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Neutrinos, Calactic Dark Matter and Flat Rotation Curves

by

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ABSTRACT

Degenerate or partially degenerate density distributions of neutrinos do not give rise to flat rotation curves. If the galactic dark matter chiefly consists of neutrinos they would have to be isothermal. Such a halo of thermal neutrinos would, however, dissipate on time scales substantially shorter than typical galaxy ages. Comparison with other forms of dark matter is also made and sensitivity to inclusion of baryons in the core is also studied.

1. Introduction

Observations imply that there is a considerable amount of dark or unseen matter on astronomical scales ranging from large clusters of galaxies to the individual galaxies themselves. For instance the masses of galaxies in a cluster as estimated from the virial theorem to account for their observed velocity dispersion $<V^2>$ (giving the so called dynamical mass, $M_d \simeq <V^2> R/G$) turns out to be at least a factor of ten higher than what one would expect from the luminosity (Faber and Gallagher 1979). Even groups of galaxies seem to have inadequate luminous mass by a similar factor to account for their dynamical dispersion, the proportion of unseen non-luminous mass seeming to increase with increasing dimensions. Again studies of the dynamics and structure of large spiral galaxies suggest that a universal feature of all the rotation curves (*i.e.* rotational velocity plotted *versus* distance away from the core) is that at large galactocentric distances they are either flat or slowly rising, there being no large spiral galaxy whose rotation curve falls (Rubin *et al.* 1982). As the rotational velocities are given by V^2 being proportional to $GM(r)r$, $M(r)$ being the mass contained within a radius $\leqslant r$, these observations of flat, $V =$ constant, rotation curves imply $M(r)$ increasing linearly with r indicating the presence of much unseen dark matter upto large distances from the centre of the spirals. The progressive increase in the dynamical mass with radius is a characteristic feature for all these galaxies, *i.e.* individual galaxies are surrounded by massive dark halos, which have as much as ten times the mass of the visible matter. It is now known that X-ray emitting hot gas (*e.g.* from clusters and galactic coronae) would account for only a small fraction of the required missing mass. Other propositions ranging from black holes to very low mass stars have met with various difficulties. A favourite candidate for the missing mass have been the relic neutrinos left over from the primordial big bang especially after recent experimental claims that neutrinos from beta decay may possess a small rest mass in the range 14–40 eV ($\sim 10^{-4}$ electron rest mass). Moreover such a gas of neutrinos would tend to collapse more easily on large scales than on small scales suggesting increase of missing mass with scale. The number density in relic neutrinos of each type is given by: $n_\nu \approx 100$ cm^{-3}, suggesting that they would dominate the matter density in the universe (*i.e.* $\varrho_\nu/\varrho_c \approx 1$; $\varrho_c = 3\ H_0^2/8\pi G = 2\times 10^{-29}$ gcm^{-3}, for $H_0 \simeq 100$ km/s/Mpc, being the critical density) even for a ν mass of 20–30 eV. Another major reason for favouring a non-baryonic component of dark matter like neutrinos is that galaxy formation in a baryon-dominated universe with adiabatic density perturbations is in conflict with upper limits for temperature fluctuations in the microwave background radiation. This conflict does not occur for non-baryonic dark matter if it is not coupled to photons, as the fluctuations of the baryons can be small during the epoch of decoupling. Of course as far as massive neutrinos are concerned there are other problems with the galaxy formation scenario as we shall discuss briefly later such as galaxies forming too late (*i.e.* $Z \approx 1$), too massive initial clumps etc. However, in this paper we would not be primarily concerned with galaxy formation *i.e.* we assume the halos are neutrinos and then point out the problems.

