SOLUTION OF RADIATIVE TRANSFER EQUATION WITH SPHERICAL-SYMMETRY IN PARTIALLY SCATTERING MEDIUM

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Abstract. We have solved the equation of radiative transfer in spherical symmetry with scattering and absorbing medium. We have set the albedo for single scattering to be equal to 0.5. We have set the Planck function constant throughout the medium in one case and in another case the Planck function has been set to vary as r^{-2}. The geometrical extension of the spherical shell has been taken as large as one stellar radius. Two kinds of variations of the optical depth are employed (1) that remains constant with radius and (2) that varies as r^{-2}. In all these cases the internal source vectors and specific intensities change depending upon the type of physics we have employed in each case.

1. Introduction

In another paper (Peraiah and Varghese, 1985, herafter referred to as Paper I) we have solved equation of radiative transfer with spherical symmetry in a medium which absorbs. In that paper we have introduced the internal sources which contribute mainly to the outward going and inward going radiation field. We found that the internal emission is maximum at μ approximately equal to 0, where μ is the cosine of the angle made by the ray with the radius vector. We must find out how the pattern of the radiation will change if we introduce scattering in addition to absorption. In this paper we have assumed isotropic scattering with the albedo for single scattering to 0.5. The Planck function has been assumed to be constant throughout atmospheres in one case and varying as r^{-2} in another case. The method of solving equation of transfer has been given in Paper I. We shall, however, give a brief account of how to solve the equation radiative transfer in spherical symmetry when scattering and absorption are simultaneously present in the medium.

2. Solution of the Equation of Transfer

The equation of radiative transfer in spherical symmetry is given by (see Peraiah, 1984)

$$\mu \frac{\partial I(r, \mu)}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I(r, \mu)}{\partial \mu} = K(r)\,[s(r, \mu) - I(r, \mu)] \tag{1}$$

and

$$-\mu \frac{\partial I(r, -\mu)}{\partial r} - \frac{1 - \mu^2}{r} \frac{\partial I(r, -\mu)}{\partial \mu} = K(r)\,[s(r, -\mu) - I(r, -\mu)]\,, \tag{2}$$

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where $I(r, \mu)$ is the specific intensity of the ray making an angle $\cos^{-1}\mu$ with the radius vector at r; $K(r)$ is the absorption coefficient; and the quantity $s(r, \mu)$ is the source function given by

$$s(r, \mu) = [1 - \varpi(r)]b(r, \mu) + \tfrac{1}{2}\varpi(r) \int_{-1}^{+1} P(r, \mu, \mu')I(r, \mu')\, d\mu' , \tag{3}$$

where $\varpi(r)$ is albedo for single scattering, $b(r, \mu)$ is the Planck function and $P(r, \mu, \mu')$ is the phase function. In this paper we consider isotropic scattering and $\varpi(r)$ set equal to 0.5.

We shall make the following substitution:

$$S(r, \mu) = 4\pi r^2 s(r, \mu) , \qquad B(r, \mu) = 4\pi r^2 b(r, \mu) ; \tag{4}$$

and obtain

$$\mu \frac{\partial U(r, \mu)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \mu}\{(1 - \mu^2)U(r, \mu)\} = K(r)\{S(r, \mu) - U(r, \mu)\} , \tag{5}$$

$$-\mu \frac{\partial U(r, -\mu)}{\partial r} - \frac{1}{r}\frac{\partial}{\partial \mu}\{(1 - \mu^2)U(r, -\mu)\} = K(r)\{S(r, -\mu) - U(r, -\mu)\} . \tag{6}$$

The integration of Equations (5) and (6) has been done as suggested in the earlier paper; and the specific intensity has been expressed by the interpolation formula

$$U(r, \mu) \approx (U_{00} + U_{01}\xi) + (U_{10} + U_{11}\xi)\eta , \tag{7}$$

where

$$\xi = \frac{r - \bar{r}}{\Delta r/2} , \qquad \eta = \frac{\mu - \bar{\mu}}{\Delta \mu/2} , \tag{8}$$

$$\bar{r} = \tfrac{1}{2}(r_i + r_{i-1}) , \qquad \bar{\mu} = \tfrac{1}{2}(\mu_j + \mu_{j-1}) , \tag{9}$$

$$\Delta r = (r_i - r_{i-1}) , \qquad \Delta \mu = (\mu_j - \mu_{j-1}) . \tag{10}$$

The interpolation coefficients can be written in terms of the nodal values of the specific intensity. The quantity $S(r, \mu)$ is also expressed by an interpolation formula similar to that given in Equation (7), and can be written as

$$S(r, \mu) = S_{00} + S_{01}\xi + S_{10}\eta + S_{11}\xi\eta . \tag{11}$$

We substitute Equations (7) and (11) in Equations (5) and (6) and apply the operators given by

$$X = \frac{1}{\Delta \mu} \int_{\Delta \mu} \ldots d\mu , \tag{12}$$

$$Y = \frac{1}{V} \int_{\Delta r} \ldots 4\pi r^2 \, \mathrm{d}r \, ; \tag{13}$$

where

$$V = \tfrac{4}{3}\pi (r_i^3 - r_{i-1}^3) \, , \tag{14}$$

to obtain

$$\left[\mu_{j-1/2}^{-}(1+p) + \tfrac{1}{2}\tau\left(1 + \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) - \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 + \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_c +$$
$$+\left[\mu_{j-1/2}^{+}(1+p) + \tfrac{1}{2}\tau\left(1 + \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) + \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 + \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_d -$$
$$-\left[\mu_{j-1/2}^{-}(1+q) - \tfrac{1}{2}\tau\left(1 - \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) + \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 - \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_a -$$
$$-\left[\mu_{j-1/2}^{+}(1+q) - \tfrac{1}{2}\tau\left(1 - \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) - \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 - \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_b =$$
$$= \tfrac{1}{2}\tau\left[\left(1 - \frac{1}{6}\frac{\Delta A}{\bar{A}}\right)(S_a + S_b) + \left(1 + \frac{1}{6}\frac{\Delta A}{\bar{A}}\right)(S_c + S_d)\right], \tag{15}$$

