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Reflection Effect in Close Binaries. I. Distribution of Radiation from a Point Source

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Abstract. The radiation field along an irradiated surface of a component in a binary system is calculated. The source of irradiation is assumed to be a point source. This is done primarily to understand easily how the incident radiation will get changed after it is being scattered by the atmosphere. It is noticed that the maximum radiation comes from intermediate points of the atmosphere, the reason being that here we have the combined radiation due to the star and incident radiation from the point source outside the star although both are diluted.

Key words. radiative transfer—rod model—close binaries

1. Introduction

In radiative transfer theory, we solve the equation of transfer by assuming a certain geometrical configuration such as plane-parallel or spherically symmetric stratification of the media. These geometrical configurations assume symmetric boundary conditions and whenever we have asymmetric incident radiation, the solutions developed in the context of symmetrical geometries as mentioned above, will have to be modified. Such problems are encountered in the evaluation of radiation from the irradiated component of a binary system. The problem of incidence of radiation from a point source is termed the Searchlight problem. Chandrasekhar (1958) has calculated the diffuse scattering function in a plane-parallel medium when a pencil of beam of radiation from a point source is incident. There have been several attempts at calculating the diffuse field in such simple geometries (Rybicki 1971).

However, the calculation of the radiation field during eclipses in close binaries is of different complexity. There are two important aspects one should take into account: (1) the physical processes that take place inside the atmosphere and (2) the geometrical shape of the illuminated surface which reflects the light. Generally, if the atmosphere of the component under consideration is extended or fills its Roche lobe, then the problem of determining the emergent radiation from such surfaces become very difficult. The process of estimating the radiation field from such surfaces becomes more complicated when the various competing physical processes are included in the calculations. Geometrical considerations alone would make it quite difficult because of the deformed shape due to tidal effects from the neighbour and due to self-rotation. The resultant shape would be an ellipsoid and the problem requires a special treatment. Solution of radiative transfer equation either in spherical symmetry or in cylindrical symmetry cannot describe appropriately the radiation field emanating from such surfaces. This requires a careful study from fundamental aspects. The problems of planetary atmospheres involve this type of radiation field but plane-parallel approximation has been employed by several authors in the past (Chandrasekhar 1960). Buerger (1969) employed plane-parallel approximation in computing the continuum and line radiation emitted by a rotationally and tidally distorted star which is irradiated by the light of the secondary component. This approach is obviously simplified to avoid the difficulties arising out of correct treatment of radiative transfer in such a situation, but is totally inadequate if we require realistic results. Although the thickness of the planetary atmosphere is quite small compared to the radius of the planet itself, the curvature of the atmosphere on which the parallel beam from the Sun is incident, would scatter and absorb this radiation differently at different points on its surface. Therefore, one must calculate the radiation from planetary atmospheres on lines similar to those described in this paper.

The aim of this series of papers is to estimate the reflected radiation (both in continuum and lines) that is incident on a binary component. Further, we would like to estimate the changes caused by the reflection effect on the light curves that are observed. In this paper, we present a method of calculating the emergent radiation from the surface irradiated by the radiation from the second component assumed as a point source.

2. Method of obtaining solution

We shall assume that the stars are spherical (the method can be extended easily to nonspherical shapes) and divide the atmosphere into several shells. In Fig. 1, we have described the model for calculating the distribution of radiation. We have assumed the albedo of single scattering to be unity, *i.e.* a pure scattering medium.

O is the centre of the component whose atmosphere is divided in spherical shells. Radiation from a point S outside the star (assuming that the radiation is coming from point source) is incident at points such as P_1, P_2 etc. These rays travel through the medium intersecting the shells at given radial points. We choose a radius vector corresponding to a given θ (the colatitude) and calculate the source functions where this radius vector meets the shell boundaries at points such as Q_1, Q_2 *etc.* The calculation of the source function is done by employing the 'rod' model of one-dimensional radiative transfer (see Sobolev 1963; Wing 1962) along the ray path inside the medium. This means we calculate the source functions at points Q_1, Q_2 *etc.* For readers who are not familiar with this approach, we shall briefly describe the method following Grant (1968). In Fig. 2, we describe the model. We assume a steady state, monochromatic ray with or without internal sources. Notice that the optical depth is measured in the opposite direction to that of the geometrical depth. The optical depth is calculated by the relation