2. Degenerate Neutrino Configurations

Several models of the missing mass for clusters and galaxies assume the neutrinos to form self-gravitating, non-relativistic (*i.e.* rest mass dominating at present epoch), fully degenerate configurations surrounding the visible main portion (Eqs. are: Cowsik and McCelland 1973, Gao and Ruffini 1981). Such a degenerate neutrino configuration would have a mass-radius (M-R) relationship of the form (known from the theory of white dwarfs, *e.g.* Landau and Lifshitz 1980):

$$MR^3 = 91.5\,\hbar^6/G^3\,\mathrm{m}_\nu^8\,\frac{1}{(g_\nu/2)^2}, \tag{1}$$

the equation of state being a polytrope, $P = k\varrho^{5/3}$, $k = (g_\nu/2)^{-2/3}(3\pi^2)^{2/3}\hbar^2/$ $/(5\,m_\nu^{8/3})$; g_ν is the degeneracy factor ($=1$ for each ν flavour and equals 2 if antineutrinos are also counted). The above relation when applied to large clusters of galaxies would imply $m_\nu \approx 5$–10 eV. However, the $MR^3 =$ $=$ constant relation as implied by Eq. (1) would not satisfy the $M \propto R$ relation seen for large clusters (Rood 1974) and also would not give the $\varrho \propto 1/R^2$ density distribution (again implying $M \propto R$) required to get a flat rotation curve for spiral galaxies. Also it has recently been shown by Zhang *et al.* (1983) that by solving the equations of hydrostatic equilibrium with the equation of state of a degenerate neutrino gas, one does not get configurations which would give the observed flat rotation curves. It turns out that the situation is not improved by the inclusion of temperature effects, *i.e.* by considering a partially degenerate system of neutrinos (Chau *et al.* 1984). (One would then have temperature corrections to the above degenerate equation of state).

3. Isothermal Neutrino Configuration

Of course, as we shall soon see, a purely thermal (isothermal) neutrino distribution would give a radial density distribution $\varrho \propto 1/r^2$ which in turn would give a flat $V^2 =$ const. rotation curve. It would be more appropriate to consider a thermal distribution from another consideration, *i.e.* the result of virialization by violent relaxation in gravitational collapse is a roughly isothermal halo. When a primordial neutrino fluctuation collapses the neutrinos will undergo collisionless violent relaxation, the process occurring rapidly in a time of the order of the gravitational dynamical time. As shown by Lynden-Bell (1967) and others, by statistical methods, one would expect the distribution function resulting from such a dissipationless violent relaxation (*via* chaotic changes in the collective gravitational field with the total mass much greater than the component particles) to be an isothermal equilibrium configuration. The mutual interactions of the neutrinos can be neglected and the phase space distribution $f(r, V)$ satisfies the collisionless Boltzmann equation:

$$\frac{\partial f}{\partial t} + \bar{v}\frac{\partial f}{\partial r} - \frac{\partial \varphi}{\partial r}\frac{\partial f}{\partial r} = 0, \tag{2}$$

where:

$$\Delta\varphi = 4\pi G\varrho(r), \qquad \varrho(r) = \int f(r, v)\,\mathrm{d}^3v.$$

Pressure P which is isotropic is given by $P = \frac{1}{3}\varrho(r)\langle V^2\rangle(r)$, where $\langle V^2\rangle(r)$ is the velocity dispersion. For an isothermal distribution

$\langle V^2 \rangle (r) = V_0^2$ = constant and for a spherically symmetric $f(r, V)$, Eq. (2) for the static case ($\partial f/\partial t = 0$) becomes:

$$1/r^2 \frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{\varrho} r^2 \frac{\mathrm{d}\varrho}{\mathrm{d}r}\right] = -\frac{12\pi G\varrho}{V_0^2} \tag{3}$$

the parameters being V_0^2 (kinetic temperature) and central density ϱ_0. Solutions to Eq. (3) have the usual scaling property:

$$\varrho'(Ar) = B\varrho(r), \quad v'(Ar) = Dv(r),$$
$$M'(Ar) = D^3 B^{-1/2} M(r), \quad A = B^{-1/2} D, \quad D = \frac{v_0'}{v_0}, \quad B = \frac{\varrho_0'}{\varrho_0}. \tag{4}$$

For large r, the asymptotic relations hold:

$$v(r) \simeq \sqrt{\frac{2}{3}}\, v_0, \quad M(r) \simeq \frac{2}{3}\frac{v_0^2 r}{G}, \quad \varrho(r) \simeq \frac{v_0^2}{6\pi G}\frac{1}{r^2}. \tag{5}$$

