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# High Resolution Imaging of the Sun and Other Extended Sources

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**Abstract:**
The functional form of the long exposure OTF $P(u,v)$ is known (Fried 1966; Kirshnakumar & Venkatakrishnan 1997a) and $i(x,y)$ is what we record. The knowledge of $P(u,v)$ is complete once $r_o$ is estimated. Our aim is to recover $o_e(x,y)$ which is close to $o(x,y)$. We present here the results of estimation of Fried's parameter $r_o$ using a parameter search method (Kirshnakumar & Venkatakrishnan 1997b) and an modified Wiener filter ($U$ filter) (Roddier 1981) for image restoration. The estimation of $r_o$ and restoration using the $U$ filter on simulations and images of an extended stellar source, globular cluster NGC 1409, is presented.

## 1. Parameter Search for Estimation of $r_o$ in Long Exposure Images

Consider the image degradation model,

$$i(x,y) = o(x,y) * p(x,y) + n(x,y) \quad . \tag{1}$$

In the Fourier domain,

$$I(u,v) = O(u,v)P(u,v) + N(u,v) \quad , \tag{2}$$

where

| Description | Real Space | Fourier Space |
|---|---|---|
| Degraded image | $i(x,y)$ | $I(u,v)$ |
| True object | $o(x,y)$ | $O(u,v)$ |
| Atmospheric psf | $p(x,y)$ | $P(u,v)$ |
| Noise | $n(x,y)$ | $N(u,v)$ |

and $(x,y)$ are real space coordinates; $(u,v)$ are the spatial frequencies. The object estimate $O_e(u,v)$ is defined as:

$$O_e(u,v) = \frac{I(u,v) - N(u,v)}{P(u,v)} \tag{3}$$

and $i(x,y)$ is the only known quantity. We restrict ourselves to long exposure images. The long exposure point spread function (psf) is:

$$P(u,v) = P_o(0,0)\exp\left(-3.44\frac{\lambda k}{r_o}\right)^{\alpha} \tag{4}$$

where: $k = (u^2 + v^2)^{1/2}$, $u$ and $v$ are the spatial frequencies, $\lambda$, the mean wavelength of observation, $r_o$, Fried's parameter, and $\alpha$, a power index = 5/3. The long exposure psf is a real function both in the real domain and in the Fourier domain. Hence, the object's Fourier phase does not get contaminated by the psf. The only source of degradation of the object's Fourier phase is by the noise.

Simple inverse filtering is performed on these degraded images in the Fourier domain and inverse Fourier transformed to the real space. The number of negative pixels ($N$) are found. This process is repeated for various values of $r_o$. The $r_o$ for which $N$ is a minimum is the true "Fried's parameter." The consistency of this technique has been checked at various levels of noise. The technique was validated inthe following way. A frame consisting of many point sources was chosen. The frame was subdivided into many subframes and the Fried's parameter was evaluated. Gaussian profiles were fitted to the stars in the field and an average seeing was obtained for each subframe. A parameter search routine was then used to get the Fried's parameter in that subframe. Figure 1 is the plot of seeing in arcseconds obtained in different subframes. For more details see Krishnakumar & Venkatakrishnan (1997b).