$$\tau = \tau(\xi) = \int_0^l \sigma(\xi')\,d\xi', \quad \tau(l) = T. \tag{1}$$

In this model, we assume that the transfer takes place along $0l$ in Fig. 2 (or P_1Q_1, P_2Q_2 *etc.* in Fig. 1) with isotropic scattering ($\mu = \pm 1$ and $p(\tau)$ = phase matrix elements $= \frac{1}{2}$). The quantities $I^+(\tau)$ and $I^-(\tau)$ represent the specific intensities in the opposite directions. The two equations for oppositely directed rays are written as

$$\frac{dI^+}{d\tau} + I^+ = S^+, \tag{2}$$

$$\frac{dI^-}{d\tau} + I^- = S^-, \tag{3}$$

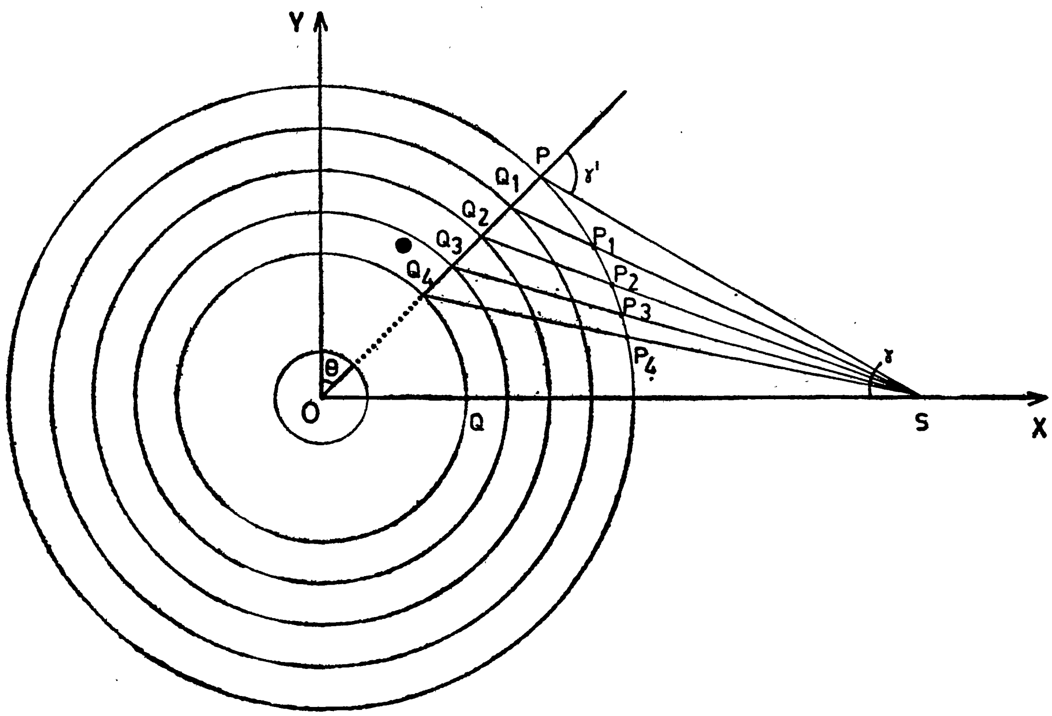

Figure 1. Model diagram showing how the radiation is calculated.