Thus the rotational curves for an isothermal distribution of neutrinos do follow the required flat behaviour $V(r)$ = constant, and the corresponding density distribution $\varrho(r) \propto 1/r^2$, with M rising linearly with r. The neutrino number density to account for the missing mass in the spiral galaxies would have to be $n_\nu \simeq 10^6$ cm^{-3}. The neutrino temperature, owing to the Hubble expansion of the Universe, will for massive neutrinos fall off with the scale factor a of expansion as $\sim a^{-2} \sim (1+Z)^2$. For $m_\nu c^2 \approx 30$ eV, this would give an energy of $\approx kT_\gamma/10^5$ where T_γ is the microwave photon background temperature.

4. Phase Space And Dispersion of Halos

However, phase space considerations (Tremaine and Gunn 1979) would show that the trapping and collapse of neutrinos would change their distribution function to an isothermal Maxwellian distribution of velocities $\sim$ few hundred kilometres per second. Although one gets density distribution functions, appropriate for obtaining flat rotation curves, as well as the required missing mass, with isothermal neutrinos it turns out that phase space considerations would give velocity dispersions contrary to what is observed over different astronomical length scales. Neutrinos being fermions their phase space density (see also Datta, Sivaram and Ghosh 1984) must satisfy the inequality: $\mathrm{d}^3x\, \mathrm{d}^3p \geqslant h^3$, which translates into the relations:

$$n_\nu/(m_\nu v_\nu)^3 \leqslant \hbar^{-3} \quad \text{or} \quad m_\nu \geqslant (h^3 \varrho_m / v_\nu^3)^{1/4}, \tag{6}$$

where ϱ_m is the density of the missing matter. For a typical spiral galaxy

$\varrho_m \simeq 10^{-24}$ g/cm^3, V_ν (the velocity dispersion) $\simeq 200$ km/sec, which constraints $m_\nu \geqslant 50$ eV. However, for dwarf spheroidal galaxies (unseen matter may dominate the gravitational potential of such systems too, *e.g.* Aaronson 1983), we have from the one rather fundamental phase space constraint that $m_\nu \geqslant 300$ eV, totally incompatible with experimentally suggested values (Lyubimov *et al.* 1980). Moreover, such large neutrino masses (*i.e.* > 100 eV) would severely violate cosmological density constraints even allowing for models with a cosmological constant (Sivaram 1985). Thus the phase space requirement shows that one would need neutrinos with widely ranging rest masses from 5 eV for clusters to 300 eV for dwarf galaxies if the same missing matter is to dominate the dynamics over all these astronomical systems. Such a range of neutrino rest masses is incompatible with particle physics. Another serious problem with the neutrino hypothesis for halos follows from the phase space constraint $m_\nu \geqslant (h^3 \varrho_m / V_\nu^3)^{1/4}$, obtained above. This relation shows that as the neutrinos cluster on smaller scales (*i.e.* alternatively $m_\nu^4 \geqslant h^3/G\sigma R_H^2$, σ is the velocity dispersion $\langle V^2 \rangle = \langle 3\sigma^2 \rangle$, R_H is the halo radius, so that $\sigma \propto 1/R_H^2$), their velocity dispersions should increase as $\sim 1/R^2$, completely contrary to what is actually observed, the smaller systems like the dwarf spheroidal galaxies having smaller velocity dispersions than larger systems. This inverse relationship would also be predicted for other fermionic particles that are dark matter candidates such as gravitinos or photinos, as these particles would also be subject to the same phase space constraints. Another problem with the neutrino halo, would be the dissipation of the thermal particles on time-scales substantially shorter than typical galaxy ages. The time taken for the dissipation of a thermal gas with a Maxwellian velocity distribution trapped in a gravitational field is given by the method due to Jeans (1925) as:

$$t_D \simeq \frac{4\cdot 4}{4\pi G n_\nu m_\nu} \frac{v_\nu^3}{R_H(v_\nu^2 + 4\pi G n_\nu m_\nu R_H^2)} \exp\left(\frac{4\pi G \varrho_\nu R_H^2}{v_\nu^2}\right). \tag{7}$$

