STATISTICAL MECHANICS OF VELOCITY AND MAGNETIC FIELDS IN SOLAR ACTIVE REGIONS

V. KRISHAN

Indian Institute of Astrophysics, Bangalore 560 034, India

(Received 19 December, 1983; in final form 29 October, 1984)

Abstract. A statistical mechanics of the velocity and magnetic fields is formulated for an active region plasma. The plasma subjected to the conservation laws emerges in a most probable state which is described by an equilibrium distribution function containing a lagrange multiplier for every invariant of the system. The lagrange multipliers are determined by demanding that the measured expectation values of the invariants be reproduced. For a numerical exercise, we have assumed some probable values for these invariants. The total energy of a coronal loop is estimated from energy balance considerations. Doppler widths of the UV and EUV lines excited in the coronal loop plasma give a measure of the root-mean-square velocities. Measurements of magnetic helicity are not available for the solar corona.

1. Introduction

Solar active regions are believed to be dominated by loop like or arch like structures in emission. The spatial structure of these loops outlines the magnetic field geometry, which may be current free or force free, Vaiana and Rosner (1978). Sakurai (1976) has studied the motion of prominences of the arch and the loop type, deriving the equations of nonlinear evolution of MHD plasma system making use of the principle of least action. The time development of the prominence plasma exhibits various phases of motion. It is the phase showing turbulent motions without any rising motion, that leads to the steady loop system. In earlier papers (Krishan, 1983a, b) a steady-state model of active region coronal loops was presented. The active region plasma is treated as a turbulent magnetofluid. This magnetofluid when subjected to the invariance of total energy, the magnetic helicity and the toroidal and poloidal magnetic fluxes acquires a temperature profile which agrees well with the observed temperature structure of the cool core and hot sheath type of loops. The spatial widths of the UV and EUV lines excited in these loops were calculated and were found to be following the observed gradation (Krishan, 1983b). The statistical theory of incompressible magnetohydrodynamic turbulence as described by Montgomery *et al.* (1978) was used in order to delineate the spatial configuration of active region coronal loops. The main features of the theory consist of using the MHD equations for an incompressible fluid. The magnetic and velocity fields are expanded in terms of Chandrasekhar–Kendall functions. A single Chandrasekhar–Kendall function represents a force-free state, the superposition does not. The pressure profile of the plasma is obtained from a poisson equation for the mechanical pressure as a function of the velocity and magnetic fields. Taylor (1974, 1975, 1976) conjectured that the decay of energy to a minimum value compatible with a conserved value of magnetic helicity leads to a force-free state

Solar Physics **95** (1985) 269–280. 0038–0938/85.15

representable by a single Chandrasekhar–Kendall function. In this state of minimum energy, one can simultaneously invoke the constancy of total energy and magnetic helicity. Montgomery *et al.* (1978) introduced the toroidal and the poloidal magnetic fluxes as additional invariants. This resulted in several states being accessible for a fixed value of the ratio of toroidal and poloidal magnetic fluxes and for a fixed value of the axial and azimuthal mode numbers (n, m), respectively. In the present paper, we present this steady state as an equilibrium ensemble. The statistical mechanics of the velocity and magnetic fields is formulated in a phase space whose coordinates are the real and imaginary parts of the expansion coefficients. The success of the lowest mode state $(m = n = 0)$ in accounting for the temperature profile of the cool core and hot sheath loops has provided the motivation for studying the statistical distribution of the velocity and the magnetic fields in this particular state $(m = n = 0)$. An equilibrium distribution is assumed in which the Lagrange multipliers are determined by requiring the expectation values of the energy, the magnetic flux and the magnetic helicity to match the observed average values of these conserved quantities. An estimate of the total energy in the coronal loop can be made using the energy balance arguments Levine and Withbroe (1977). There are no direct measurements of the magnetic helicity and magnetic fluxes in the coronal loops. Therefore, the values of these quantities assumed here to be indicative of the actual values could serve as a prediction to be verified by possible future observations. It may be appropriate to point out that the measurement of the invariants of MHD turbulence in the solar wind has been achieved Matthaeus and Goldstein (1982). In the next section, the canonical distributions for the expansion coefficients of the fields and for the conserved quantities are given. The statistical distribution of velocity fields has been derived for the prominence plasma Jensen (1982). The present work, in addition describes the distributions of magnetic helicity and magnetic fluxes. Assuming a Gibbs distribution for the system enables us to determine the magnitude as well as the probability distribution of fluctuations in the velocity and magnetic fields. The role of these fluctuations in producing large-scale coherent structures is one of the most important revelations of the MHD turbulence theory. The study of correlations between fluctuations give us clues about the kinds of MHD modes like Alfvén waves excited in the plasma. According to one suggestion, the heating and acceleration of plasma particles in a coronal loop is achieved through Alfvén waves propagating in the opposite legs of the loop. Such a situation corresponds to a finite velocity fluctuation associated with a zero magnetic field fluctuation as discussed by Matthaeus and Goldstein (1982). The presence of coherent loop like structures in the solar corona provides an appropriate system for the applications of the results of MHD turbulence theory.