$$-\left[\mu_{j-1/2}^{-}(1-p) - \tfrac{1}{2}\tau\left(1 + \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) - \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 + \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_c -$$
$$-\left[\mu_{j-1/2}^{+}(1-p) - \tfrac{1}{2}\tau\left(1 + \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) + \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 + \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_d +$$
$$+\left[\mu_{j-1/2}^{-}(1-q) + \tfrac{1}{2}\tau\left(1 - \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) + \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 - \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_a +$$
$$+\left[\mu_{j-1/2}^{+}(1-q) + \tfrac{1}{2}\tau\left(1 - \frac{1}{6}\frac{\Delta A}{\bar{A}}\right) - \frac{1}{2}\frac{1-\overline{\mu^2}}{\Delta\mu}\frac{\Delta A}{\bar{A}}\left(1 - \frac{1}{6}\frac{\Delta r}{\bar{r}}\right)\right]U_b =$$
$$= \tfrac{1}{2}\tau\left[\left(1 - \frac{1}{6}\frac{\Delta A}{\bar{A}}\right)(S_a + S_b) + \left(1 + \frac{1}{6}\frac{\Delta A}{\bar{A}}\right)(S_c + S_d)\right], \tag{16}$$

where

$$\overline{\mu^2} = (\bar{\mu})^2 + \frac{(\Delta\mu)^2}{12} \, , \tag{17}$$

$$\bar{A} = \frac{V}{\Delta r} \, , \tag{18}$$

$$\Delta A = 4\pi(r_i^2 - r_{i-1}^2)\,; \tag{19}$$

and obtain

$$\tau = K\Delta r\,, \tag{20}$$

$$\mu_{j-1/2}^{+} = \bar{\mu}\left(1 + \frac{1}{6}\frac{\Delta\mu}{\bar{\mu}}\right) = \tfrac{1}{3}(2\mu_j + \mu_{j-1})\,, \tag{21}$$

$$\mu_{j-1/2}^{-} = \bar{\mu}\left(1 - \frac{1}{6}\frac{\Delta\mu}{\bar{\mu}}\right) = \tfrac{1}{3}(\mu_j + 2\mu_{j-1})\,, \tag{22}$$

$$p = \frac{\bar{r}}{\Delta r}\frac{\Delta A}{\bar{A}} - \frac{1}{2}\frac{\Delta A}{\bar{A}} - 2\,, \tag{23}$$

$$q = 2 - \frac{1}{2}\frac{\Delta A}{\bar{A}} - \frac{\bar{r}}{\Delta r}\frac{\Delta A}{\bar{A}}\,, \tag{24}$$

$$t = \frac{\Delta r}{\bar{r}} = (r_i - r_{i-1})/\tfrac{1}{2}(r_i + r_{i-1})\,; \tag{25}$$

and if $t \ll 1$, then

$$\frac{\Delta A}{\bar{A}} = 2t/(1 + \tfrac{1}{12}t^2) \approx 2t\,, \tag{26}$$

$$1 \pm \frac{1}{6}\frac{\Delta A}{\bar{A}} = (1 \pm \tfrac{1}{3}t + \tfrac{1}{12}t^2)/(1 \pm \tfrac{1}{12}t^2) \approx 1 \pm \tfrac{1}{3}t\,, \tag{27}$$

$$p \approx q \approx -t\,; \tag{28}$$

and

$$\begin{aligned} U(r_i, \mu_j) &= U_{00} + U_{01} + U_{10} + U_{11} = U_d\,, \\ U(r_{i-1}, \mu_j) &= U_{00} - U_{01} + U_{10} - U_{11} = U_b\,, \\ U(r_i, \mu_{j-1}) &= U_{00} + U_{01} - U_{10} - U_{11} = U_c\,, \\ U(r_{i-1}, \mu_{j-1}) &= U_{00} - U_{01} - U_{10} + U_{11} = U_a\,. \end{aligned} \tag{29}$$

By inverting Equations (29) we obtain

$$\begin{aligned} U_{00} &= \tfrac{1}{4}(+U_a + U_b + U_c + U_d)\,, \\ U_{01} &= \tfrac{1}{4}(-U_a - U_b + U_c + U_d)\,, \\ U_{10} &= \tfrac{1}{4}(-U_a + U_b - U_c + U_d)\,, \\ U_{11} &= \tfrac{1}{4}(+U_a - U_b - U_c + U_d)\,, \end{aligned} \tag{30}$$

the quantities U_a, U_b, U_c, and U_d and S_a, S_b, S_c, and S_d are replaced by U_{j-1}^{i-1}, U_j^{i-1}, U_{j-1}^i, and U_j^i and S_{j-1}^{i-1}, S_j^{i-1}, S_{j-1}^i, and S_j^i, respectively. We thus obtain

$$[\mathbf{M}_p^+ - \boldsymbol{\rho}^+ + \tfrac{1}{2}\tau^+ \mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{++})]\,\mathbf{U}_i^+ - [\mathbf{M}_q^+ + \boldsymbol{\rho}^- - \tfrac{1}{2}\tau^- \mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{++})]\,\mathbf{U}_{i-1}^+ = = (1 - \overline{\omega})\mathbf{Q}\mathbf{B}^+ + \tfrac{1}{2}\tau^+ \mathbf{Q}\boldsymbol{\gamma}^{+-}\,\mathbf{U}_{i-1}^- + \tfrac{1}{2}\tau^- \mathbf{Q}\boldsymbol{\gamma}^{+-}\,\mathbf{U}_i^-\,, \tag{31}$$

and

$$-[\mathbf{M}_p^- - \boldsymbol{\rho}^+ - \tfrac{1}{2}\tau^+ \mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{--})]\,\mathbf{U}_i^- + [\mathbf{M}_q^- + \boldsymbol{\rho}^- + \tfrac{1}{2}\tau^- \mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{--})]\,\mathbf{U}_{i-1}^- = = (1 - \overline{\omega})\mathbf{Q}\mathbf{B}^- + \tfrac{1}{2}\tau^+ \mathbf{Q}\boldsymbol{\gamma}^{-+}\,\mathbf{U}_i^+ + \tfrac{1}{2}\tau^- \mathbf{Q}\boldsymbol{\gamma}^{-+}\,\mathbf{U}_i^+\,, \tag{32}$$

where

$$\tau_{i-1/2}^+ = \tau_{i-1/2}\left(1 + \frac{1}{6}\frac{\Delta A}{\overline{A}}\right) = \tau_{i-1/2}\,\frac{3r_i^2 + 2r_i r_{i-1} + r_{i-1}^2}{2(r_i^2 + r_i r_{i-1} + r_{i-1}^2)}\,, \tag{33}$$

$$\tau_{i-1/2}^- = \tau_{i-1/2}\left(1 - \frac{1}{6}\frac{\Delta A}{\overline{A}}\right) = \tau_{i-1/2}\,\frac{r_i^2 + 2r_i r_{i-1} + 3r_{i-1}^2}{2(r_i^2 + r_i r_{i-1} + r_{i-1}^2)}\,, \tag{34}$$