For typical halo sizes, $R_H \simeq 5\times 10^{23}$ cm, $V_\nu \simeq 500$ km/s (from virial relaxation), $m_\nu \simeq 50$ eV, we have: $t_D < 10^{16}$ s, substantially smaller than a galaxy life time. In the case of the large clusters t_D would be comparable. Using the two characteristic time parameters, *i.e.* the free fall (or dynamical) time t_{ff} given by: $t_{ff} \simeq (4\pi G \varrho_\nu)^{-1/2}$ and the crossing time $t_c \simeq R_H/V$, and defining $Z = t_c/t_{ff}$, we can express the above formula for t_D in the functional form:

$$t_d/t_{ff} = \frac{4.4}{Z(1+Z^2)} e^{Z^2}. \tag{8}$$

This is minimized for $Z = 1$, giving $(t_D)_{min} \simeq 2e t_{ff}$. In order to account for the progressive increase in dark matter over larger and larger scales we would need a density of at least $\approx 10^{-26}$ g cm^{-3} over cluster sizes ~ 1 Mpc (Mass $\simeq 10^{16}\ M_\odot$). It follows that $(t_D)_{min}$ for such large clusters is only $\simeq 2\times10^9$ yrs, $t_{ff} < (5\times10^8 - 10^9$ yrs.). This would also imply that the time scale for collapse of the infalling neutrino matter to the core would also be of the same order. In other words, extended neutrino halos made up of such thermal particles are unlikely to exist for time scales much longer than $\simeq 2\times10^9$ years, substantially shorter than the lifetimes of the individual galaxies. However, if the neutrino density continues to drop off as $\sim 1/r^2$ beyond R_H even in intergalactic space, then the amount of dark matter between galaxies would be more than sufficient for closure density. In such a case, the dissipating halo neutrinos could be replenished by neutrinos falling into the galaxy from intergalactic space, so that the halos would remain for longer periods. However, with a larger neutrino mass to accommodate the dark matter in dwarf galaxies, such a density distribution in intergalactic space, would lead to too small a value for the age of the universe.

It should be remarked that as far as the total mass is concerned, both the degenerate and isothermal distributions give more or less the same value (to within a factor of two). The infinite extension of the isothermal model may not be really a shortcoming as observations of hierarchies of cosmological structures seen to show that somehow galaxies may be embedded in larger structures.

5. Other Forms of Dark Matter

We can also consider the addition of baryons chiefly in the centre of the galaxy with a fixed mass density acting as a background on the neutrinos. We can introduce a distribution of $\varrho_B(r)$ as:

$$\varrho_B(r) \sim \varrho_0 \left[\frac{1}{(1+r/a)^2} - \frac{1}{(1+(R_B/a)^2)}\right],$$

where R_B is the extent of the baryonic matter and ϱ_0 is the central density. Again scaling relations similar to Eq. (4) are obtained. For $m_\nu \geqslant 50$ eV, $R_B \simeq 6$ kpc we set a maximal rotation $V_m \simeq 250$ km/s and a total mass $\simeq 10^{12}\ M_\odot$. Roughly one-tenth of this would be in the form of baryons. Inclusion of higher proportion of baryons tends to change the rotation curves giving a peak and a fall off rather than a flat curve.

