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Reflection Effect in Close Binaries. II. Distribution of Emergent Radiation from the Irradiated Component along the Line of Sight

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Abstract. We have calculated the effects of irradiation from a point source observed at infinity. Plane-parallel approximation and spherically-symmetric approximations are employed in calculating the self-radiation field for the sake of comparison. It is found that there are considerable changes in the radiation received at infinity between the approximation of plane-parallel stratification and spherical symmetry.

Key words: reflection effect—irradiation—spherical symmetry

1. Introduction

In an earlier paper (Peraiah 1982, henceforth called Paper 1) we have presented an approach to obtain the radiation reflected from the component of a binary system. We have considered the incident radiation coming from an external point and estimated how this radiation field has changed the total radiation coming from the irradiated surface. We have noticed several important changes in the radiation field emergent from the irradiated part of the component. The intermediate regions of the irradiated part of the atmosphere become brighter than the extreme regions. This result suggests that the law of limb darkening that is generally used in the light curve analysis (see Kopal 1959) should be replaced by accurate calculations of the distribution of radiation from centre to limb.

In paper I, we have calculated the angular distribution along the surface of the irradiated surface as it is important to understand the basic problem. Now, in this paper, we shall calculate the distribution of radiation from the centre to limb as received at infinity.

2. Procedure to calculate the radiation field from centre to limb

In Fig. 1, we have shown the portion of the star illuminated by radiation from a point

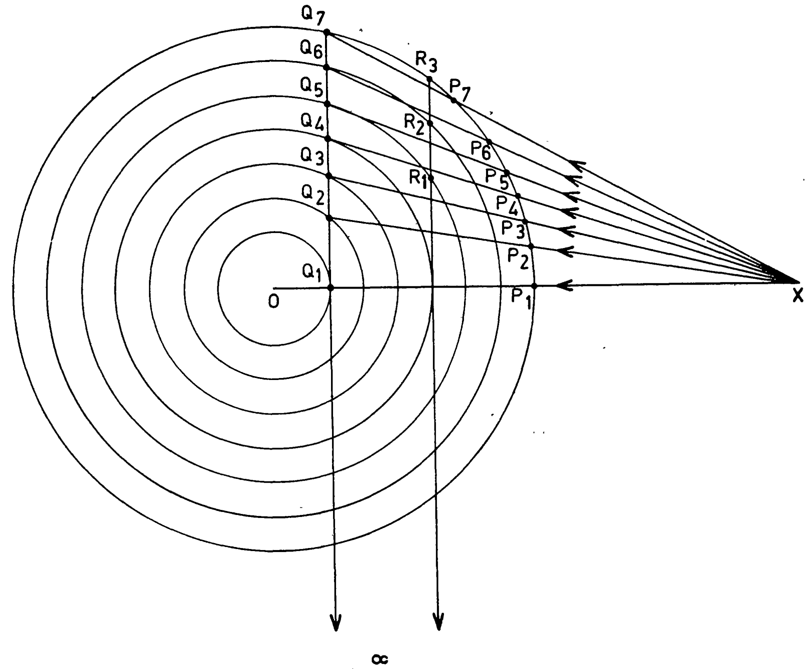

Figure 1. Schematic diagram showing the irradiated section of the component. X is the point source of radiation. O is the centre of the component. The specific intensities are calculated along the line of sight. ($Q_7 Q_6$ *etc.*, $R_3 R_2$ *etc.*)

source X. The atmosphere is divided into several shells of equal radial thickness. The radiation from this irradiated part of the atmosphere is received at ∞. We have chosen a set of parallel rays tangential to the shell boundaries at a point on the axis OX where O is the centre of the component. Let one of these rays meet the shell boundaries at Q_1, Q_2, Q_3 *etc.* Join these points to X. Let these lines meet the surface of the atmosphere at P_1, P_2 *etc.* The intensity along the parallel rays is calculated first by obtaining the combined source functions (self + irradiated) at points Q_1, Q_2 *etc.* For this purpose we have to calculate (1) the source function at these points due to self-radiation S_S and (2) the source function due to irradiation S_I. The latter is calculated by using the Rod model explained in Paper I. The radiation field is calculated along the lines Q_1P_1, Q_2P_2 *etc.* We then add the two source functions to obtain the total source function S_T. Therefore,