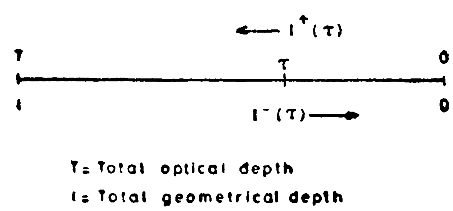

Figure 2. Rod model.

where

$$S^+ = \varpi(\tau)\,[p(\tau)\,I^+(\tau) + (1 - p(\tau))\,I^-(\tau)], \tag{2a}$$

$$S^- = \varpi(\tau)\,[(1 - p(\tau))\,I^+(\tau) + p(\tau)\,I^-(\tau)]. \tag{3a}$$

The boundary conditions at $\tau = 0$ and $\tau = T$ are given by

$$I^+(0) = I_1$$

and

$$I^-(T) = I_2, \tag{4}$$

where I_1 shall be specified later. The total source function that includes the diffuse radiation field can be written as,

$$S_d^+(\tau) = S^+(\tau) + \varpi(\tau)\,[p(\tau)\,I_1 \exp(-\tau) + (1 - p(\tau))\,I_2 \exp\{-(T - \tau)\}]$$

$$S_d^-(\tau) = S^-(\tau) + \varpi(\tau)\,[(1 - p(\tau))\,I_1 \exp(-\tau) + p(\tau)\,I_2 \exp\{-(T - \tau)\}]. \tag{5}$$

In this case the boundary conditions are

$$I^+(0) = I^-(T) = 0.$$

The mathematical aspects of the problem have been fully discussed in Grant (1968) and we shall simply quote his results. When $\varpi = 1$, we have

$$I^+(\tau) = I_1 \frac{1 + (T - \tau)(1 - p)}{1 + T(1 - p)},$$

and

$$I^-(\tau) = I_1 \frac{(T - \tau)(1 - p)}{1 + T(1 - p)}. \tag{6}$$

From Equations (6) one can write

$$I^+(\tau = T) = I_1 \frac{1}{1 + T(1 - p)},$$

$$I^-(\tau = 0) = I_1 \frac{T(1 - p)}{1 + T(1 - p)}. \tag{7}$$

Notice that

$$r(T) = \frac{T(1 - p)}{1 + T(1 - p)} \to 1 \text{ as } T \to \infty$$

and

$$t(T) = \frac{1}{1 + T(1-p)} \to 0 \text{ as } T \to \infty, \tag{8}$$

where $r(T)$ and $t(T)$ are the reflection and transmission co-efficients. Therefore in the limit when $T \to \infty$ we have

$$r(T) + t(T) = 1 \tag{9}$$

which is nothing but conservation of energy.

Using the results of the above analysis we can calculate the source functions according to one-dimensional rod model at points where the radii corresponding to each θ meet the shell boundaries.

Our aim is, therefore, to obtain the source functions described in Equations (5). Here, we calculate the optical depth along the ray path *e.g.* P_1Q_1, P_2Q_2 *etc.* and employ this optical depth to estimate the specific intensities and source functions at these points.

In addition to the incident radiation, we have the radiation of the star itself. This radiation can be calculated easily by employing the spherically-symmetric approximation of the radiative transfer equation. This is done by solving the equation in spherical symmetry in discrete space theory as described in Peraiah and Grant (1973). The total source function S, at each radial point Q_1, Q_2 *etc.* is given by the sum of the source functions due to star and irradiation from the point S. That is

$$S(r, \theta) = S_1(r, \theta) + S_2(r) \tag{10}$$

where $S_1(r, \theta)$ is due to irradiation from outside and $S_2(r)$ is due to self radiation field.

Next step is to calculate the distribution of intensities at the internal points like Q_1, Q_2 *etc.* along the radius and find out the distribution of the emergent radiation field at points like P_1, P_2 *etc.* The radiation field is given by the formal solution of radiative transfer equation in plane-parallel approximation, (See Chandrasekhar 1960, Equations 64 and 65 on p. 12, and Fig. 3 here),

$$I(\tau, +\mu) = I(\tau_1, \mu) \exp[-(\tau_1 - \tau)/\mu] + \int_{\tau}^{\tau_1} S(t) \exp[-(t-\tau)/\mu]\, dt/\mu \tag{11}$$