2. The Equilibrium Distribution Function

The equations describing an incompressible ideal MHD turbulent plasma are:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \mathbf{V})\mathbf{V} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla P, \qquad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) , \tag{2}$$

$$\nabla \cdot \mathbf{V} = 0 , \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0 . \tag{4}$$

The three quadratic invariants of an incompressible MHD turbulent plasma as discussed by Frisch *et al.* (1975) are:

$$\text{total energy } E = \int \mathrm{d}^3 x \, (V^2 + B^2) , \tag{5}$$

$$\text{the magnetic helicity } H_m = \int \mathrm{d}^3 x \, \mathbf{A} \cdot \mathbf{B} , \tag{6}$$

$$\text{the cross helicity } H_c = \int \mathrm{d}^3 x \, \mathbf{V} \cdot \mathbf{B} , \tag{7}$$

where **V**, **B**, and **A** are respectively the velocity, the magnetic field, and the vector potential. The magnetic field B in defined in Alfvén speed units, i.e. $\mathbf{B} = \mathbf{B}/(4\pi\rho_0)^{1/2}$, and the total energy E is given per unit density. The invariants, Equations (5), (6), and (7) can be derived by using the MHD equations and by converting the time derivatives of the invariants to surface terms. The invariance is assured for those boundary conditions for which the surface terms vanish. This point is discussed in detail by Matthaeus and Goldstein (1982). This system of constrains in an equilibrium ensemble gives the average values of the linear quantities like magnetic field and velocity field to be zero. In order to deal with systems which necessarily have net magnetic and velocity fields, it is essential to demand a non-zero value of these linear quantities. This has been achieved by proposing to constrain the toroidal and poloidal magnetic fluxes ψ_t and ψ_p as well as the corresponding fluxes of the velocity field. In the present study we restrict ourselves to the systems with $\langle \mathbf{V} \rangle = 0$. The magnetic fluxes are defined as

$$\psi_t = \frac{R}{L} \int_0^L \mathrm{d}z \int_0^{2\pi} \mathrm{d}\theta A_\theta \tag{8}$$

and

$$\psi_p = \int_0^L \mathrm{d}z \int_0^{2\pi} \mathrm{d}\theta A_z . \tag{9}$$

Here, a cylindrical geometry of the plasma has been taken with R and L as the radius and the length of the cylinder. This geometry is most appropriate for studying coronal loops. The magnetic and velocity fields are expanded in terms of Chandrasekhar–Kendall functions. In the present study, we represent the loop plasma by the lowest state corresponding to $m = n = 0$ since this state has been able to account for the radial temperature structure of the active region coronal loops quite well. The fields are expanded as

$$\mathbf{B} = \sum_q \xi(0, 0, q)\lambda(0, 0, q)\mathbf{F}(0, 0, q),$$
$$\mathbf{V} = \sum_q \eta(0, 0, q)\lambda(0, 0, q)\mathbf{F}(0, 0, q), \tag{10}$$

where

$$\mathbf{F}(0, 0, q) = c_{0,0,q}\left[\hat{e}_\theta\left\{-\frac{\partial}{\partial r} J_0(\gamma_{00q} r)\right\} + \hat{e}_z \lambda_{00q} J_0(\gamma_{00q} r)\right],$$
$$\lambda_{00q} = \pm \gamma_{00q}. \tag{11}$$

$c_{0,0,q}$ are normalization constants, $\xi(0, 0, q)$ and $\eta(0, 0, q)$ are the expansion coefficients. Here r is the radial coordinate in cylindrical geometry and n and m are the axial and azimuthal wave numbers. Now $\gamma_{00q} > 0$ and are determined from boundary conditions. The geometry of a coronal loop is shown in Figure 1. For a closed coronal