$$S_T = S_S + S_I . \tag{1}$$

The element QP is given by

$$QP = \left\{ a^2 + b^2 + 2ab \quad \cos\,(O\hat{Q}P + O\hat{P}Q) \right\}^{1/2} \tag{2}$$

where $a = OP$ (OP_1, OP_2 *etc.*), $b = OQ$ (OQ_1, OQ_2 *etc.*), $OX = R$ and $OQ_1 = h$ (where h is measured along OX). The angles $O\hat{Q}P$ and $O\hat{P}Q$ are given by

$$\sin\, O\hat{Q}P = \frac{R}{b}\left(\frac{b^2 - h^2}{b^2 + R^2 - 2hR}\right)^{1/2}$$

and

$$\sin O\hat{P}Q = \frac{R}{a}\left(\frac{b^2 - h^2}{b^2 + R^2 - 2hR}\right)^{1/2}$$

We have obtained the source terms S_I at Q's by calculating the optical depths along QP's. This is done by assuming an electron scattering atmosphere and the electron density ρ varying as

$$\rho \sim \frac{1}{r^2}$$

and

$$\rho \sim \frac{1}{r^3}. \qquad (3)$$

The quantity S_S, the source function due to self radiation, is calculated by solving the equation of radiation transfer in spherical symmetry given as (see Peraiah & Grant 1973)

$$\mu \frac{\partial u(r,u)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\, u(r,\mu)\right] + \sigma(r)\, u(r,\mu)$$

$$= \sigma(r)\left\{\left[1 - \varpi(r)\right] B(r) + \frac{1}{2}\varpi(r)\int_{-1}^{+1} P(r,\mu,\mu')\, u(r,\mu')\, d\mu'\right\},$$

for outward-going rays, and

$$\mu \frac{\partial u(r,\mu)}{\partial r} - \frac{1}{r}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\, u(r,-\mu)\right] + \sigma(r)\, u(r,-\mu)$$

$$= \sigma(r)\left\{\left[1 - \varpi(r)\right] B(r) + \frac{1}{2}\varpi(r)\int_{-1}^{+1} P(r,-\mu,\mu')\, u(r,\mu')\, d\mu'\right\},$$

for inward-going rays, $\mu \in (0,1)$ being the cosine of the angle made by the ray with radius vector r. $u(r,\mu)$ is given by

$$u(r,\mu) = 4\pi r^2 I(r,\mu), \qquad (6)$$

$I(r,\mu)$ being the specific intensity. $\varpi(r)$ is the albedo for single scattering and $P(r,\mu,\mu')$ is the phase function assumed to be isotropic in this investigation. $B(r)$ is the internal source term and is set equal to 0, so that we have $\varpi = 1$ which means that we are treating a purely scattering medium. $\sigma(r)$ is the scattering coefficient. We have to ensure that

$$\frac{1}{2}\int_{-1}^{+1} P(r,\mu,\mu')\, d\mu' = 1, \quad -1 \leqslant \mu,\ \mu \leqslant 1. \qquad (7)$$