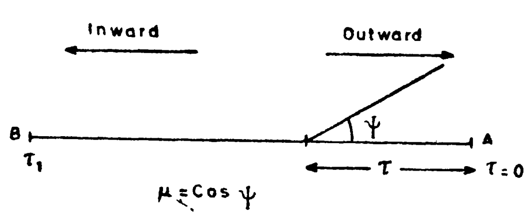

Figure 3. Plane-parallel model.

for outward intensities, and

$$I(\tau, -\mu) = I(0, -\mu) \exp(-\tau/\mu) + \int_0^\tau S(t) \exp[-(\tau - t)/\mu] \, dt/\mu \tag{12}$$

for the intensities directed inwards. In Equations (11) and (12), the optical depths are always measured from A to B, and $1 \geqslant \mu > 0$ where $\cos^{-1} \mu$ is the angle made by the ray with AB. Equations (11) and (12) describe the radiation field emerging from the irradiated surface which receives the incident radiation coming from the point S.

3. Results and discussion

We have considered a component whose radius is 10^{11} cm with half the radius as its atmosphere, and assume that the incident radiation is coming from a point S at a distance of $4{\cdot}5 \times 10^{11}$ cm from the centre of the component O. We also assume the point S is on the axis OX and the axis OY is perpendicular to OX. We calculte the source functions at points Q_1, Q_2 *etc.* on a radius vector with co-latitude θ. This is done by employing the rod model described in Equations (2a), (3a), (5) and (6). We have assumed isotropic scattering and the albedo for single scattering is set equal to unity. We calculte the line segments P_1Q_1, P_2Q_2 *etc.* and estimate the optical depths according to the law that the density varies as $1/r$. While calculating the source functions due to self radiation, we have used the spherically symmetric approximations.

Let I_Q be the intensity of radiation incident spherically symmetrically at Q inside

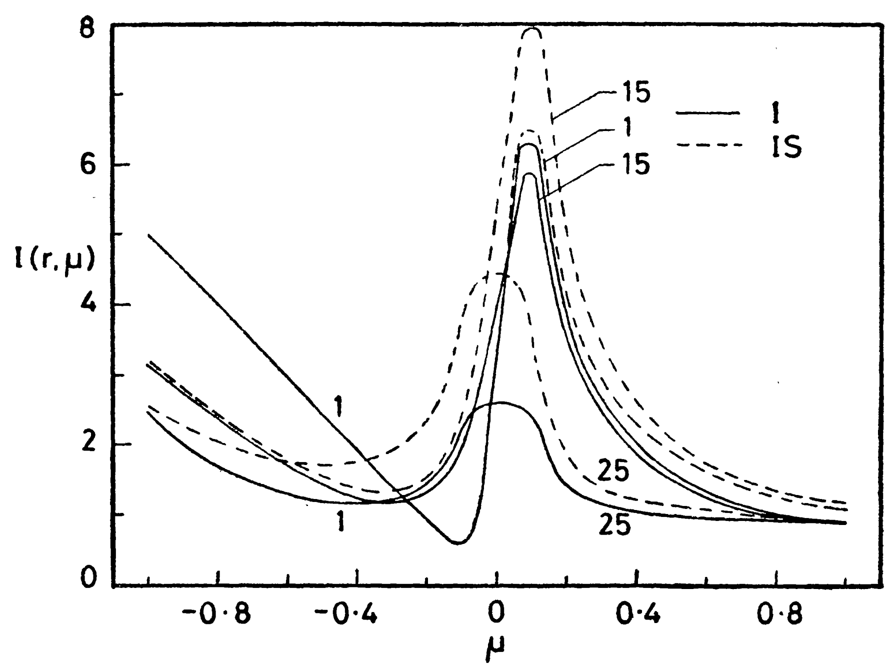

Figure 4. Distribution of radiation field at $\theta = 0°$, Case 1, for the shell numbers shown in the figures. I stands for the irradiation, and IS stands for irradiation plus selfradiation.