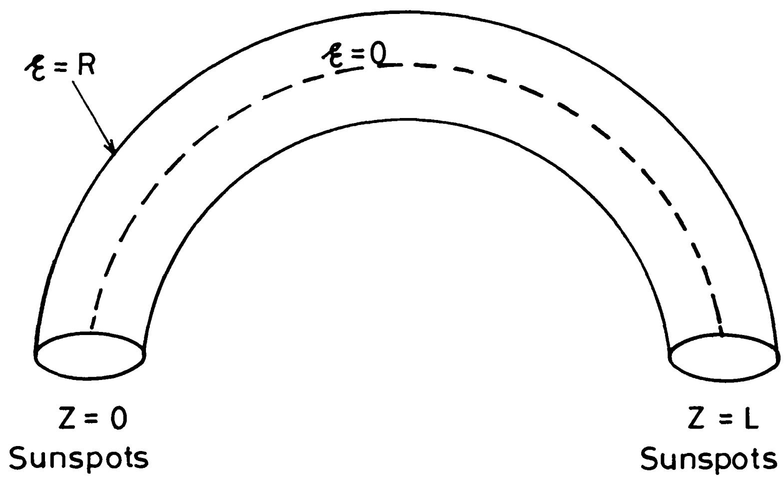

Fig. 1. Geometry of a coronal loop.

loop, we choose the boundary conditions for a rigid and perfectly conducting surface i.e. $B_r(r = R) = 0 = V_r(r = R)$. The eigenvalues for $m = n = 0$ have to be determined from different considerations since the radial component of $\mathbf{F}$ vanishes for $m = n = 0$. One notices that for each individual $(0, 0, q)$ mode, the ratio of the toroidal flux ψ_t to the poloidal flux ψ_p is

$$\frac{\psi_t}{\psi_p} = -\frac{R}{L}\frac{\gamma_{00q} J_0'(\gamma_{00q} R)}{\lambda_{00q} J_0(\gamma_{00q} R)}. \tag{12}$$

Since both ψ_t and ψ_p are constants of motion, λ_{00q} can be determined from Equation (12) for all $q = 1, 2, 3 \ldots$ where $q = 1$ corresponds to the lowest value of

$\lambda(0, 0, q)$. Thus we take two values of q and the corresponding eigenvalues are λ_0 and λ_1. The integrals of motion take the form:

$$E = \lambda_0^2[|\xi_0|^2 + |\eta_0|^2] + \lambda_1^2[|\xi_1|^2 + |\eta_1|^2], \tag{13}$$

$$H_m = \lambda_0|\xi_0|^2 + \lambda_1|\xi_1|^2, \tag{14}$$

$$\psi_t = -2\pi R[\xi_0\gamma_0 c_0 J_0'(\gamma_0 R) + \xi_1\gamma_1 c_1 J_0'(\gamma_1 R)], \tag{15}$$

and

$$\psi_p = -2\pi L[\xi_0\lambda_0 c_0 J_0(\gamma_0 R) + \xi_1\lambda_1 c_1 J_0(\gamma_1, R)], \tag{16}$$

where E is the total energy and H_m is the magnetic helicity. One chooses the canonical distribution subjected to the constraints E, H_m, ψ_t, and ψ_p as

$$D = \text{constant}\exp[-\alpha E - \beta H_m - \delta\psi_t], \tag{17}$$

where α, β, and δ are the Lagrange multipliers. We known that the Gibb's distribution is applicable for a subsystem which is in statistical equilibrium with a larger closed system. Here, in the present situation, each $(0, 0, q)$ represents one subsystem. From Equation (17) we can factor out the probability distribution for a particular subsystem in the presence of remaining subsystems which act like a medium. It is in this sense that the fluctuations in physical quantities for a subsystem are derived from Gibb's distribution. If we represent the whole system by only one mode say (0, 0, 1), there are no other subsystems and thus no medium, the fluctuations vanish as the chosen state (0, 0, 1) forms a completely closed system, Lifshitz and Pitaevskii (1980). One can factor out the probability distribution for expansion coefficients $\xi(0, 0, q)$ and $\eta(0, 0, q)$ as

$$P_\eta(0, 0, q) = K^\eta_{0,0,q}\exp[-\alpha\lambda^2(0, 0, q)|\eta(0, 0, q)|^2] \tag{18}$$

which gives

$$\langle|\eta(0, 0, q)|^2\rangle = \tfrac{1}{2}\alpha^{-1}\lambda^{-2}(0, 0, q) \tag{19}$$

and

$$\langle|\eta(0, 0, q)|\rangle = 0\,.$$

The probability distribution for $\xi(0, 0, q)$ is

$$P_\xi(0, 0, q) = K^\xi_{0,0,q}\exp[-\{\alpha\lambda^2(0, 0, q) + \beta\lambda(0, 0, q)\}\times \times|\xi(0, 0, q)|^2 - \delta\rho(0, 0, q)\xi(0, 0, q)], \tag{20}$$

where

$$\rho(0, 0, q) = 2\pi Rc(0, 0, q)\gamma_{00q}J_1(\gamma_{00q}R);$$

$K^\xi_{0,0,q}$ and $K^\eta_{0,0,q}$ are normalization constants.