$$\rho_{j-1/2}^{i-1/2,\,\pm} = \rho_{j-1/2}^{i-1/2}\left(1 \pm \frac{1}{6}\frac{\Delta r}{\overline{r}}\right), \tag{35}$$

$$\rho_{j-1/2}^{i-1/2} = \frac{1}{2}\,\frac{1-\mu^2}{\Delta\mu}\,\frac{\Delta A}{\overline{A}} = = \left[\frac{1 - \frac{1}{3}(\mu_j^2 + \mu_j\mu_{j-1} + \mu_{j-1}^2)}{2(\mu_j - \mu_{j-1})}\right]\left[\frac{3(r_i^2 - r_{i-1}^2)}{(r_i^2 + r_i r_{i-1} + r_{i-1}^2)}\right], \tag{36}$$

$$S^{i,\,+} = (1 - \overline{\omega}_i)B^{i,\,+} + \tfrac{1}{2}\overline{\omega}_i[P_i^{++}\,CU^{i,\,+} + P_i^{+-}\,CU^{i,\,-}]\,, \tag{37}$$

$$S^{i-1,\,+} = (1 - \overline{\omega}_{i-1})B^{i-1,\,+} + \tfrac{1}{2}\overline{\omega}_{i-1}[P_{i-1}^{++}\,CU^{i-1,\,+} + P_{i-1}^{+-}\,CU^{i-1,\,-}]\,, \tag{38}$$

$$P_i^{++} = P(r_i, +\mu_j, +\mu_j)\,, \qquad P_i^{+-} = P(r_i, +\mu_j, -\mu_j)\,, \tag{39}$$

$$B^{i,\,+} = B^{i,\,-}\,, \qquad B^{i-1,\,+} = B^{i-1,\,+}\,, \tag{40}$$

$$U^{i,\,\pm} = U(r_i, \pm\mu_j)\,; \tag{41}$$

and

$$\mathbf{M}_p^+ = (1+p)\mathbf{M}\,, \qquad \mathbf{M}_p^- = (1-p)\mathbf{M}\,;$$

$$\mathbf{M}_q^+ = (1+q)\mathbf{M}\,, \qquad \mathbf{M}_q^- = (1-q)\mathbf{M}\,; \tag{42}$$

$$\mathbf{M} = \begin{bmatrix} \mu_{1/2}^- & \mu_{1/2}^+ & & & \\ & \mu_{3/2}^- & \mu_{3/2}^+ & & \\ & & \mu_{j-1/2}^- & \mu_{j-1/2}^+ & \\ & & & \mu_j & \end{bmatrix} \tag{43}$$

$$\boldsymbol{\rho}^{\pm} = \begin{bmatrix} \rho^{\pm}_{1/2} & -\rho^{\pm}_{1/2} & & & \\ & \rho^{\pm}_{3/2} & -\rho^{\pm}_{3/2} & & \\ & & \rho^{\pm}_{j-1/2} & -\rho^{\pm}_{j-1/2} \\ & & & 0 \end{bmatrix} \tag{44}$$

$$\mathbf{Q} \begin{bmatrix} 1 & 1 & & & & \\ & 1 & 1 & & & \\ & & 1 & 1 & & \\ & & & 1 & 1 & \\ & & & & 1 & \ddots 1 \\ & & & & & \ddots 1 \end{bmatrix} \tag{45}$$

and

$$\begin{aligned} &\boldsymbol{\gamma}^{++} = \tfrac{1}{2}\overline{\omega}\mathbf{P}^{++}\mathbf{C}, \qquad \boldsymbol{\gamma}^{--} = \tfrac{1}{2}\overline{\omega}\mathbf{P}^{--}\mathbf{C}, \\ &\boldsymbol{\gamma}^{+-} = \tfrac{1}{2}\overline{\omega}\mathbf{P}^{+-}\mathbf{C}, \qquad \boldsymbol{\gamma}^{-+} = \tfrac{1}{2}\overline{\omega}\mathbf{P}^{-+}\mathbf{C}, \end{aligned} \tag{46}$$

$$\begin{aligned} &\overline{\boldsymbol{\rho}}_{\pm} = \mathbf{Q}^{-1}\boldsymbol{\rho}^{\pm}, \\ &\mathbf{M}_1 = \mathbf{Q}^{-1}\mathbf{M}_p^{+}, \qquad \mathbf{M}_2 = \mathbf{Q}^{-1}\mathbf{M}_p^{-}, \\ &\mathbf{M}_3 = \mathbf{Q}^{-1}\mathbf{M}_q^{+}, \qquad \mathbf{M}_4 = \mathbf{Q}^{-1}\mathbf{M}_q^{-}. \end{aligned} \tag{47}$$

If we assume that U_{i-1}^{+} and U_i^{-} are the intensities of the incident radiation and U_i^{+} and U_{i-1}^{-} are output intensities then we can write

$$\begin{aligned} &[\mathbf{M}_p^{+} - \boldsymbol{\rho}^{+} + \tfrac{1}{2}\tau^{+}\mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{++})]\mathbf{U}_i^{+} - [\mathbf{M}_q^{+} + \boldsymbol{\rho}^{-} - \tfrac{1}{2}\tau^{-}\mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{++})]\mathbf{U}_{i-1}^{+} = \\ &\quad = (1 - \overline{\omega})\mathbf{Q}\mathbf{B}^{+} + \tfrac{1}{2}\tau^{+}\mathbf{Q}\boldsymbol{\gamma}^{+-}\mathbf{U}_{i-1}^{-} + \tfrac{1}{2}\tau^{-}\mathbf{Q}\boldsymbol{\gamma}^{+-}\mathbf{U}_i^{-}, \end{aligned} \tag{48}$$

$$\begin{aligned} &-[\mathbf{M}_p^{-} - \boldsymbol{\rho}^{+} - \tfrac{1}{2}\tau^{+}\mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{--})]\mathbf{U}_i^{-} + [\mathbf{M}_q^{-} + \boldsymbol{\rho}^{-} + \tfrac{1}{2}\tau^{-}\mathbf{Q}(\mathbf{I} - \boldsymbol{\gamma}^{--})]\mathbf{U}_{i-1}^{-} = \\ &\quad = (1 - \overline{\omega})\mathbf{Q}\mathbf{B}^{-} + \tfrac{1}{2}\tau^{+}\mathbf{Q}\boldsymbol{\gamma}^{-+}\mathbf{U}_i^{+} + \tfrac{1}{2}\tau^{-}\mathbf{Q}\boldsymbol{\gamma}^{-+}\mathbf{U}_i^{+}. \end{aligned} \tag{49}$$