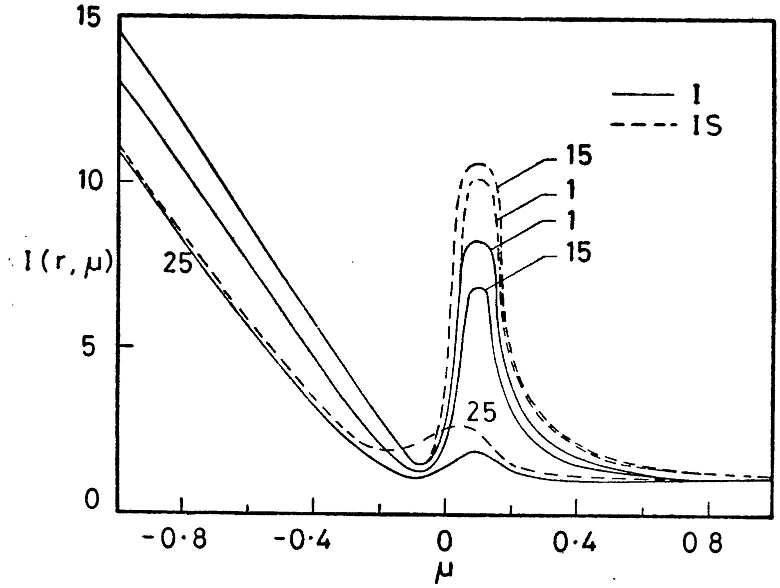

Figure 5. Distribution of radiation field at $\theta = 60°$, Case 1.

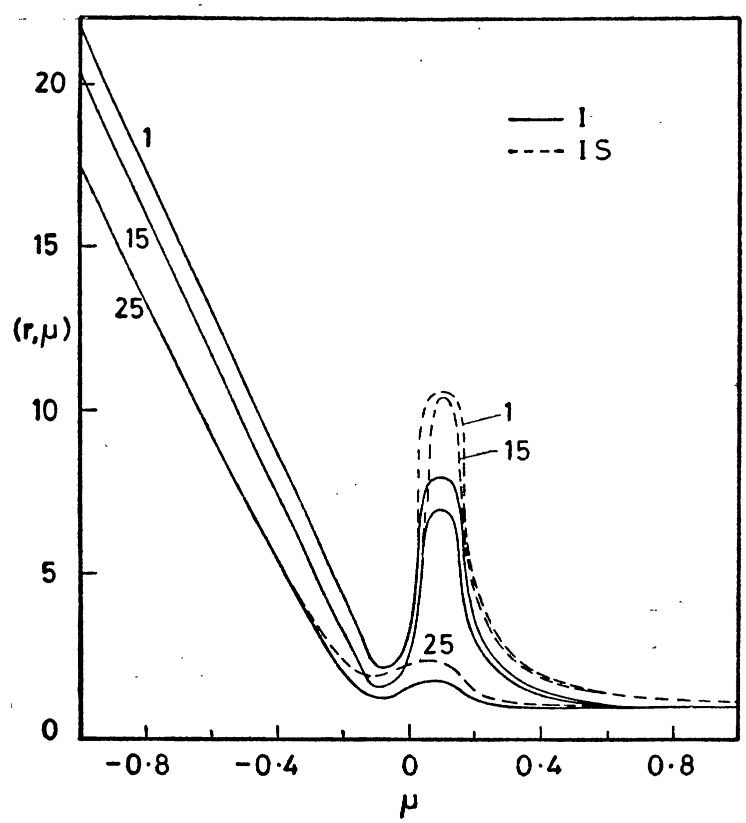

Figure 6. Distribution of radiation field at $\theta = 90°$, Case 1.

