# SUPERCRITICAL WINDS FROM COOL 'CANONICAL' STARS CAUSED BY EVOLUTION ON THE MAIN SEQUENCE*

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**Abstract.** Even the very slow expansion of a star's radius due to evolution on the Main Sequence is shown to be supercritical for cool stars without coronae. Since steady spherically-symmetric supercritical solutions are theoretically impossible, unsteady supercritical solutions are studied. It is seen that smooth sonic transitions are possible in the unsteady case, but are accompanied by enhancement of pressure over the critical values.

## 1. Introduction

Stellar winds are driven by gradients of pressure between the atmosphere and the interstellar medium. Conventional sources of pressure heads are (1) thermal energy of hot coronae (significant for cool stars) and (2) radiation pressure (significant for hot stars). A possible third source is thermonuclear energy generated in stellar cores. This energy is known to increase the star's radius on the Main Sequence at a rate of $\approx 10^{-7}$ cm s$^{-1}$ (Strömgren, 1965). Even this apparently negligible expansion becomes supercritical for cool stars without coronae. This paper is a preliminary effort to understand the dynamics of the star's environment in the presence of such an evolutionary expansion.

## 2. The Magnitude of Supercriticality

Isothermal wind solutions suffice for a simple estimation of supercriticality. The parameter representing the critical isothermal wind is $r_c/r_*$ where $r_c = GM_*/2S_*^2$ is the sonic point, $r_*$, $M_*$, and $S_*$ being, respectively, the star's radius, mass, and isothermal sound speed of its atmosphere. From the well-known solution of an isothermal wind, one obtains

$$\left(\frac{V_*}{S_*}\right) \exp\left\{-\frac{1}{2}\left(\frac{V_*}{S_*}\right)^2\right\} = \left(\frac{r_c}{r_*}\right)^2 \exp\left\{-\frac{2r_c}{r_*} + 1.5\right\}; \tag{1}$$

where $V_*$ is the wind speed at the stellar surface. In Table I we can see the values of $V_*/S_*$ corresponding to various values of $r_c/r_*$.

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TABLE I

Values of critical surface Mach numbers $V_*/S_*$ for various values of $r_c/r_*$

| $r_c/r_*$ | $V_*/S_*$ |
|---|---|
| 0.5 | 0.45 |
| 1.0 | 1.00 |
| 5.0 | $5 \times 10^{-3}$ |
| 10.0 | $9 \times 10^{-7}$ |
| 100.0 | $6 \times 10^{-83}$ |
| 1000.0 | 0 |

A typical 'canonical ' cool star ($T_* = 6000$ K) without a hot corona would have $r_c/r_* \approx 100$, for which $V_*/S_* = 6 \times 10^{-83}$! Thus we see that any non-static behaviour of the stellar surface would lead to extreme supercriticality. It is also interesting to ponder over the fact that evolutionary expansion velocities of $10^{-7}$ cm s$^{-1}$ are critical only at $T_* \approx 10^5$ K for a star with solar mass and radius.

## 3. Impossibility of Steady Spherically-Symmetric Supercritical Expansion

Wolfson and Holzer (1975) have shown that steady supercritical solutions are not theoretically possible. This is because the transition near the singularity $V = S_*$ would make the solution jump from a branch of higher entropy to one of lower entropy. Wolfson and Holzer, however, do not explicitly answer the question as to what could be a result of imposing a supercritical mass flux on a steady critical wind. They suggest either breakdown of the steady condition or the adjustment of surface conditions to the new value of the mass flux. In this paper we numerically examine unsteady supercritical solutions.

