THE INTERACTION PRINCIPLE IN RADIATIVE TRANSFER

(Letter to the Editor)

A. PERAIAH
Indian Institute of Astrophysics, Bangalore, India

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Abstract. We describe the interaction principle which is of fundamental importance to the theory of radiative transfer in one-, two-, and three-dimensional geometry. We also describe the practical difficulties associated with this principle in these geometries.

1. Introduction

When radiation is incident upon material medium, it is absorbed, scattered or both and the radiation that emerges out is related to that incident on the medium. This relationship is described as the interaction principle (see Preisendorfer, 1965). This is of fundamental importance in the theory of invariant imbedding process of radiative transfer. For a one-dimensional medium, Redheffer (1962) described the relationship between the input and output radiation fields from a given medium. In this paper, we develop the interaction principle in two and three dimensions.

2. The Interaction Principle

Let I represent the specific intensity and the letters i and o represent the incident and output intensities at the boundaries A, B in one-dimensional geometry A, B, C, D in two-dimensional geometry and at surfaces A, B, C, D, E, F in three-dimensional geometry. Let t and r represent the transmission and reflection operators and S represents the internal source term.

In one-dimensional geometry (see Figure 1) we can write the output intensities in terms of the input intensities as

$$I(A, o) = t(B, A)I(B, i) + r(A, A)I(A, i) + S(A)\,, \tag{1}$$

$$I(B, o) = t(A, B)I(A, i) + r(B, B)I(B, i) + S(B)\,, \tag{2}$$

or

$$\begin{bmatrix} I(A, o) \\ I(B, o) \end{bmatrix} = \begin{bmatrix} r(A, A) & t(B, A) \\ t(A, B) & r(B, B) \end{bmatrix} \begin{bmatrix} I(A, i) \\ I(B, i) \end{bmatrix} + \begin{bmatrix} S(A) \\ S(B) \end{bmatrix}. \tag{3}$$

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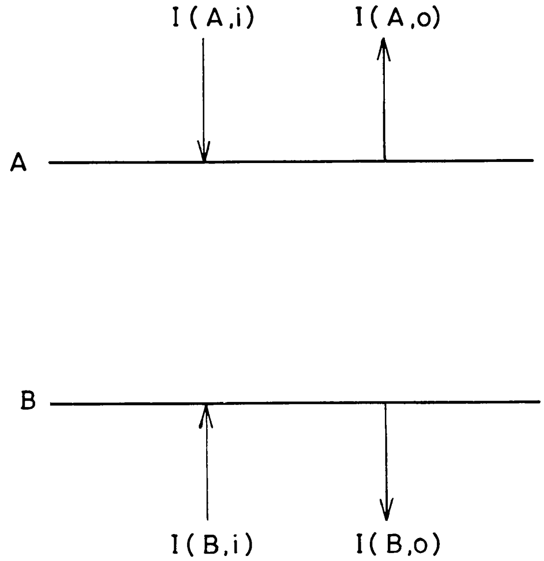

Fig. 1. One-dimensional slab.