We find:

$$\langle \xi(0,0,q)\rangle = \frac{-\delta\rho(0,0,q)}{2[\alpha\lambda^2(0,0,q)+\beta\lambda(0,0,q)]}\,,$$

$$\langle |\xi(0,0,q)|^2\rangle = [\alpha\lambda^2(0,0,q)+\beta\lambda(0,0,q)]^{-1}\times \times\left[\frac{1}{2}+\frac{\delta^2\rho^2(0,0,q)}{4\{\alpha\lambda^2(0,0,q)+\beta\lambda(0,0,q)\}}\right]. \tag{21}$$

The average values of the invariants for the state $(0, 0, q)$ are

$$\overline{E} = \frac{\lambda_0^2}{2x_0} + \frac{\lambda_0^2\delta^2\rho_0^2}{4x_0^2} + \frac{\lambda_1^2}{2x_1} + \frac{\lambda_1^2\delta^2\rho_1^2}{4x_1^2} + \frac{1}{\alpha}\,,$$

$$\overline{H}_m = \frac{\lambda_0}{2x_0} + \frac{\lambda_0\delta^2\rho_0^2}{4x_0^2} + \frac{\lambda_1}{2x_1} + \frac{\lambda_1\delta^2\rho_1^2}{4x_1^2}\,, \tag{22}$$

$$\overline{\psi}_t = -\frac{\delta\rho_0^2}{2x_0} - \frac{\delta\rho_1^2}{2x_1}\,,$$

where

$$x_0 = \alpha\lambda_0^2 + \beta\lambda_0\,; \qquad x_1 = \alpha\lambda_1^2 + \beta\lambda_1\,,$$

$$\rho_0 \equiv \rho_{001} = 2\pi R c_0 \gamma_0 J_1(\gamma_0 R)\,; \qquad \gamma_0 = \lambda_0\,,$$

$$\rho_1 \equiv \rho_{002} = 2\pi R c_1 \gamma_1 J_1(\gamma_1 R)\,; \qquad \gamma_1 = -\lambda_1\,,$$

$$c^2(0,0,q) = [\pi L \gamma_{00q}^2 R^2 \{J_0^2(\gamma_{00q}R) + 2J_1^2(\gamma_{00q}R) - - J_0(\gamma_{00q}R)J_2(\gamma_{00q}R)\}]^{-1}\,;$$

$\lambda(0, 0, q)$ are determined from Equation (12) for $R = 10^9$ cm. $L = 5 \times 10^9$ cm and $\overline{\psi}_t/\overline{\psi}_p = 1$. We find $\lambda(0, 0, 1) \equiv \lambda_0 = 2.2/R$ and $\lambda(0, 0, 2) \equiv \lambda_1 = -2.6/R$. We neglect higher roots of Equation (12) since according to Montgomery *et al.* (1978), for $\lambda^2 \geq (3.1/R)^2$ $m = 0 = n$ is not the most probable state. So, now the system is composed of two subsystems corresponding to two values of $\lambda(0, 0, q)$ and we go on to determine the Lagrange multipliers in terms of the average quantities $\overline{E}$, $\overline{H}_m$, and $\overline{\psi}_t$ from Equations (22) one finds:

$$x_0 = \frac{1 \pm [1 + 4A_0\delta^2\rho_0^2]^{1/2}}{4A_0}\,, \tag{23}$$

$$x_1 = \frac{1 \pm [1 + 4A_1\delta^2\rho_1^2]^{1/2}}{4A_1}\,, \tag{24}$$

where

$$A_0 = \frac{\overline{E} - \lambda_1 \overline{H}_m}{\lambda_0(\lambda_0 - \lambda_1)} ,$$

$$A_1 = \frac{\overline{E} - \lambda_0 \overline{H}_m}{\lambda_1(\lambda_1 - \lambda_0)} .$$

It is not possible to solve for α, β, and δ without resorting to numerical methods since these from algebraic equations of very high degree. We have found analytical solutions under two limiting cases: (i) $4A_0 \delta^2 \rho_0^2 \ll 1$ and $4A_1 \delta^2 \rho_1^2 \ll 1$; and (ii) $4A_0 \delta^2 \rho_0^2 \gg 1$ and $4A_1 \delta^2 \rho_1^2 \gg 1$. These conditions translate into a relationship between the magnetic energy and the kinetic energy for each mode i.e. for every q. Thus $4A_0 \delta^2 \rho_0^2$ can be expressed as

$$4A_0 \delta^2 \rho_0^2 = p(p - 1) ,$$

where

$$p \simeq \frac{\langle \lambda_0^2 |\xi(0, 0, 1)|^2 \rangle}{\langle \lambda_0^2 |\eta(0, 0, 1)|^2 \rangle} .$$

Therefore, $4A_0 \delta^2 \rho_0^2 \ll 1$ says that $p \sim 1$ and hence an approximate equipartition of magnetic and kinetic energy for each mode $(0, 0, q)$. The opposite limit $4A_0 \delta^2 \rho_0^2 \gg 1$ says that $p \gg 1$ and hence the magnetic energy is much larger than the kinetic energy. Similarly one can conclude about the mode $(0, 0, 2)$ corresponding to the eigenvalue λ_1 from the quantity $4A_1 \delta^2 \rho_1^2$. We discuss these cases below.