## 4. The Unsteady Solutions

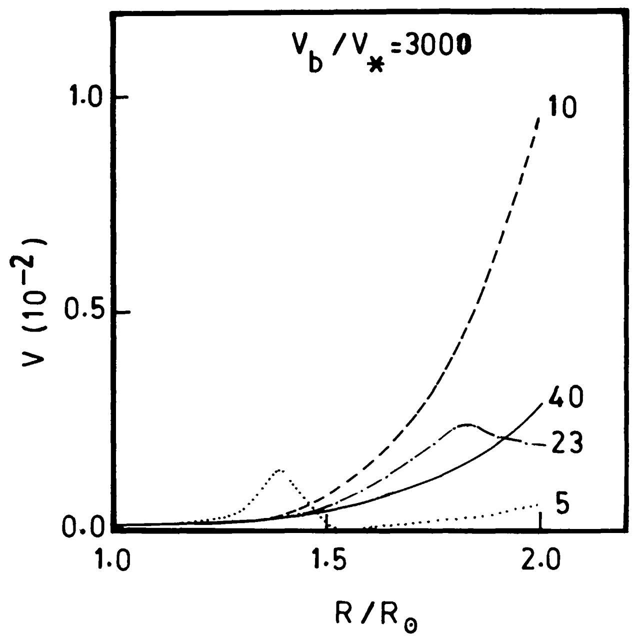
The following equations for 'isothermal' unsteady flow in a gravitational field were integrated in time using the method of characteristics (Zucrow and Hoffman, 1976):

$$\frac{\partial}{\partial t}\rho + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v) = 0\,, \tag{2}$$

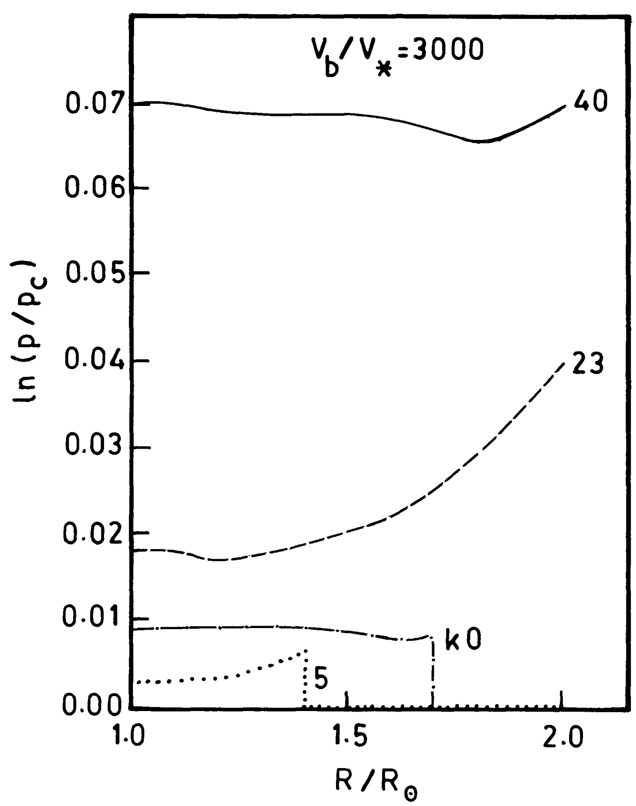
$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial r}p + \frac{GM}{r^2} = 0\,, \tag{3}$$

$$p = S_*^2\rho\,. \tag{4}$$

At $t = 0$, a supercritical base velocity $V_{\text{base}}$ was imposed on the steady critical solution and the resulting response was followed numerically in time. Figures 1 and 2 show the