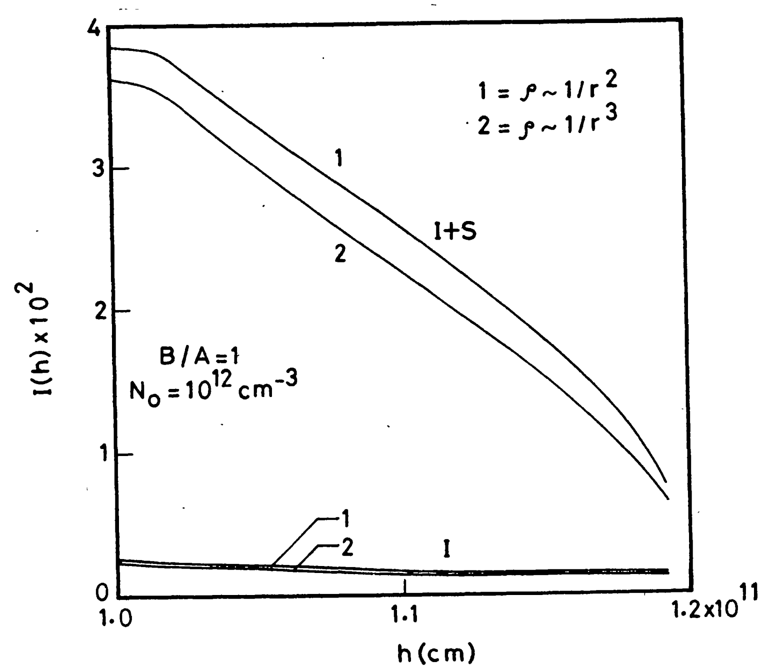

Figure 2. The specific intensities $I(h)$ (h = OQ) are plotted with respect to h. The curves labelled I correspond to only irradiation and those with I + S correspond to irradiation plus self radiation. N_0 is the electron density at A. $B/A = 1$, $N_0 = 10^{12}\,cm^{-3}$.

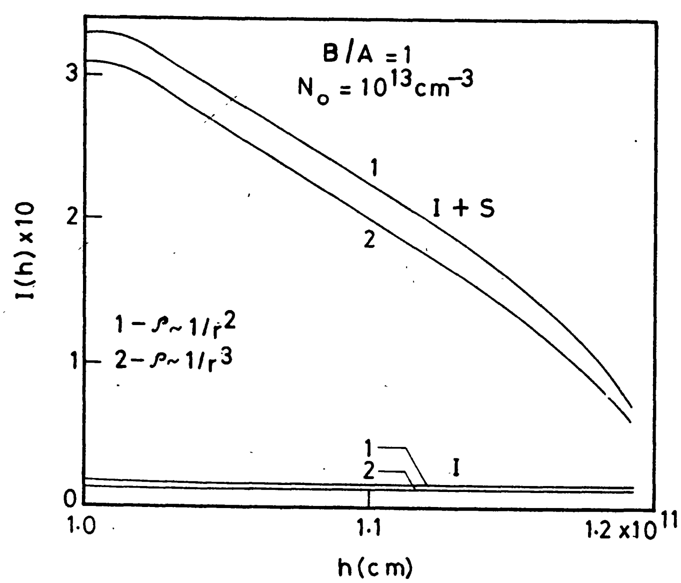

Figure 3. $I(h)$ versus h for $B/A = 1$, $N_0 = 10^{13}\,cm^{-3}$.

For the solution of Equations (4) and (5) see Peraiah & Grant (1973). We have obtained the source function due to self radiation given by the solution,

$$S_S(r) = \frac{1}{2}\int_{-1}^{+1} I(r,\mu)\, d\mu\,, \tag{8}$$

which is nothing but the mean intensity. The boundary conditions have been assumed in such a way that if I_I is the intensity of the irradiation from point X and I_S is the incident self radiation on the inner boundary of the atmosphere, then we have set

$$I_S/I_I = I\,. \tag{9}$$

The radiation incident on the surface at points P_1, P_2 *etc.* is taken to be $I_I \cos O\hat{P}Q$. We have set $I_S = 1$.

We have considered both plane-parallel and spherically-symmetric media for the sake of comparison and set $B/A = 1$ and 1.5 where B and A are the outer and inner radii of the atmosphere.

In the case of plane-parallel stratification, we have considered a small geometrical thickness for the purpose of calculating the optical depth. However, in obtaining the quantity S_S in plane-parallel approximation, we have solved Equations (4) and (5) by suppressing the spherical terms

$$\pm\frac{1}{r}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\, u(r, \pm\mu)\right]$$

To calculate the spherical solution, we have set $B/A = 1.5$ and $R(=\mathrm{OX}) = 4.5 \times 10^{11}$ cm. The radius of the component is taken to be 10^{11} cm. We have varied the electron densities from $10^{12}\,\mathrm{cm}^{-3}$ to $10^{14}\,\mathrm{cm}^{-3}$.

