hawking formula we can get

$$(\Delta S_{bh})_{\min} = (1/4)\,K_B\frac{c^3}{G\hbar}(\Delta A)_{\min} \geq 2\pi K_B, \tag{6}$$

which means the lower bound of the entropy S increase of black holes is $2\pi K_B$. If the black hole at the final stage of evaporation is of Planck mass, its energy is also $2\pi K_B$ by Bekenstein-Hawking formula. So it seems reasonable that the entropy of black holes can be quantized in units of K_B.

It is to be noted that only black holes with spin can transform into a massive string (also an object of negative specific heat) as massive strings inevitably have angular momentum proportional to M^2, the same as for black holes with extremal spins [7]. Otherwise we would have violation of angular momentum conservation. In all such transitions, entropy would change in discrete units of K_B.

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