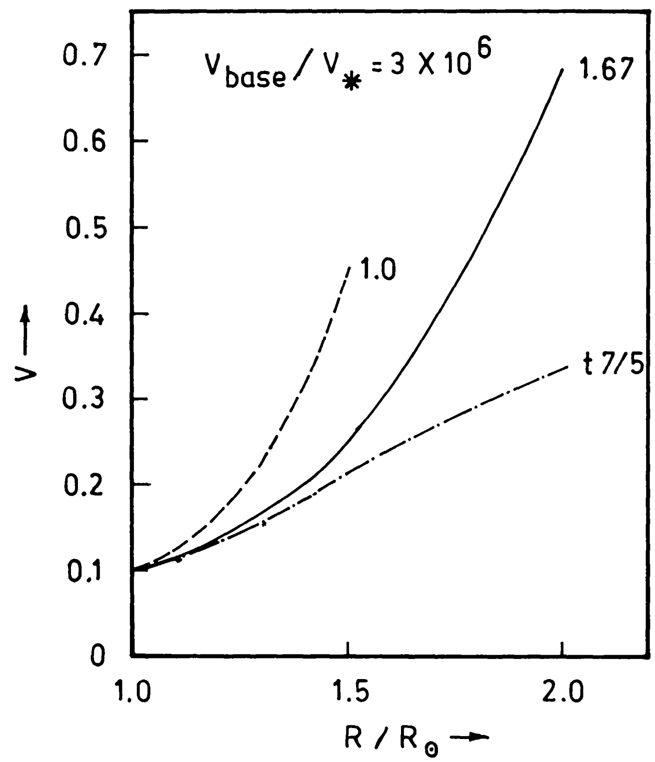



(1a)

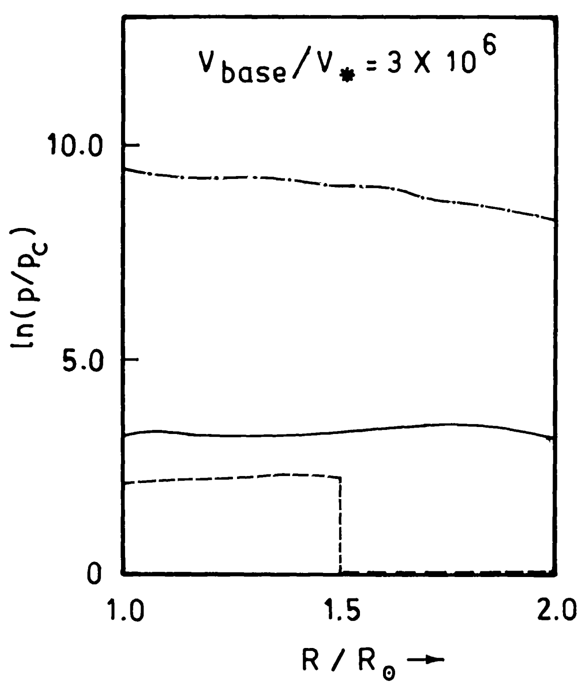



(1b)

Fig. 1. Spatial profile of (a) velocity and (b) $\ln P/P_{\text{critical}}$ at different instants of time for $T_* = 10^6$ K, $r_* = 1\, R_\odot$, $M_* = 1\, M_\odot$, and for $V_{\text{base}}/V_* = 3 \times 10^3$.




(2a)




(2b)

Fig. 2. Same as in Figure 1 but for $V_{base}/V_* = 3 \times 10^6$.

velocity and pressure profiles for two values of the base velocity ($V_{\text{base}}/V_* =$ (a) $3 \times 10^3$ and (b) $3 \times 10^6$) for a million degree isothermal flow. The high value of $T_*$ was chosen for computational convenience. We can see that in case (a) the solutions remain unsteady even after $\approx 40$ free-fall times while for (b) an asymptotic steady state for the velocity is approached. In both the cases pressure enhancements commensurate with the magnitude of supercriticality are seen, without approaching any asymptotic state for pressure.

## 5. Summary and Conclusions

The evolutionary expansion of a star's radius on the Main Sequence results in a supercritical wind for cool stars without coronae. Steady solutions are theoretically impossible. The present limited numerical study for unsteady supercritical winds from a million degree atmosphere points out the possibility of a smooth sonic transition. The price to be paid, however, is to tolerate an increase in the pressure (or density, in this isothermal case) commensurate with the amount of supercriticality. The implications for actual situations in stars with $T_* \approx 6000$ K will be seen after further calculations.

## References

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