Equations (48) and (49) can be re-written in the form of interaction principle given by

$$\begin{bmatrix} U_i^{+} \\ U_{i-1}^{-} \end{bmatrix} = \begin{bmatrix} t(i, i-1) & r(i-1, i) \\ r(i, i-1) & t(i-1, i) \end{bmatrix} \begin{bmatrix} U_{i-1}^{+} \\ U_i^{-} \end{bmatrix} + \begin{bmatrix} \Sigma_{i-1/2}^{+} \\ \Sigma_{i-1/2}^{-} \end{bmatrix}, \tag{50}$$

from which we obtain the quantities of transmission and reflection matrices and these operators are used to calculate the internal radiation field by using the algorithm given in Peraiah (1984). We calculate the solutions at the internal points by use of the following formulae:

$$\mathbf{U}_{n+1}^{+} = \mathbf{r}(1, n+1)U_{n+1}^{-} + \mathbf{V}_{n+1/2}^{+}, \qquad \mathbf{U}_n^{-} = \hat{\mathbf{t}}(n, n+1)\mathbf{U}_{n+1}^{-} + \mathbf{V}_{n+1/2}^{-}, \tag{51}$$

$$\mathbf{r}(1, n+1) = \mathbf{r}(n, n+1) + \mathbf{t}(n+1, n)\mathbf{r}(1, n)\mathbf{T}_{n+1/2}\,\mathbf{t}(n, n+1)\,, \tag{52}$$

$$\mathbf{V}^{+}_{n+1/2} = \hat{\mathbf{t}}(n+1, n)\mathbf{V}^{+}_{n-1/2} + \boldsymbol{\Sigma}^{+}(n+1, n) + \mathbf{R}_{n+1/2}\,\boldsymbol{\Sigma}^{-}(n, n+1)\,,$$

$$\mathbf{V}^{-}_{n-1/2} = \hat{\mathbf{r}}(r+1, n)\mathbf{V}^{+}_{n+1/2} + \mathbf{T}_{n+1/2}\,\boldsymbol{\Sigma}^{-}(n, n+1)\,; \tag{53}$$

with the initial conditions

$$U^{-}_{N+1} = U^{-}(A) \tag{54}$$

and

$$r(1, 1) = 0\,, \tag{55}$$

$$V^{+}_{1/2} = U^{+}(B)\,; \tag{56}$$

where N represents the number of shells into which the medium is divided. For details see Peraiah (1984).

3. Results and Discussion

We have presented the results in Figures 1 to 10. The boundary conditions are described in the previous section and we have not given any incident radiation on either side of the atmospheres. This means that $U^{-}(A) = U^{+}(B) = 0$, where A and B are the inner and outer radii of the medium.

We have calculated the internal source vectors and the intensities at a different point inside the medium. We have considered several types of atmospheres with the Planck function changing as r^{-2} in one case and remaining constant in another case. Similar variation of optical depth is assumed. The results of the spherical symmetry are compared with those in plane-parallel symmetry.

In Figure 1 the angular distribution of the source vectors V^{+} and V^{-} are described at different points inside the atmospheres. The albedo for single scattering $\overline{\omega}$ is set equal to 0.5. We notice that the radiation directed towards the centre of the sphere has a higher value than that directed away from the centre. However, in the outermost layer the outward- and inward-directed source vector are symmetric at $\mu = 0$. This is true both in plane parallel and spherically-symmetric approximation. The peaking of the source vectors is largest in the case of spherically-symmetric approximation in the internal layers. In Figure 2 the angular distribution of the specific intensities corresponding to the source vectors given in Figure 1 are described. These intensities show a variation which is slightly similar to the variation shown by the corresponding internal source vectors. However, we note one important feature: namely, that the intensities show a maximum around $\mu = 0$ in the case of spherically-symmetric geometry at some of the internal points. The differences between plane parallel and spherically symmetric cases are not as large as expected. In Figure 3 the angular distribution of source vectors is described for an atmosphere in which the optical depth is kept constant and the Planck function is set to vary as r^{-2}. In this case the differences between the plane parallel geometry and spherically-symmetric geometry are very well brought out. The plane

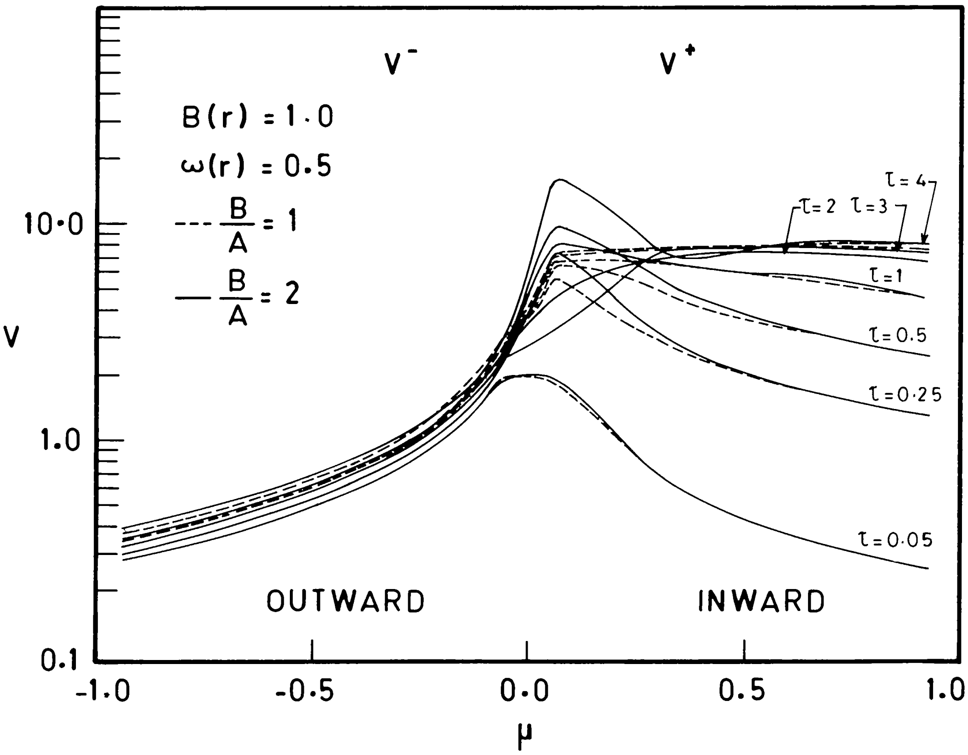

Fig. 1. Angular distribution of source vectors V^+ and V^- corresponding to $B(r) = 1\overline{\omega}(r) = 0.5$.