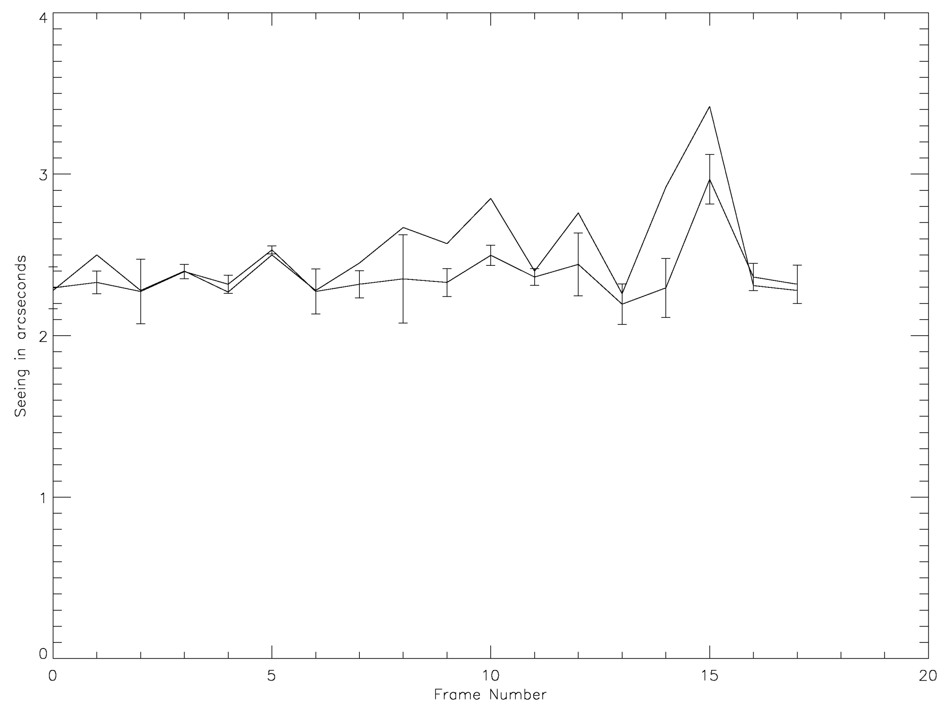


Figure 1. Seeing in arcseconds estimated using a parameter search method and seeing estimated by fitting Gaussian profiles.

## 2. Image Restoration Using the $U$ Filter

From the assumed degradation model, the estimate of the object can be written as:

$$O_e(u, v) = \frac{I(u, v) - N(u, v)}{P(u, v)} \quad . \tag{5}$$

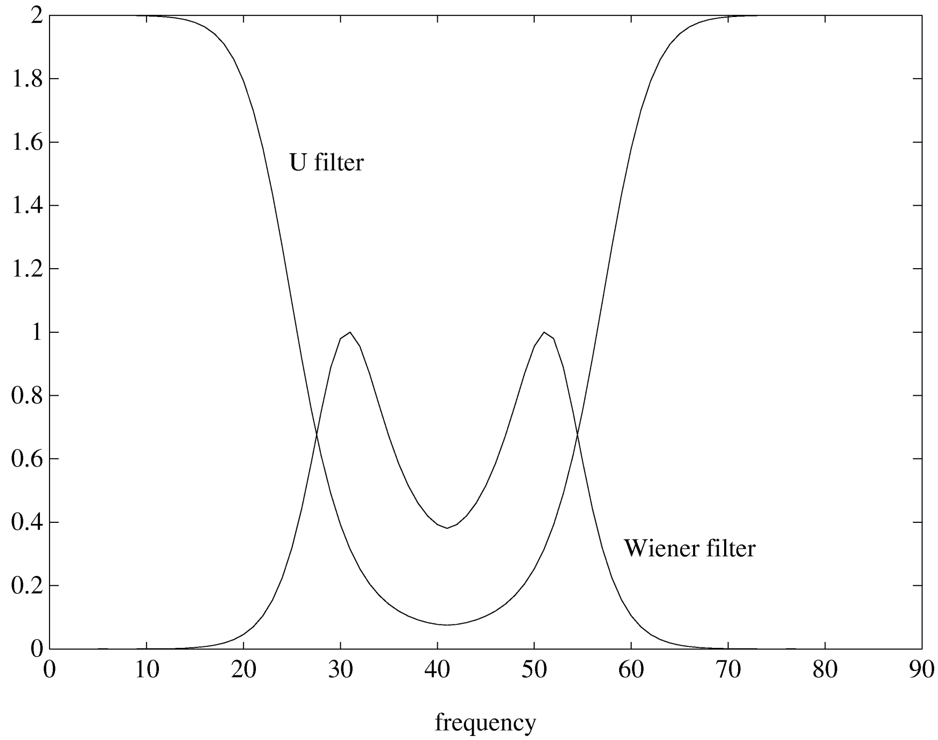


Figure 2. Wiener filter and $U$ filter as a function of frequency. The axes are in arbitrary units. We can see the distinct difference in the high frequency response of the filters. The origin in the plot is at 41 on the x axis and the filter functions are symmetric about this point.