2.1. Case I: $4A_0 \delta^2 \rho_0^2 \ll 1$ and $4A_1 \delta^2 \rho_1^2 \ll 1$.

This gives:

$$\alpha\lambda_0^2 + \beta\lambda_0 = \frac{1}{2A_0} \tag{25}$$

and

$$\alpha\lambda_1^2 + \beta\lambda_1 = \frac{1}{2A_1} . \tag{26}$$

Here, we assume $A_0 > 0$ and $A_1 > 0$ and take only the positive values of $\alpha\lambda^2(0, 0, q) + \beta\lambda(0, 0, q)$ since this is a measure of the width of the probability distribution of $\langle |\xi(0, 0, q)|^2 \rangle$. The values of α, β, and δ are determined from the following relationships:

$$\alpha = [2(\overline{E} - \lambda_1 \overline{H}_m)(\overline{E} - \lambda_0 \overline{H}_m)]^{-1} [-\{3\overline{E} + \tfrac{3}{2}(\lambda_0 + \lambda_1)\overline{H}_m\} \pm \pm \{\overline{E}^2 + \tfrac{9}{4}(\lambda_0 + \lambda_1)^2 \overline{H}_m^2 - 8\lambda_1\lambda_0 \overline{H}_m^2 - \overline{E}(\lambda_0 + \lambda_1)\overline{H}_m\}^{1/2}] , \tag{27}$$

$$\beta = \frac{-\lambda_1}{2\left(\bar{E} - \dfrac{1}{\alpha} - \lambda_1 \bar{H}_m\right)} - \frac{\lambda_0}{2\left(\bar{E} - \dfrac{1}{\alpha} - \lambda_0 \bar{H}_m\right)}, \tag{28}$$

and

$$\delta = -\bar{\psi}_t \left[\frac{\rho_0^2 \left(\bar{E} - \dfrac{1}{\alpha} - \lambda_1 \bar{H}_m\right)}{\lambda_0(\lambda_0 - \lambda_1)} + \frac{\rho_1^2 \left(\bar{E} - \dfrac{1}{\alpha} - \lambda_0 \bar{H}_m\right)}{\lambda_1(\lambda_1 - \lambda_0)} \right]. \tag{29}$$

Now, $\bar{E}$, $\bar{H}_m$, and $\bar{\psi}_t$ are to be obtained from observations. Since for the case of coronal loops, there are no direct measurements of these quantities, we shall have to fix their values from other considerations. For example, the value of $\bar{E}$ can be obtained from energy balance arguments which give $\bar{E} \sim 10^{28}$ ergs (Levine and Withbroe, 1977). We choose

$$\bar{E} = \bar{E}_{28} \times 10^{28} \text{ ergs}$$

and

$$\bar{H}_m = \bar{H}_{m37} \times 10^{37} \text{ ergs cm}.$$

The values of $\bar{E}_{28}$ and $\bar{H}_{m37}$ are fixed such that the positivity of $\alpha\lambda^2 + \beta\lambda$ is preserved. As an example we take $\bar{E}_{28} = 1$, $\bar{H}_{m37} = 0.05$. The two values of α determined from Equation (39) are

$$\alpha = 2.04 \times 10^{-28} \text{ (ergs)}^{-1}$$

and

$$\alpha = 0.98 \times 10^{-28} \text{ (ergs)}^{-1}.$$

For the above chosen numbers $\alpha > 1$ is the proper choice satisfying $A_1 > 0$ and $A_0 > 0$. Equation (28) provides $\beta = 0.2 \times 10^{-37}$ (ergs cm)$^{-1}$. From Equation (24) we find

$$\delta = 10^{-36}\, \bar{\psi}_t \text{ (G cm}^2)^{-1}.$$

This combined with the conditions $4A_0 \delta^2 \rho_0^2 \ll 1$ and $4A_1 \delta^2 \rho_1^2 \ll 1$ gives $\bar{\psi}_t \ll 0.65 \times 10^{18}$ G cm^2. A representative choice of $\bar{\psi}_t \sim 10^{17}$ maxwell gives $\delta = 10^{-19}$. Thus the distribution function for this particular indicative numerical example is

$$D \sim e^{-2.04E/\bar{E}}\, e^{-0.01H_m/\bar{H}_m}\, e^{-0.01\psi_t/\bar{\psi}_t}. \tag{30}$$

We can factor out probability distribution for each subsystem and study their widths as shown in Figures 2, 3, and 4, where E stands for E_1 or E_2 and same is true of H_m and $\bar{V}^2$. The validity of the conditions $4A_0 \delta^2 \rho_0^2 \ll 1$ and $4A_0 \delta^2 \rho_1^2 \ll 1$ can be checked by observing the relative contributions of the magnetic energy and the kinetic energy.