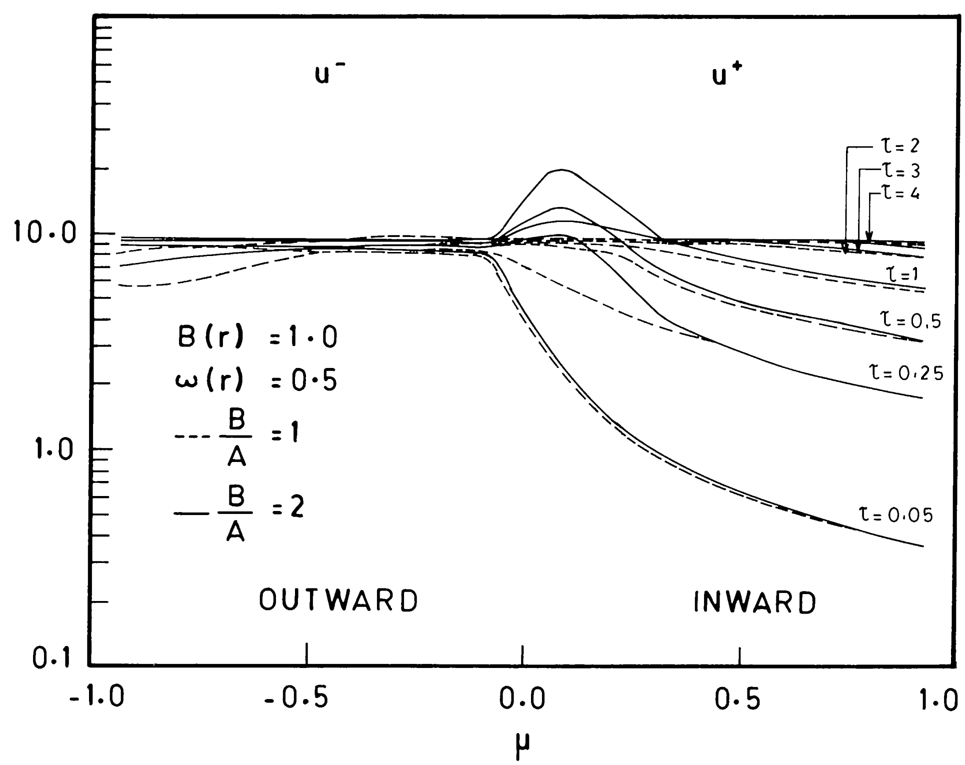

Fig. 2. Angular distribution of intensities U^- and U^+ corresponding to source vectors are described in Figure 1.

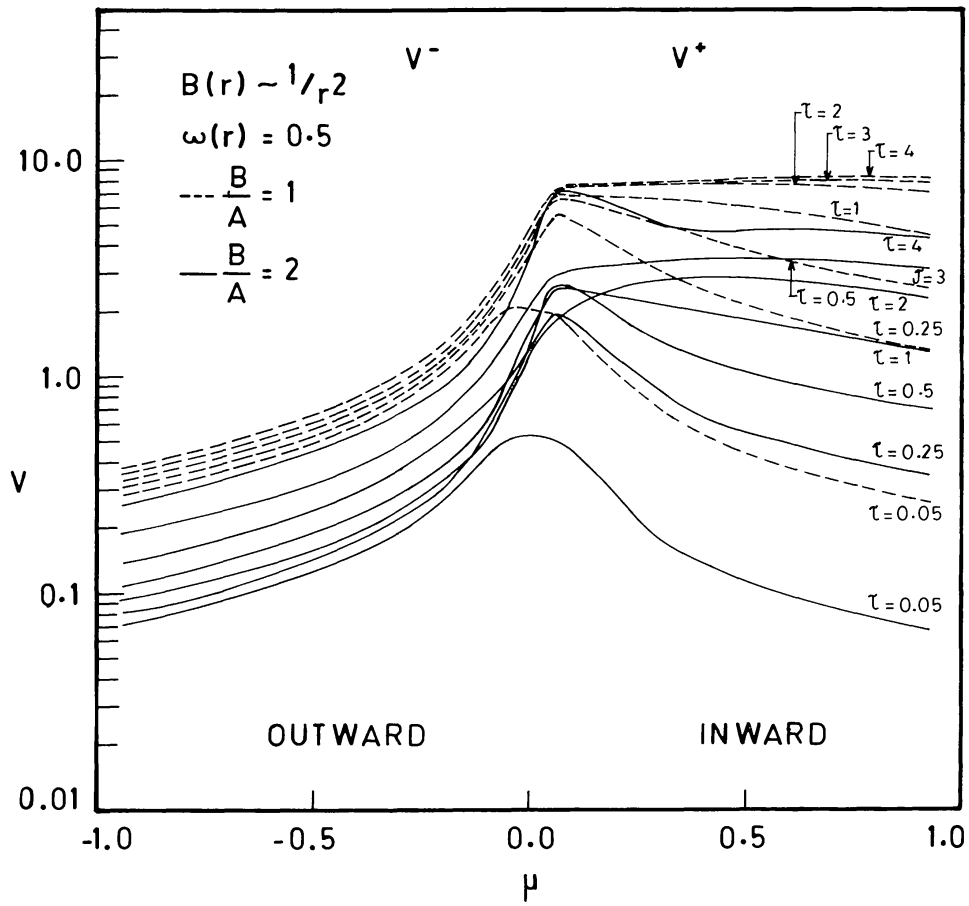

Fig. 3. Angular distribution of the source vectors for a medium with constant optical depth and the Planck function changing as r^{-2}.

parallel geometry exhibits the radiation field which is quite large compared to that of spherically-symmetric geometry. The source vectors directed inwards have higher values compared to those directed outwards, and those in the outer most layer are symmetric about $\mu = 0$.