The filter function is given as:

$$U(u,v) = \frac{P(u,v)}{|P(u,v)|^2 + W_{\rm p}\ P(u,v)} \quad . \tag{6}$$

Figure 2 shows the frequency response of the $U$ and Wiener filters. The $U$ filter function is similar to the Wiener filter except that the Wiener parameter $W_{\rm p}$ is weighted by the psf itself. The mean square error is less dependent on the Wiener parameter in the $U$ filter than the Wiener filter. This filter has been rigorously tested on simulations of real images. For more details on the simulations and application of this filter on real images see Roddier (1981).

The parameter search technique and the image restoration filter was applied to the image of Globular Cluster NGC 1409, observed at Vainu Bappu observatory, Kavalur, India. Figure 3 is the contour map of the globular cluster and Figure 4 is the contour map of the restored image.

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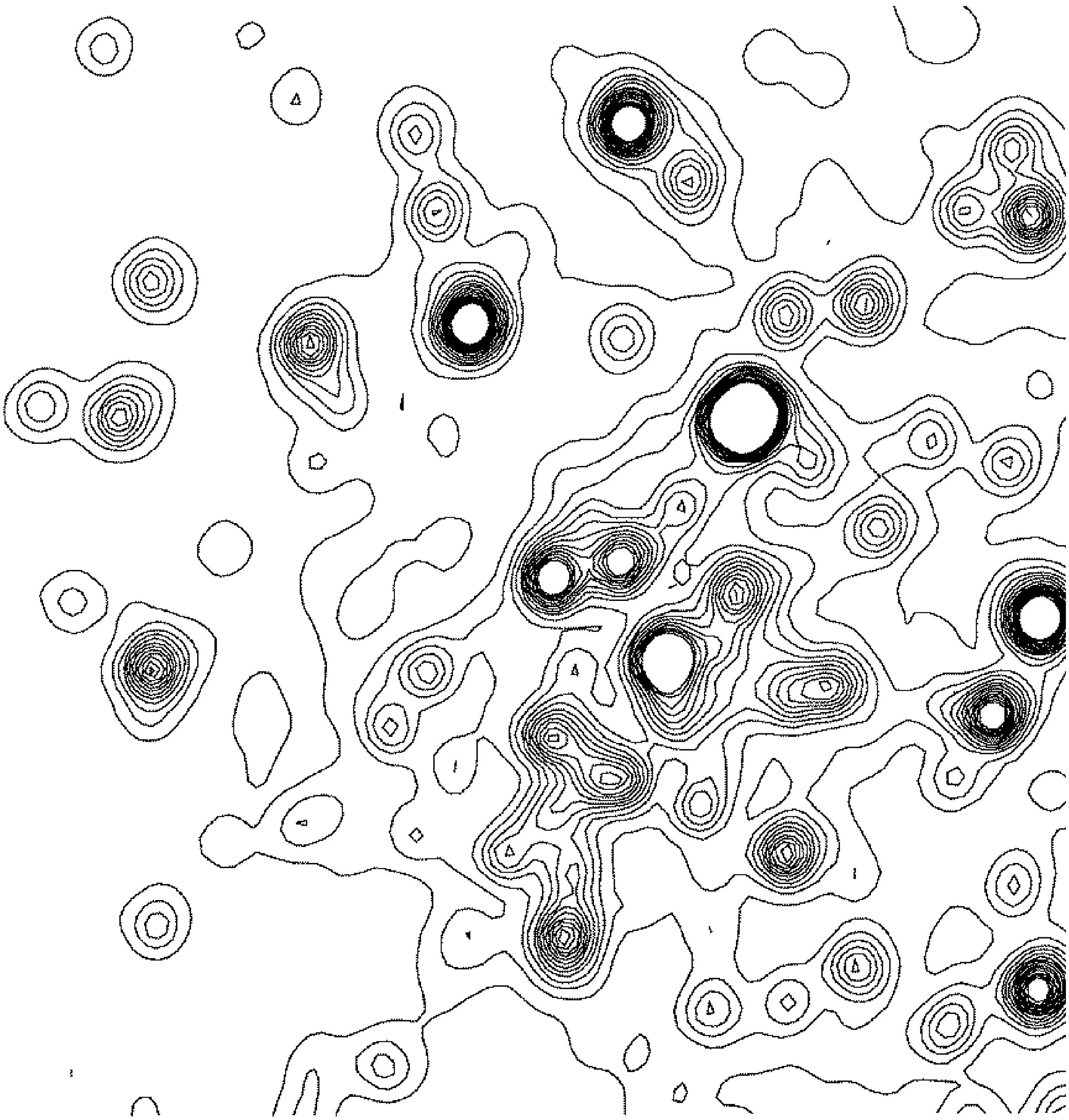

Figure 3. Contour map of globular cluster NGC 1409, obtained from Vainu Bappu observatory, Kavalur, India.