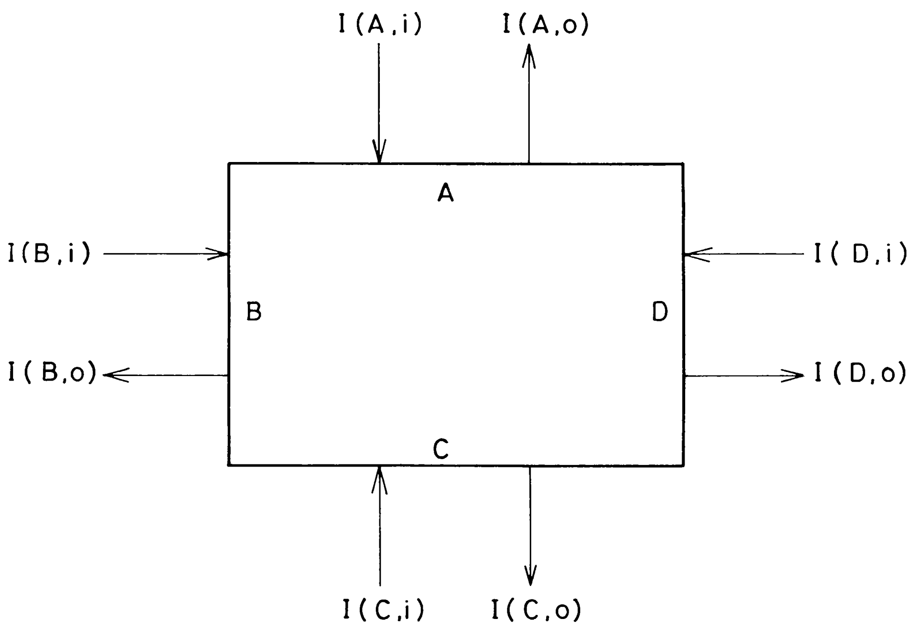

Fig. 2. Two-dimensional plane.

In two dimensions (see Figure 2) we can write the output intensities in terms of the input intensities as follows:

$$I(A, o) = r(A, A)I(A, i) + t(B, A)I(B, i) + \\ + t(C, A)I(C, I) + t(D, A)I(D, i) + S(A)\,, \tag{4}$$

$$I(B, o) = t(A, B)I(A, i) + r(B, B)I(B, i) + \\ + t(C, B)I(C, I) + t(D, B)I(D, i) + S(B)\,, \tag{5}$$

$$I(C, o) = t(A, C)I(A, i) + t(B, C)I(B, i) + \\ + r(C, C)I(C, i) + t(D, C)I(C, i) + S(C)\,, \tag{6}$$

$$I(D, o) = t(A, D)I(A, i) + t(B, D)I(B, i) + \\ + t(C, D)I(C, i) + r(D, D)I(D, i) + S(D)\,, \tag{7}$$

or

$$\begin{bmatrix} I(A, o) \\ I(B, o) \\ \\ I(C, o) \\ I(D, o) \end{bmatrix} = \begin{bmatrix} r(A, A) & t(B, A) & t(C, A) & t(D, A) & I(A, i) \\ t(A, B) & r(B, B) & t(C, B) & t(D, B) & I(B, i) \\ \\ t(A, C) & t(B, C) & r(C, C) & t(D, C) & I(C, i) \\ t(A, D) & t(B, D) & t(C, D) & r(D, D) & I(D, i) \end{bmatrix} + \begin{bmatrix} S(A) \\ S(B) \\ \\ S(C) \\ S(D) \end{bmatrix}. \tag{8}$$

Similarity in a three-dimensional geometry (see Figure 3) we can write the input and output intensities as

$$I(A, o) = t(A, B)I(B, i) + t(A, C)I(C, i) + t(A, D)I(D, i) + \\ + t(A, E)I(E, i) + t(A, F)I(F, i) + r(A, A)I(A, i) + S(A)\,, \tag{9}$$

$$I(B, o) = t(B, A)I(A, i) + t(B, C)I(C, i) + t(B, D)I(D, i) + \\ + t(B, E)I(E, i) + t(B, F)I(F, i) + r(B, B)I(B, i) + S(B)\,, \tag{10}$$

$$I(C, o) = t(C, A)I(A, i) + t(C, B)I(B, i) + t(C, D)I(D, i) + \\ + t(C, E)I(E, i) + t(C, F)I(F, i) + r(C, C)I(C, I) + S(C)\,, \tag{11}$$