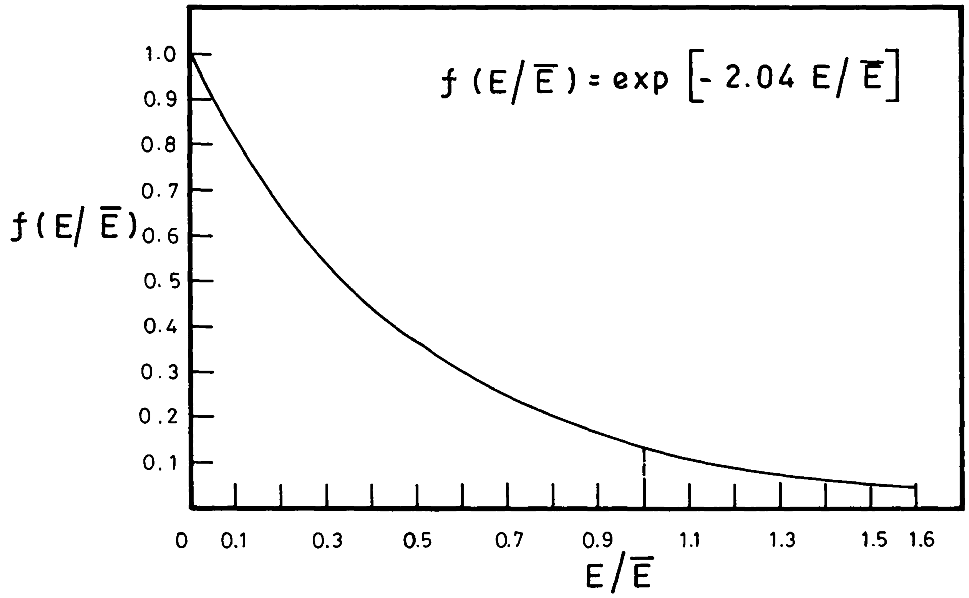

Fig. 2. Distribution function of the energy E.

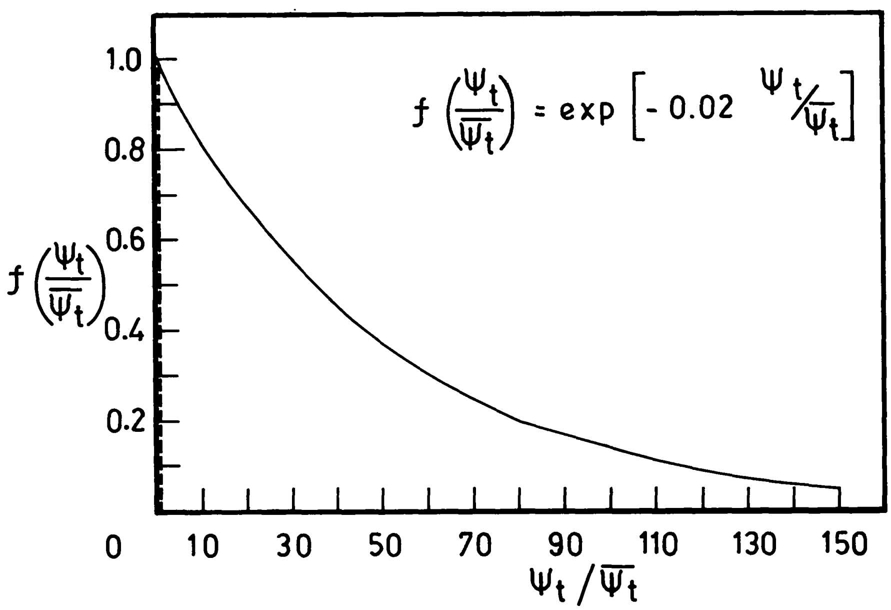

Fig. 3. Distribution function of the magnetic helicity H_m and the toroidal flux ψ_t.