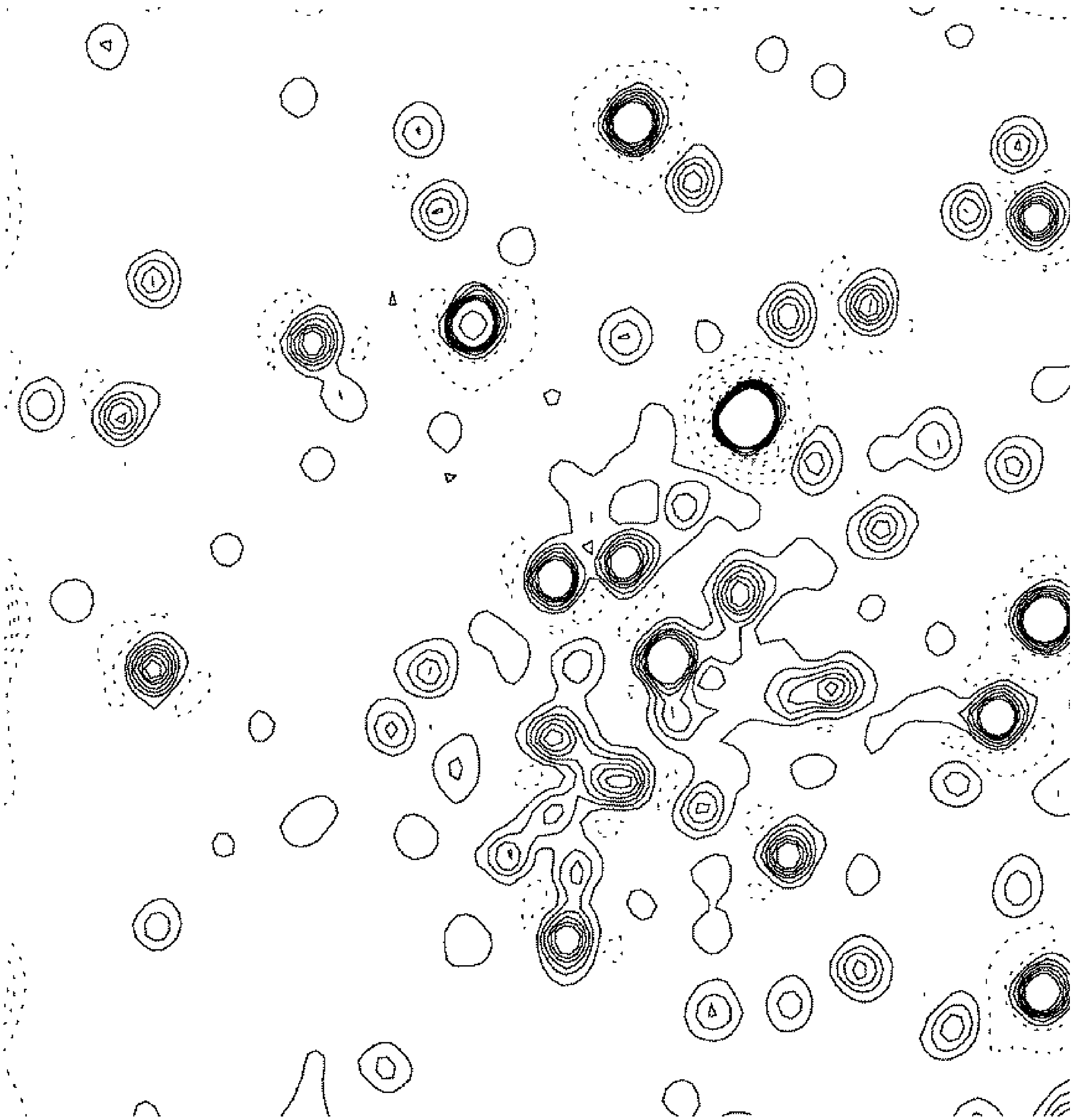

Figure 4. Contour map of the restored image of the globular cluster.

## References

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