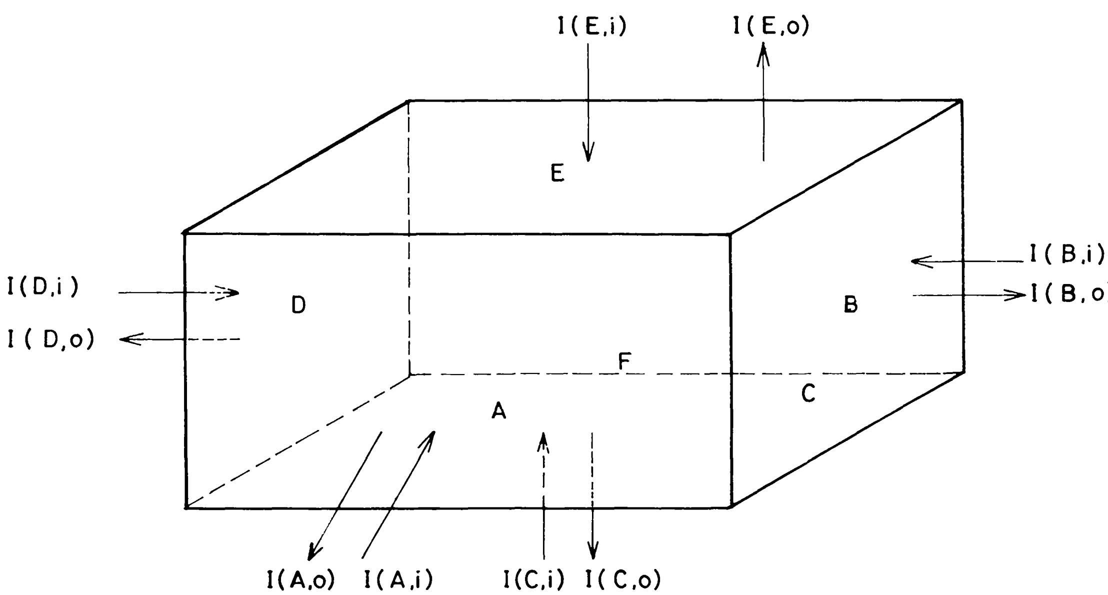

Fig. 3. Three-dimensional parallelopipedon.

$$I(D,o) = t(D,A)I(A,i) + t(D,B)I(B,i) + t(D,C)I(C,i) + \\ + t(D,E)I(E,i) + t(D,F)I(F,i) + r(D,D)I(D,i) + S(D)\,, \quad (12)$$

$$I(E,o) = t(E,A)I(A,i) + t(E,B)I(B,i) + t(E,C)I(C,i) + \\ + t(E,D)I(D,i) + t(E,F)I(F,i) + r(E,E)I(E,i) + S(E)\,, \quad (13)$$

$$I(F,o) = t(F,A)I(A,i) + t(F,B)I(B,i) + t(F,C)I(C,i) + \\ + t(F,D)I(D,i) + t(F,E)I(E,i) + r(F,F)I(F,i) + S(F)\,; \quad (14)$$

or, more succinctly, we can write

$$\begin{bmatrix} I(A,o) \\ I(B,o) \\ I(C,o) \\ I(D,o) \\ I(E,o) \\ I(F,o) \end{bmatrix} = \begin{bmatrix} r(A,A) & t(A,B) & t(A,C) & t(A,D) & t(A,E) & t(A,F) \\ t(B,A) & r(B,B) & t(B,C) & t(B,D) & t(B,E) & t(B,F) \\ t(C,A) & t(C,B) & r(C,C) & t(C,D) & t(C,E) & t(C,F) \\ t(D,A) & t(D,B) & t(D,C) & r(D,D) & t(D,E) & t(D,F) \\ t(E,A) & t(E,B) & t(E,C) & t(E,D) & r(E,E) & t(E,F) \\ t(F,A) & t(F,B) & t(F,C) & t(F,D) & t(F,E) & r(F,F) \end{bmatrix} \times$$

$$\times \begin{bmatrix} IA,i) \\ I(B,i) \\ I(C,i) \\ I(D,i) \\ I(E,i) \\ I(F,i) \end{bmatrix} + \begin{bmatrix} S(A) \\ S(B) \\ S(C) \\ S(D) \\ S(E) \\ S(F) \end{bmatrix} . \quad (15)$$

In Equations (3), (8), and (15) we have formulated the interaction principle. From one-dimensional principle to three-dimensional principle the size of the matrix of operators grew nine times.

If we wish to multiply the layers or planes or cuboids, we need to add one layer at a time in one-dimensional geometry, 8 planes at a time in two-dimensional geometry, and 26 cuboids in three-dimensional geometry. The last problem is called 26-point problem (cf. Preisendorfer, 1965). This problem is currently under study.

References

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Redheffer, R. M.: 1962, *J. Math. Phys.* **41**, 1.