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NON-ZERO REST MASS NEUTRINOS AND THE COSMOLOGICAL CONSTANT

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Abstract. Recently it was pointed out that a non-zero cosmological constant can play a role in the formation of neutrino halos only in the case of neutrinos of very low rest mass ($m_\nu \leq 0.1$ eV). However, phase-space considerations would require $m_\nu > 50$ eV if neutrinos dominate the missing mass in halos of large spiral galaxies and moreover $m_\nu > 200$ eV is implied in the case of dwarf spheroidals. These larger neutrino masses would be in conflict with observed constraints on the age of the Universe unless a cosmological constant is invoked.

In a recent paper, Stuchlík and Calvani (1984) concluded that a non-zero cosmological constant (Λ) can play a role in the formation of neutrino halos only for very light neutrino rest masses. Using observational upper limits of $\Lambda \leq 10^{-55}$ cm^{-2} on a possible non-zero cosmological constant they find it is relevant only for very low neutrino rest masses of $m_\nu \leq 0.1$ eV. However, if, as seems probable, the dynamics of large spiral galaxies are dominated by dark unseen matter (the flat rotation curves suggesting a progressive increase in such matter with radius) and if this non-luminous matter was chiefly non-zero rest mass neutrinos then phase space considerations would imply that $m_\nu > 50$ eV for galactic halos (see, e.g., Datta *et al.*, 1985). Put briefly – neutrinos being fermions – their phase space density must satisfy the inequality $d^3 \times d^3p \geq h^3$ (h being the Planck constant) which translates into $n_\nu/(m_\nu V_\nu)^3 \geq h^{-3}$ or $m_\nu \geq (h^3 \rho_m / V^3)^{1/4}$ where ρ_m is the density in missing matter with velocity V. For a typical spiral galaxy, $\rho_m \approx 10^{-24}$ g cm^{-3}, $V \approx 200$ km s^{-1}, which gives $m_\nu \geq 50$ eV. Further, it appears that unseen matter may dominate the gravitational potential of dwarf spheroidal galaxies also (Aaronson, 1983) in which case the rather fundamental phase space constraint would imply $m_\nu \geq 300$ eV!, which is rather incompatible with the Russian experiment (Lubimov *et al.*, 1980), which gives the range for m_ν as 14–46 eV. Neutrino rest masses with $m_\nu \approx 50$–100 eV, are incompatible with the known ages of Friedmann universes without a cosmological constant. This applies to both open or closes Universes. Thus, a cosmological constant would have to be invoked if one has to accommodate the rather large neutrino rest masses implied by phase space considerations. We shall discuss this in some detail below.

The number density of relic neutrinos is given by $n_\nu = 7.5 g_\nu T_\nu^3$ and the mass density as $\rho_\nu = 7.5 g_\nu T_\nu^3 \times \Sigma m_\nu$, $g_\nu = 2$ for each flavour of Dirac neutrinos and three flavours are known (i.e., those corresponding to the electron, muon, and tau leptons), T_ν is the neutrino background temperature which from statistics is expected to be $(\frac{4}{11})^{1/3}$ × photon background temperature (for the electron neutrino). The sum Σm_ν denotes that of the masses of the neutrino species; so that the constraint is on the combined rest mass

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of the three flavours. To give some numbers: if the Hubble constant H_0 is $50\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$, then the critical density for a closed universe is $\rho_c = 3H_0^2/8\pi G \approx 5 \times 10^{-30}\ \mathrm{g\ cm^{-3}}$, for $H_0 = 100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$; corresponding $\rho_c = 2 \times 10^{-29}\ \mathrm{g\ cm^{-3}}$. For T_ν in units of 3 K, $\rho_\nu = 2.8 \times 10^{-31}\ (T_\nu/3)^3\ \Sigma m_\nu$ and if all the neutrino flavours have the same value of the rest mass given by $m_\nu c^2 = 30$ eV, then $\rho_\nu = 2.5 \times 10^{-29}\ (T_\nu/3)^3\ \mathrm{g\ cm^{-3}}$. Then the Hubble age of a neutrino-dominated universe would be given by (without the cosmological term):

$$t_H = \frac{1}{H_0} \int_0^\infty \frac{\mathrm{d}z}{(1+z)^2\,[1 + (\rho_\nu/\rho_c)z]^{1/2}} \quad (z = \text{redshift})$$

$$= 2 \times 10^{10}\ \mathrm{yr}/[(H_0/50) + \{(T_\nu/3)^2\ \Sigma m_\nu c^2/40\ \mathrm{eV}\}^{1/2}]\,. \tag{1}$$