In Figure 4 the angular distribution of the source vectors is described for a medium in which the optical depth and Planck function are changing as r^{-2}. The vectors show a similar variation as shown by the vectors described in Figure 3. The outer most layer shows a symmetric distribution of the vectors both in plane parallel and in spherically-symmetric geometry. But, however, the vector which is directed towards the centre of the sphere shows more radiation than the vector which is directed towards the outer surface. In Figure 5 we have plotted the angular distribution of the specific intensities corresponding to the vector given in Figure 4. These vectors show that the outward radiation field is more than inward directed field as shown in Figure 2. Although the corresponding source vectors given in Figure 4 show an exactly opposite trend. In Figures 6 and 7, source vectors and their corresponding specific intensities are plotted for an atmosphere in which the Planck function remain constant while the optical depth changes as r^{-2}. The source vectors show the usual trend that the inward radiation is

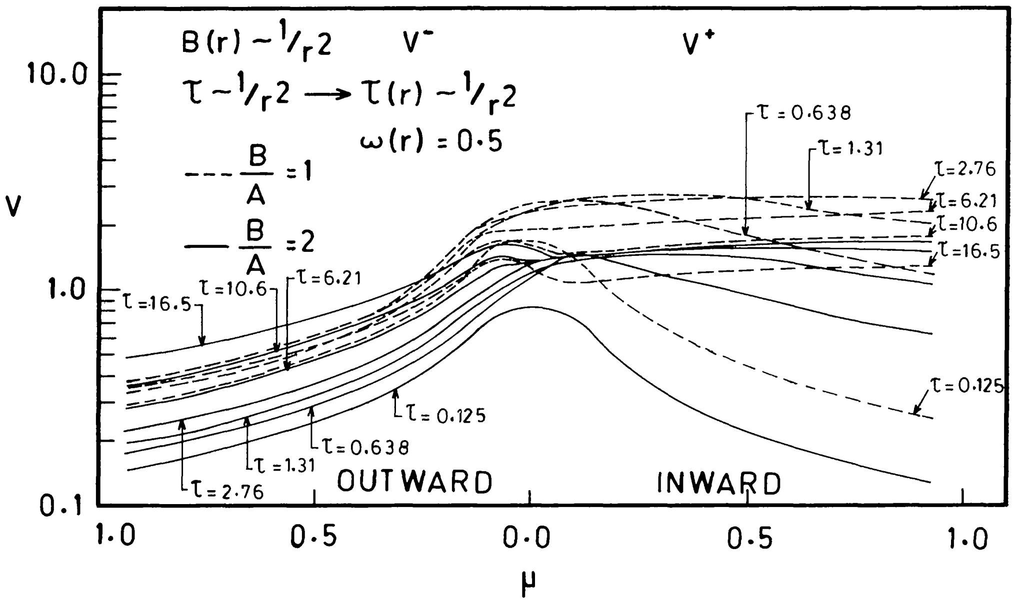

Fig. 4. Angular distribution of the source vectors V^+ and V^- for a medium in which the Planck function and the optical depth change as r^{-2}.

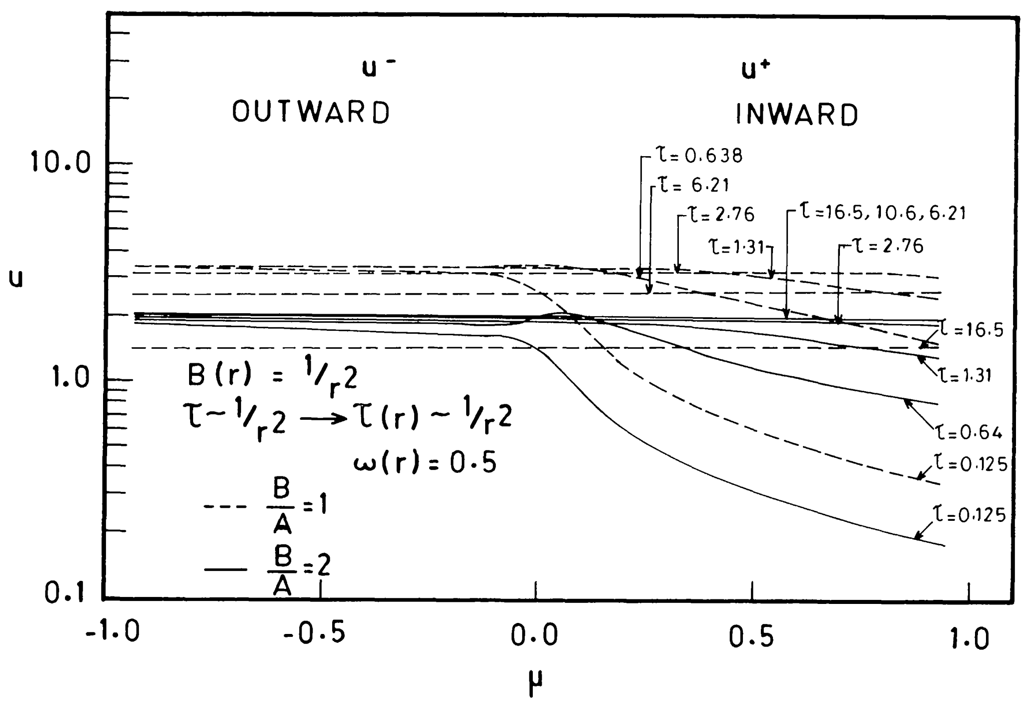

Fig. 5. Angular distribution of the specific intensities U^- and U^+ to the parameters shown in the figure.