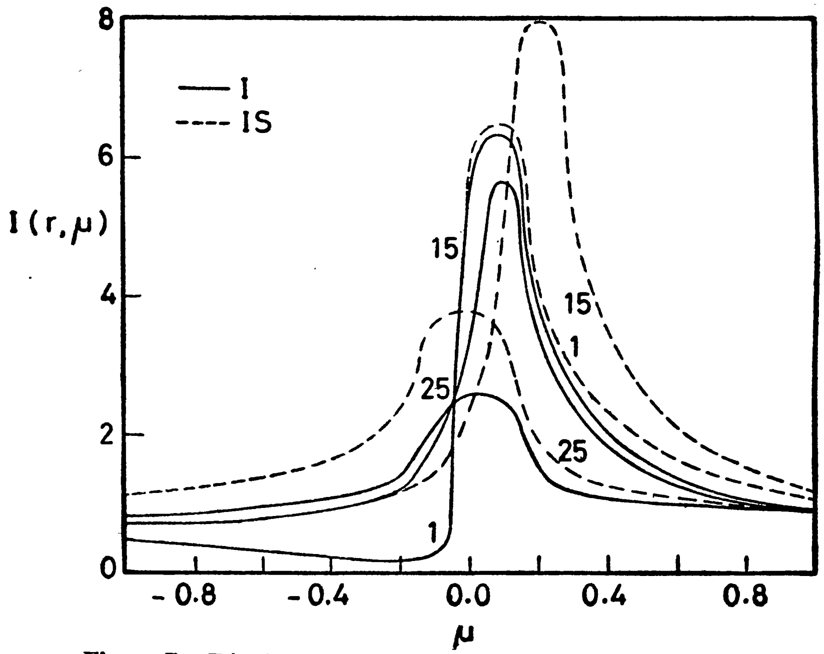

Figure 7. Distribution of radiation field at $\theta = 0°$, Case 2.

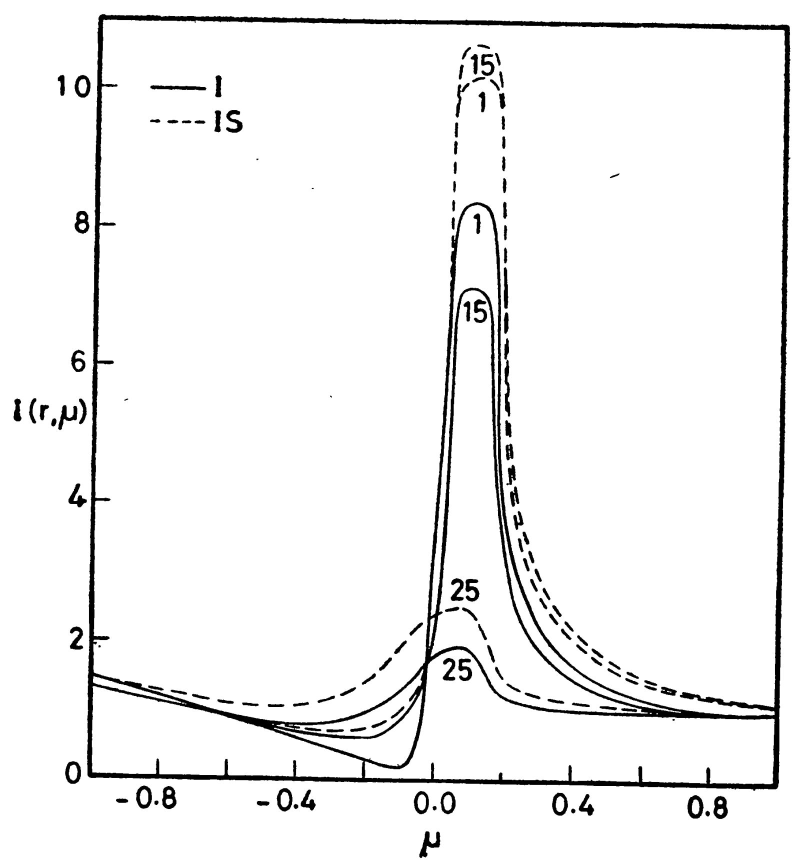

Figure 8. Distribution of radiation field at $\theta = 60°$, Case 2.