3. Results and discussion

We have presented the results in Figs 2–8. These results represent the specific intensities observed at infinity (see Fig. 1) emanating from the irradiated portion of the component. In Figs 2–4, we have shown the specific intensities from centre to limb in the case of plane-parallel approximation. The curves labelled I correspond to irradiation and those with I + S correspond to irradiation together with self radiation. The contribution from irradiation to the intensities $I(h)$ is several times smaller than the total contribution from both self and irradiation. We notice in Figs 2 and 3 that the limb is darker than the centre. When the electron density is increased (see Fig. 4) the irradiance brightens the limb but when combined with self radiation, the same variation is noticed as seen in Figs 2 and 3.

In Figs 5–7 we have plotted $I(h)$ with $B/A = 1.5$. The limb darkening is noticed in all these cases. One can also notice that the intensities fall sharply when compared to those in plane-parallel approximation. When the electron density is increased, the situation changes (see Fig. 8 and compare these results with those given in Fig. 4). The irradiation definitely shows limb brightening and the intensities corresponding to I + S fall gradually, but at the limb, they start increasing.

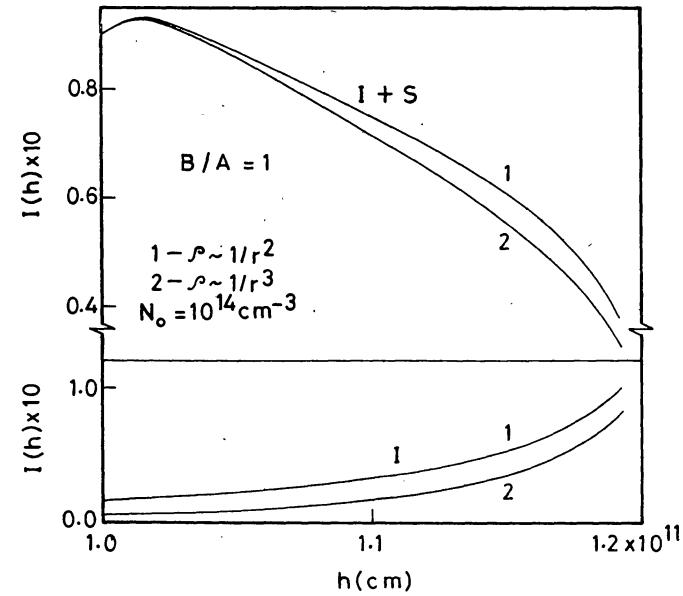

Figure 4. $I(h)$ versus h for $B/A = 1$, $N_0 = 10^{14}\,\mathrm{cm}^{-3}$.

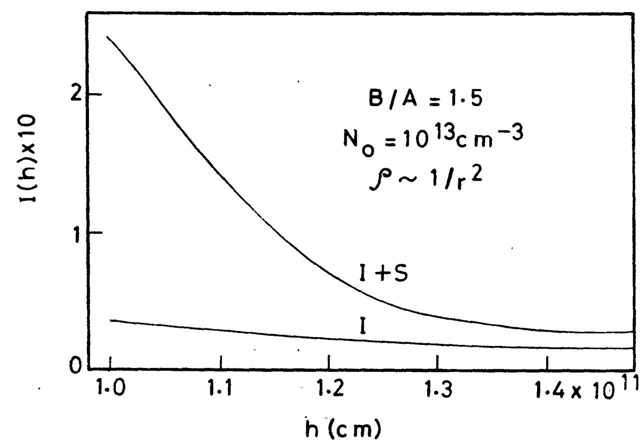

Figure 5. $I(h)$ versus h for $B/A = 1.5$, $N_0 = 10^{13}\,\mathrm{cm}^{-3}$, $\rho \sim 1/r^2$.