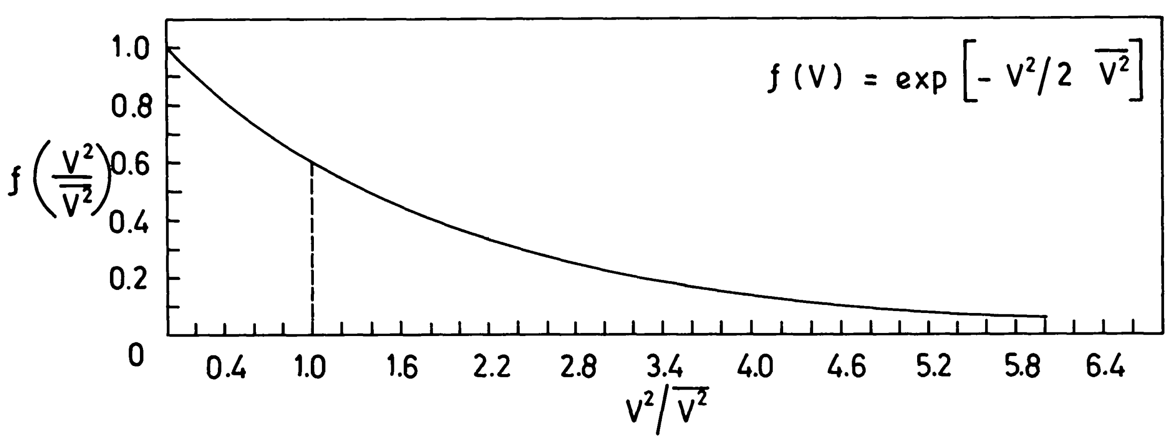

Fig. 4. Distribution function of the velocity.

This can be seen as follows:

$$\bar{E} = \bar{E}_H + \bar{E}_V ,$$

$$\bar{E}_H = \frac{\lambda_0^2}{2x_0}\left[1 + \frac{\delta^2\rho_0^2}{2x_0}\right] + \frac{\lambda_1^2}{2x_1}\left[1 + \frac{\delta^2\rho_1^2}{2x_1}\right]$$

$$= \frac{\lambda_0^2}{2(\alpha\lambda_0^2 + \beta\lambda_0)}[1 + \delta^2\rho_0^2 A_0] + \frac{\lambda_1^2}{2(\alpha\lambda_1^2 + \beta\lambda_1)}[1 + \delta^2\rho_1^2 A_1] .$$

Since $\alpha\lambda_0^2 \gg \beta\lambda_0$ and $\alpha\lambda_1^2 \gg \beta\lambda_1$ we thus get

$$\bar{E}_H \simeq \frac{1}{2\alpha} + \frac{1}{2\alpha}$$

$$\simeq \frac{1}{\alpha} ;$$

$$\bar{E}_V = \sum_q \lambda^2(0, 0, q)\langle |\eta(0, 0, q)|^2 \rangle$$

$$= \frac{1}{2\alpha} + \frac{1}{2\alpha} = \frac{1}{\alpha} .$$

Thus there is equipartition of energy.

2.2. Case II: $4A_0\delta^2\rho_0^2 \gg 1$ and $4A_1\delta^2\rho_1^2 \gg 1$.

We find

$$x_1 = \frac{\delta\rho_1}{2\sqrt{A_1}} , \tag{31}$$

$$x_0 = \frac{\delta\rho_0}{2\sqrt{A_0}} \tag{32}$$

and Equation (22) gives

$$\rho_0\sqrt{A_0} + \rho_1\sqrt{A_1} = -\bar{\psi}_t . \tag{33}$$

α can be determined from Equation (33) for given $\bar{E}$, $\bar{H}_m$, and $\bar{\psi}_t$. It is found that for $\bar{\psi}_t = 1.9 \times 10^{18}$ maxwells, $\bar{E} = 10^{28}$ ergs and $\bar{H}_m = 0.05 \times 10^{37}$ ergs cm, $\alpha = 233 \times 10^{-28}$ (ergs)$^{-1}$. A large value of α and $\bar{\psi}_t$ is needed in order to satisfy the conditions for this case.

α is found to be very sensitive to the value of $\bar{\psi}_t$. For example for $\bar{\psi}_t = 1.8 \times 10^{18}$ maxwell, $\bar{E} = 10^{28}$ ergs, and $\bar{H}_m = 0.05 \times 10^{37}$ ergs cm, $\alpha = 7.72 \times 10^{-28}$ (ergs)$^{-1}$. For this case β and δ are given by

$$\beta = -\alpha\lambda_0 + \frac{\alpha}{2\sqrt{A_0}} \frac{\lambda_1(\lambda_1 - \lambda_0)}{[(\lambda_0/2\sqrt{A_1}) - (\lambda_1/2\sqrt{A_0})]} \tag{34}$$

and

$$\delta = \frac{\alpha\lambda_1\lambda_0(\lambda_1 - \lambda_0)}{\left[\frac{\rho_1\lambda_0}{2\sqrt{A_1}} - \frac{\rho_0\lambda_1}{2\sqrt{A_0}}\right]} . \tag{35}$$