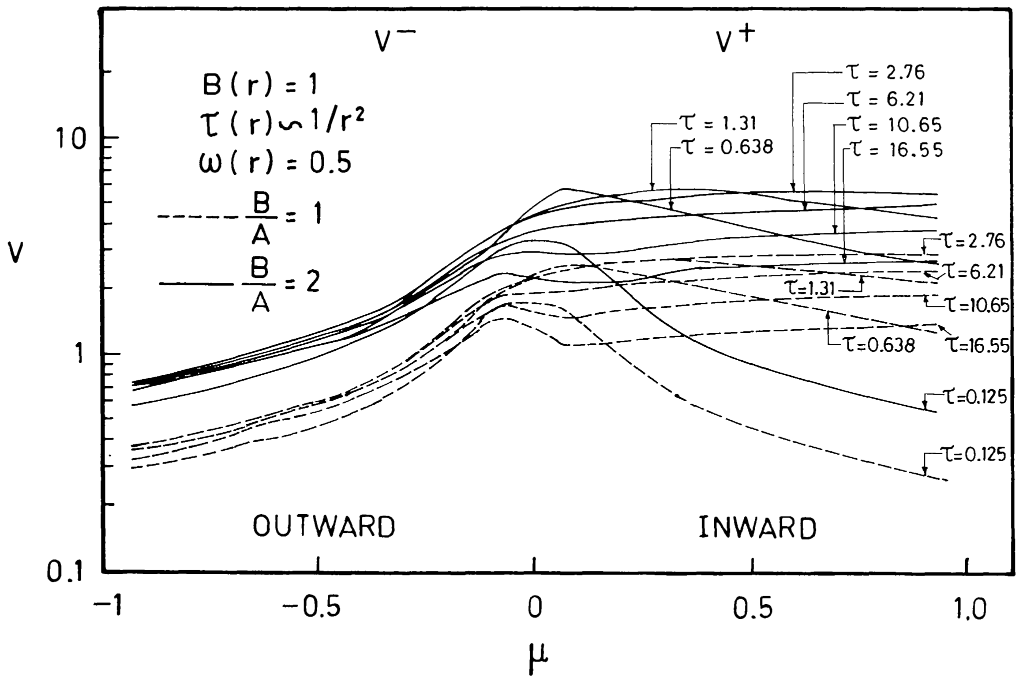

Fig. 6. Angular distribution of the source vectors corresponding to the parameters shown in the figure.

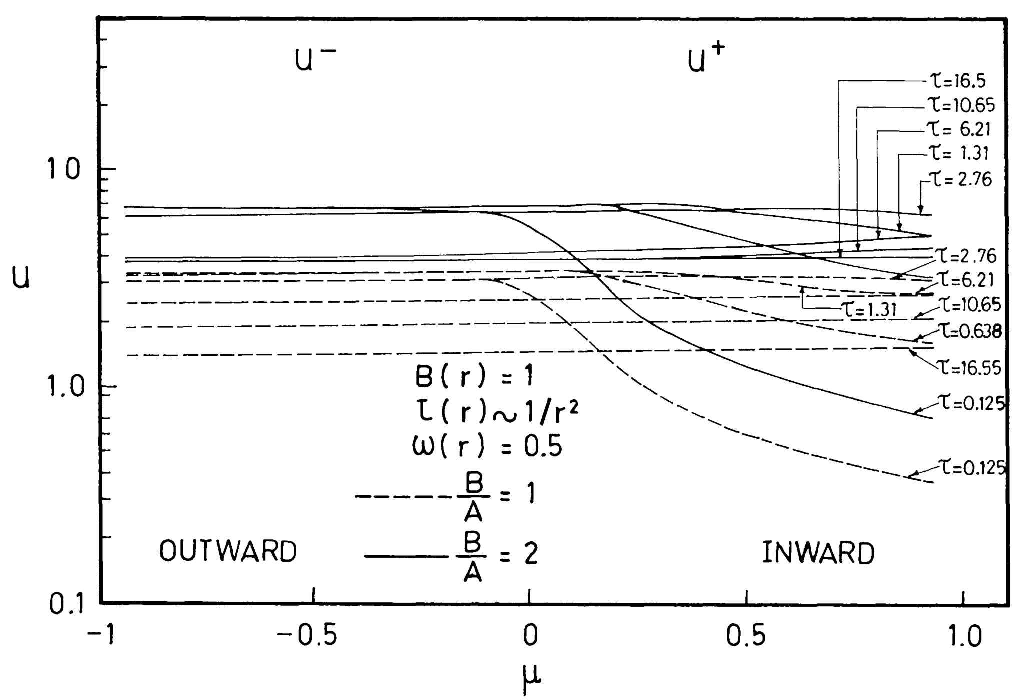

Fig. 7. Angular distribution of the intensities corresponding to the parameters shown in the figure.

larger than the outward radiation while the specific intensities show that at least in the outermost layer the inward radiation is less than the radiation directed outward. However, at some of the internal points the intensities remain almost constant with respect to angle variable.

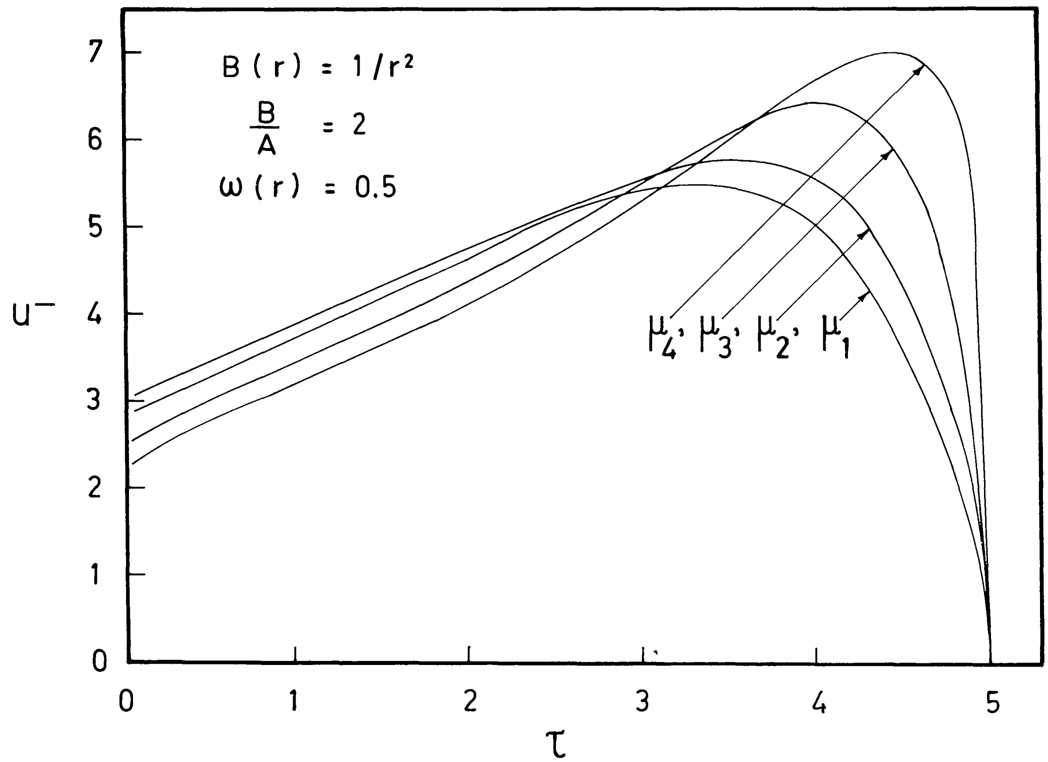

Fig. 8. Radial distribution of the specific intensities U^- corresponding to the angles μ_1, μ_2, μ_3, and μ_4.