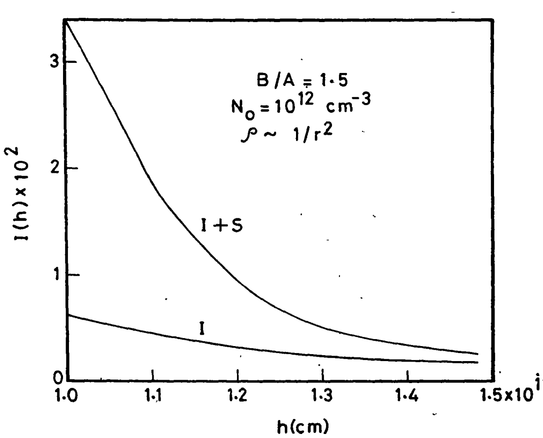

Figure 6. $I(h)$ versus h for $B/A = 1.5$, $N_0 = 10^{12}\,\mathrm{cm}^{-3}$, $\rho \sim 1/r^3$.

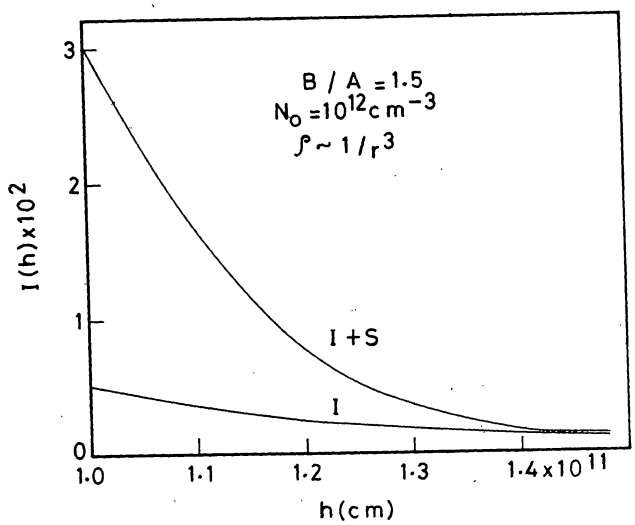

Figure 7. $I(h)$ versus h for $B/A = 1.5$, $N_0 = 10^{12}\,\mathrm{cm}^{-3}$, $\rho \sim 1/r^3$.

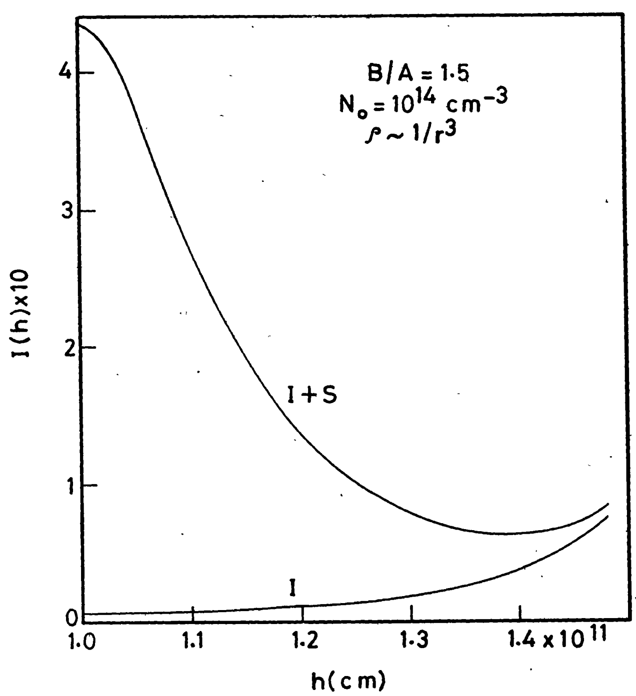

Figure 8. $I(h)$ versus h for $B/A = 1.5$, $N_0 = 10^{14}\,\mathrm{cm}^{-3}$, $\rho \sim 1/r^3$.

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