We find that $\alpha = 233 \times 10^{-28}$ (ergs)$^{-1}$ is appropriate for satisfying $4A_0\delta^2\rho_0^2 \gg 1$ and $4A_1\delta^2\rho_1^2 \gg 1$. The corresponding values of β and δ are:

$$\beta = 10^{-34}\,(\text{ergs cm})^{-1} \quad \text{and} \quad \delta = 2.5 \times 10^{-16}\,(\text{maxwell})^{-1} .$$

The distribution function of the ensemble can be written as

$$D \sim \exp[-233E/\bar{E}]\exp[-50H_m/\bar{H}_m]\exp[-450\psi_t/\bar{\psi}_t] .$$

Again one can calculate the average values of the magnetic energy and kinetic energy separately. We find since $\alpha \gg 1$, most of the energy is in the form of magnetic energy. The value of the quantity $4A_0\delta^2\rho_0^2 = 2.75 \times 10^5$ and $4A_1\delta^2\rho_1^2 = 1.75 \times 10^5$. For this particular numerical example we find $\bar{E}_H/\bar{E}_V = 10^5$. Thus this case may not represent a realistic situation in the coronal loop since one observes root mean square velocities of 30–50 km s^{-1} in the line widths. Besides the equipartition of energy between the magnetic and the kinetic energies is a well trusted hypothesis in many astrophysical situations. More quantitative insight into the distribution of magnetic and kinetic energies can only be obtained for a general case without resorting to the limiting cases I and II presented here which necessarily involves lot of numerical work. The present study has, however, contributed in a positive manner in delineating the probability distributions of the magnetic fluxes and the total energy of the system. This study, on one hand proves the validity of the statistical treatment of magneto-hydrodynamical turbulence and predicts the values of the invariants which cannot be measured at present. Once the probability distributions are known, one can proceed to study the evolution of fluctuations and their correlations as for example has been done by Dobrowolny *et al.* (1980a, b), for the case of solar wind. The availability of data for the solar corona similar to the solar wind could scrutinize some of these theoretical concepts. This could further lead one to consider stellar atmospheres where such data may never be available.

3. Conclusion

The statistical mechanics of the velocity and magnetic field is formulated in a phase space whose coordinates are the expansion coefficients of these fields. The distribution function for the equilibrium ensemble is calculated. Results are applied to the steady state of the active region coronal loop. The average values of the magnetic helicity and the poloidal and toroidal fluxes are predicted assuming the average value of the total energy being known. For analytical progress, two limiting cases are studied out of which one represents the equipartition of energy and the other represents the dominance of magnetic energy over the kinetic energy. The stage is set for studying the nature of magnetic field and velocity field fluctuations, their interrelationship, their correlations and their temporal behaviour in the solar coronal loops.

References

Dobrowolny, M., Mangeney, A., and Veltri, P.: 1980a, *Astron. Astrophys.* **83**, 26.
Dobrowolny, M., Mangeney, A., and Veltri, P.: 1980b, *Phys. Rev. Letters* **45**, 144.
Frisch, U., Pouquet, A., Leorat, J., and Mazure, A.: 1975, *J. Fluid Mech.* **68**, 769.
Jensen, E.: 1982, *Solar Phys.* **77**, 109.
Krishan, V.: 1983a, in *Proceedings of the Spring College on Radiation in Plasmas*, Trieste, Italy.
Krishan, V.: 1983b, *Solar Phys.* **88**, 155.
Levine, R. H. and Withbroe, G. L.: 1977, *Solar Phys.* **51**, 83.
Lifshitz, E. M. and Pitaevskii, L. P.: 1980, *Statistical Physics*, 3rd Edition, Part 1, p. 79.
Matthaeus, W. H. and Goldstein, M. L.: 1982, *J. Geophys. Res.* **87**, 6011.
Montgomery, D., Turner, L., and Vahala, G.: 1978, *Phys. Fluids* **21**, 757.
Sakurai, T.: 1976, *Publ. Astron. Soc. Japan* **28**, 177.
Taylor, J. B.: 1974, *Phys. Rev. Letters* **33**, 1139.
Taylor, J. B.: 1975, *Plasmas Physics and Controlled Fusion*, Tokyo, IAEA, Vienna, Vol. 1., p. 161.
Taylor, J. B.: 1976, in D. E. Evans (ed.), *Pulsed High Beta Plasmas*, Pergamon, Oxford, p. 59.
Vaiana, G. S. and Rosner, R.: 1978, *Ann. Rev. Astron. Astrophys.* **16**, 393.