In Figure 8 the radial distribution of the specific intensities directed outwards is described for an atmosphere in which the optical depth remain constant and the Planck function changes as r^{-2}. We have considered four angles in each quarter and these are μ_1, μ_2, μ_3, and μ_4 taken as the roots of the Gauss–Legendre polynomial. The intensity corresponding to these four angles increase from the minimum value at $\tau = \tau_{\max}$ to a maximum value at about τ slightly less than $\tau_{\max}$ and then start falling towards the $\tau = 0$. In Figure 9 the radial distribution of intensities directed outwards are described for the four angles for an atmosphere with a constant Planck function and a constant optical depth. The intensities remain almost constant throughout the medium but they have a minimum at $\tau = \tau_{\max}$ and this is because no radiation was incident from outside at $\tau = \tau_{\max}$.

In Figure 10 we have plotted the radiation distribution of intensities directed outward for the angles μ_1, μ_2, μ_3, and μ_4 with the constant Planck function and optical depth change as r^{-2}. The intensities corresponding to the four angles cannot be graphically resolved. The change in this case is not similar to the change in the intensities described

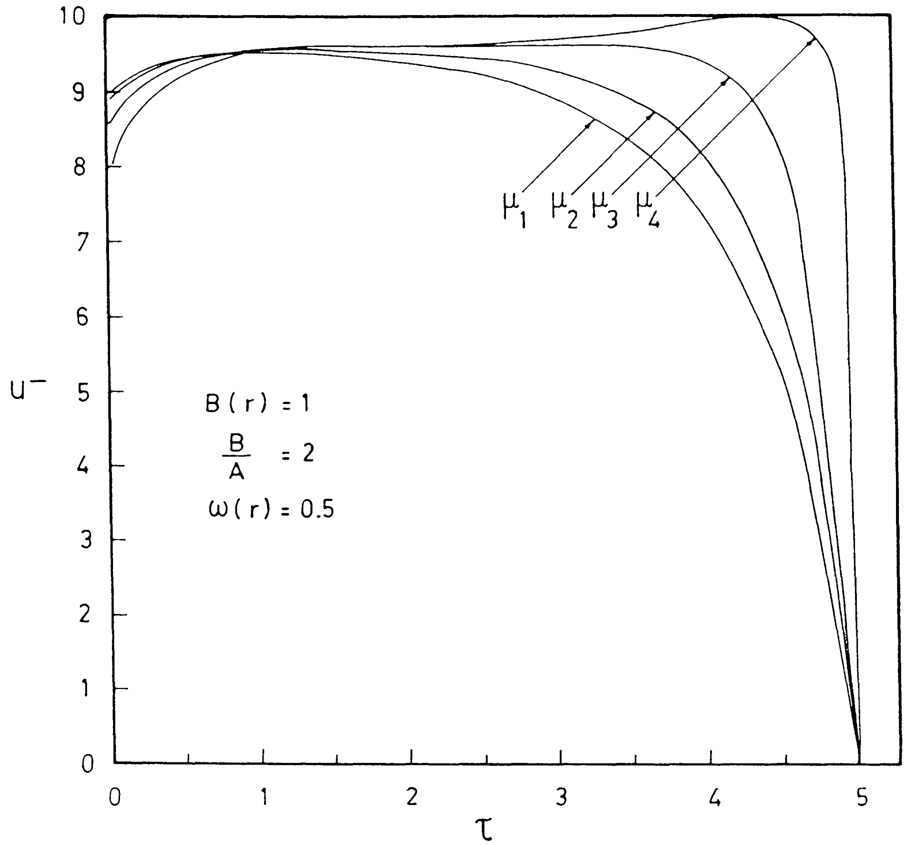

Fig. 9. Radial distribution of the specific intensities U^- corresponding to the angles μ_1, μ_2, μ_3, and μ_4 for the parameters shown in the figure.

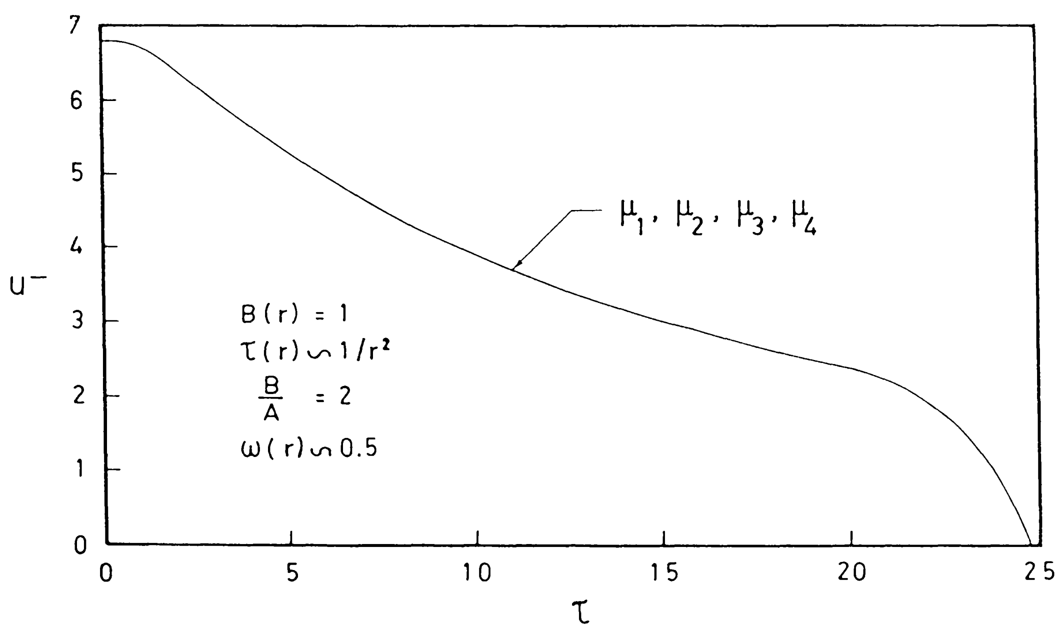

Fig. 10. Angular distribution of the intensities U^- for the parameters shown in the figure.

in Figures 8 and 9 at $\tau = 0$. They have maximum intensities while the intensities described in Figure 8 have minimum values at both $\tau = \tau_{max}$ and $\tau = 0$ and the intensities described in Figure 9 vary in a dissimilar way to those described in Figure 10.

In conclusion, we notice that there is more of back-scattered radiation in the case of spherical symmetry than in plane-parallel geometry. This aspect should be taken into account in the calculation of blanketed models.

References

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