For a low-density open universe, where $\rho_\nu \ll \rho_c$, $t_H = 1/H_0 = 2 \times 10^{10}\ (50/H_0)$ yr. For a closed universe with

$$\rho_\nu = \rho_c\,, \qquad t_H = 1/H_0 \int_0^\infty \mathrm{d}z/(1+z)^{5/2} = \tfrac{2}{3} H_0^{-1}.$$

The implication is that for $\Sigma m_\nu c^2 = 90$ eV (i.e., each species of neutrino has a mass of 30 eV and three species), we have $\rho_\nu/\rho_c = 5$, $t_H \approx 0.45 H_0^{-1} \approx 8.5 \times 10^9$ yr, which is rather too short a time for stars and galaxies to evolve.

A recent estimate for the age of the Universe based on quasar double images (caused by gravitational lensing) gives $t_H = 1.3 \times 10^{10}$ yr (Borgeest and Refsdal, 1985). Now the maximum possible age t_m for closed Friedmann universes (with deceleration parameter $q_0 > 0.5$) is given (cf., for example, Joshi and Chitre, 1981) by

$$t_m = \pi \left(\frac{3}{32\pi G \rho}\right)^{1/2};$$

and for open models ($0 < q_0 < 0.5$), $t_m = (\frac{1}{6}\pi G \rho)^{1/2}$. With $t_H = 1.3 \times 10^{10}\ \mathrm{yr} < t_m$, and with $n_\nu \approx 100 \times$ no. of flavours $\mathrm{cm^{-3}}$; this would imply upper bounds on the neutrino rest mass of 40 eV and 7 eV for closed and open Friedmann models, respectively (without a cosmological term).

Thus, for neutrino rest masses > 50 eV required for galactic halos as implied from phase space considerations it would be difficult to reconcile with the ages of Friedmann universe models. The contradiction with the Hubble age can be ameliorated to some extent if universe models with a cosmological constant are allowed (Doroshkevich *et al.*, 1980). This can be seen as follows: the Hubble rate of expansion in models with a Λ-term is given by

$$H_0^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi G \rho}{3} + \frac{\Lambda c^2}{3} - \frac{K c^2}{R^2}\,; \tag{2}$$

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with a corresponding age given by the expression

$$t_H = H_0^{-1} \int_0^{\infty} \frac{dz}{(1+z)\left[(1+z)^2\left(1+\frac{\rho_v}{\rho_c} z\right) - \left(\frac{\Lambda c^2}{8\pi G\rho_c}\right) z(2+z)\right]^{1/2}}; \tag{3}$$

thus indication that for an arbitrary (ρ_v/ρ_c) (i.e., for larger and larger ρ_v values) we can still get as large an age as required by allowing an increased value of Λ. So an increase in ρ_v can be offset by a Λ-term, and still giving a lower value for the age t_H. Of course there would be a critical value for Λ, which occurs when the expression within the square brackets vanishes and as we require the Universe to have evolved from an initial dense state, we must have $\Lambda < \Lambda_{\text{crit}}$. The value of m_v corresponding to Λ_{crit} would be given by $\Sigma m_v c^2 \approx 180$ eV and is thus a stringent upper limit even allowing for the cosmological constant. For $\Sigma m_v c^2 \approx 50$ eV, $\rho_v/\rho_c \approx 3$ and $\Lambda c^2/8\pi G\rho_c$ would be between 4.5 and 5, so as not to contradict data on the Z-distribution of galaxies and quasars. As an example of a $K = +1$ model with Λ close to Λ_{crit} and, hence, a very prolonged period (large t_H) of slow expansion near a certain radius R_{crit} we have the Eddington–Lemaître model which was briefly revived to account for a clustering of quasar redshifts at around $z = 2$ suggesting they formed around that epoch of nearly constant radius. Interestingly, this model can be used to predict the matter density ρ_m at the present epoch along with the value of Λ. As Λ is close to Λ_{crit} (Λ_c) we can write $\Lambda = \Lambda_{\text{crit}}(1+\varepsilon)^2$, with ε a small positive number. The period of constant radius would have radius $R \approx \Lambda_c^{-1/2}$ and density $\Lambda_c c^2/8\pi G$. With $\rho R^3 = \text{const.} = c^2(1+\varepsilon)/4\pi\Lambda^{1/2} G$; we have: $(K = +1)$; $\dot{R}^2 = \frac{1}{3}\Lambda c^2 R^2 - c^2 + 2c^2(1+\varepsilon)/3\Lambda^{1/2} RG \equiv F(R)$, which for small R has $\dot{R}^2 \approx 1/R$ and $R \propto t^{2/3}$. $F(R)$ has a minimum when $R = R_{\text{min}} = (1+\varepsilon)^{1/3}\Lambda^{-1/2}$. Expanding $F(R)$ for R close to R_{min} as Taylor-series and integrating, we obtain