As remarked earlier, there are other problems with neutrino dark matter, even at the stage of galaxy formation. Thus the initial fluctuations which form, at the epoch when the neutrinos become non-relativistic (a 100 eV neutrino becoming non-relativistic at $Z \simeq 2\times10^5$) and cluster gravitationally have masses $\sim (\hbar c/Gm_\nu^2)^{3/2} \simeq 10^{16}\ M_\odot$, too large for galaxies. Hypothetical particles like the gravitino or photino if they have masses ~ 1 keV, would produce galaxy size fluctuations $\sim 10^{11}\ M_\odot$. Whereas in a ν dominated universe the first objects to form would have typical masses $\sim 10^{16}\ M_\odot$ and sizes $\sim 50\ (1+z)^{-1}$ Mpc for $m_\nu \simeq 30$ eV (Bond and Szalay 1984). Smaller scale structures can form only after the initial collapse of such superclusters, and studies show that in this picture galaxies and smaller structures would have formed only around $Z \approx 1$ or later (Kaiser 1983). However, the fact that galaxies are already now been recently detected around $Z = 3$, puts this picture into serious conflict with observations. Gravitinos and photinos (so called warm dark matter) would have similar problems arising from phase space constraints as they are fermions if they constitute the dark matter in halos, but they have the advantage of forming galaxy size fluctuations much earlier. However, the 1 keV gravitino mass required to form such fluctuations now already appears phenomenologically incorrect as far as current supersymmetry models of particle physics are concerned, which now predict the lowest mass of such particles to be in the range of several GeV!

6. Conclusions

Another favourite dark matter candidate is the so called axion initially suggested (Wilczek 1978) to render the theory of strong interactions (*i.e.* quantum chromodynamics, QCD) invariant with time reversal or CP transformations. By introducing a new pseudoscalar field, the time reversal violating phases can be removed by rotating them into the complex phase of the new field. As axions can couple to photons and charged matter they are expected to be produced copiously in the cores of stars and especially in red giants and other evolved stars. In order not to drastically alter red giant evolution, the axion mass is bounded from above (Dicus *et al.* 1978) as $m_a \leqslant 10^{-3}$ eV and currently $m_a \approx 10^{-5}$ eV is accepted. There are other constraints based on the axion flux from the Sun and giant planets like Jupiter (Sivaram 1986). $\sim 10^{-5}$ eV mass axions must have a number density $\sim 10^{14}\ \mathrm{cm}^{-3}$ to account for the dark matter in galaxy halos. Since, they are bosons, not following the exclusion principle, the stringent phase space constraints noted earlier for fermionic dark matter do not apply to axions. Moreover, as they are non-relativistic to begin with, they constitute the so called cold dark matter and would not dissipate from the halos like neutrinos. Again they would form smaller scale structures $\sim 10^{8}\ M_\odot$ in the earlier epochs, thus removing some of the difficulties with neutrinos where galaxy size structures form too late. However, a major problem with axions is that unlike neutrinos, they would tend to cluster more densely on smaller and smaller scales, so that one would expect even individual galaxies to contain amounts of dark matter implying densities close to the closure density, *i.e.* $\varrho = \varrho_c$ or $\Omega = \varrho/\varrho_c = 1$, even on Kpc scales, whereas cluster dynamics and the cosmic virial theorem would imply $\Omega \simeq 0.1$ or at most 0.2 even on scales of the order of 1 Mpc (Peebles 1980). Again with this form of dark matter it would be difficult to produce voids, and neither the Einstein-de Sitter ($\Omega = 1$) nor the open universe structure formations of the axion model agree with observations of galaxy spatial and velocity distribution (Uson and Wilkinson 1984).

Finally we may remark that the recent recommendation of IAU Commission on Galactic Structure to reduce the solar distance to the galactic centre from 10 Kpc to 8.5 Kpc (15 × reduction) and rotation speed around it to 220 km/s from 250 km/s (10% slower rotation) would imply an increase in the ratio of dark to luminous matter in the local group. The local group is dynamically dominated by the Milky Way and Andromeda which are approaching each other and as the universe as a whole is expanding, these two galaxies must be in a gravitationally bound orbit. The total mass needed to produce the binding depends on the velocity of approach and since the observed radial motion of Andromeda is the vector sum of the velocity of approach and the Sun's motion (the circular rotation velocity) any slower rotation would imply larger approach speed for the two galaxies and, therefore, larger combined mass. This would have the effect of increasing the ratio of non-luminous to luminous mass by about a tenth. It would not much alter the constraints on neutrinos, the phase space constraint now giving $m_\nu \geqslant 60$ eV.

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