$$R = \frac{(1+\varepsilon)^{1/3}}{\Lambda^{1/2}}\left[1 + \{1-(1+\varepsilon)^{-2/3}\}^{1/2} \sinh \Lambda^{1/2} c(t - t_{\text{min}})\right], \tag{4}$$

where $R = R_{\text{min}}$ at $t = t_{\text{min}}$. For small ε,

$$R \approx R_{\text{min}}\left[1 + \tfrac{1}{3}\varepsilon \sinh \Lambda^{1/2} c(t - t_{\text{min}})\right];$$

implying R remains at around R_m till $\varepsilon \sinh \Lambda^{1/2} c(t - t_{\text{min}}) \approx 1$, that is for a time $(t - t_{\text{min}}) \approx \Lambda^{-1/2} \log(1/\varepsilon)$, which means for sufficiently small ε, the time of expansion can be arbitrarily large (for $\Lambda = \Lambda_c$ it is ∞). If quasars form at $z = 2$, then $R = R_{\text{min}}$ at $z = 2$, i.e., $R_0 = R_{\text{min}}(1+z) \simeq 3R_{\text{min}} = 3\Lambda^{-1/2}$, where R_0 refers to the present value. Then with $\varepsilon \ll 1$, the *present* density of matter must be

$$\rho_0 = (4\pi G R_0^3 \Lambda^{1/2}/c^2)^{-1}$$

and present H_0 is given by

$$H_0 = (\dot{R}/R)_0 = \left(\tfrac{1}{3}\Lambda c^2 - \frac{c^2}{R_0^2} + \frac{2c^2}{3\Lambda^{1/2} R_0^3 G}\right)^{1/2},$$

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and substituting for R_0 in terms of Λ from above we have

$$H_0 = \frac{(20\Lambda)^{1/2}\, c}{9}$$

which for an observed $H_0 \approx 10^{-28}\ \text{cm}^{-1}$, gives $\Lambda = 4 \times 10^{-56}\ \text{cm}^{-2}$, corresponding to a *present* density of $\rho_0 \approx 1.5 \times 10^{-30}\ \text{g cm}^{-3}$. This would imply a neutrino mass of $m_\nu c^2 < 6$ eV. Again for $K = 0$ and $K = -1$, the solutions of Equation (2) have a minimum value for R as $R_{\text{min}} = (4\pi GM/3c^2\Lambda)^{1/3}$ which again gives a constraint for the present density from known observational limits on Λ ($\leq 10^{-57}\ \text{cm}^{-2}$).

In general, one can write for the evolution of a Universe dominated by low mass non-relativistic neutrinos, the equation

$$\frac{\dot{R}}{R} = \left[\frac{8\pi G(7.5) g_\nu T_\nu^3 \Sigma m_\nu}{3c^2} + \frac{\Lambda c^2}{3} - \frac{Kc^2}{R^2}\right]^{1/2} . \tag{5}$$

Assuming adiabatic expansion ($\dot{R}/R = -\dot{T}/T$), we can obtain a solution of Equation (5) for the temperature T which for $K = 0$ (without loss of generality) is of the form

$$T_\nu = \left[\frac{\Lambda c^4}{60\pi G g_\nu \Sigma m_\nu}\right]^{1/3} [\text{cosech}\,(\tfrac{3}{2}t/\tau)]^{2/3} ; \tag{6}$$

where $\tau = (\Lambda c^2/3)^{-1/2}$. With known present limits on Λ and T_ν(now), one can get a constraint on Σm_ν. As an example for $\Lambda \approx 10^{-57}\ \text{cm}^{-2}$, cosech$(t/\tau) \approx 1$, and

$$\Sigma m_\nu c^2 = \frac{\Lambda c^4}{60\pi G g_\nu T_\nu^3} \approx 100\ \text{eV} \quad (T_\nu = 2\ \text{K};\, g_\nu = 6)\,.$$

If only one of the neutrino types has mass, this would give $m_\nu c^2 \approx 300$ eV ($\Lambda \approx 10^{-55}\ \text{cm}^{-2}$).

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