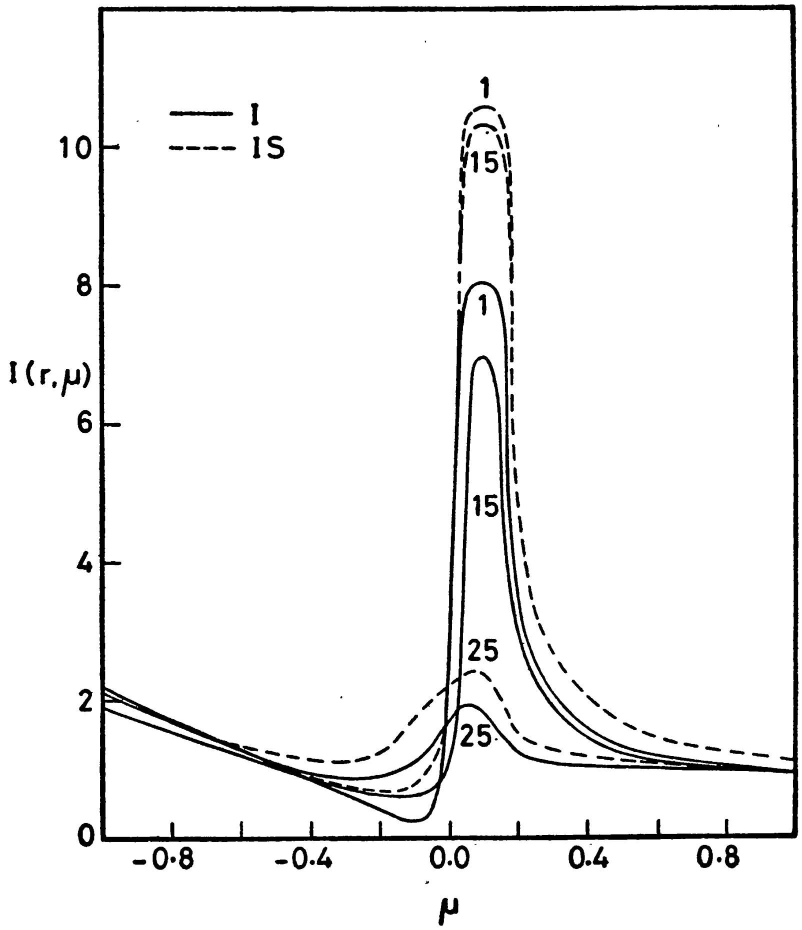

Figure 9. Distribution of radiation field at $\theta = 90°$, Case 2.

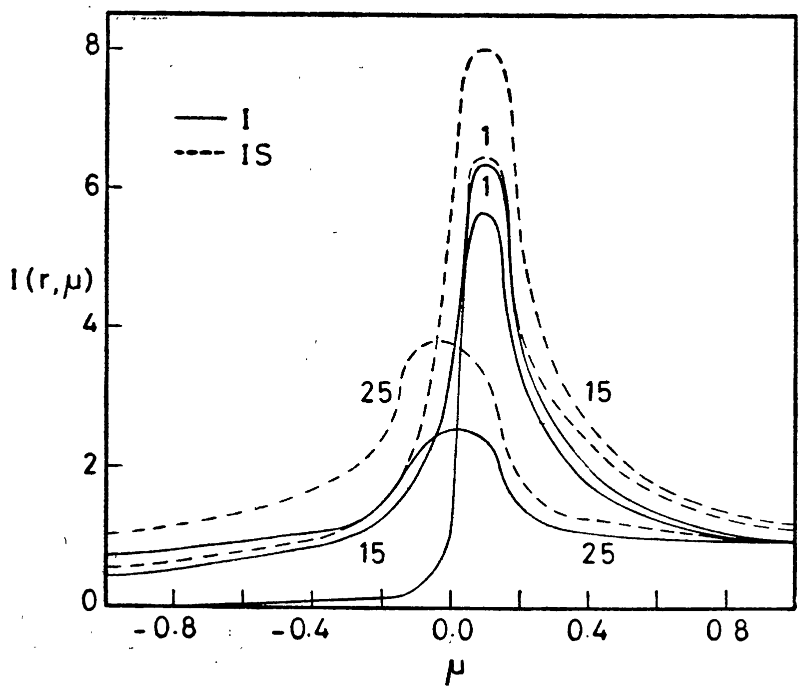

Figure 10. Distribution of radiation field at $\theta = 0°$, Case 3.

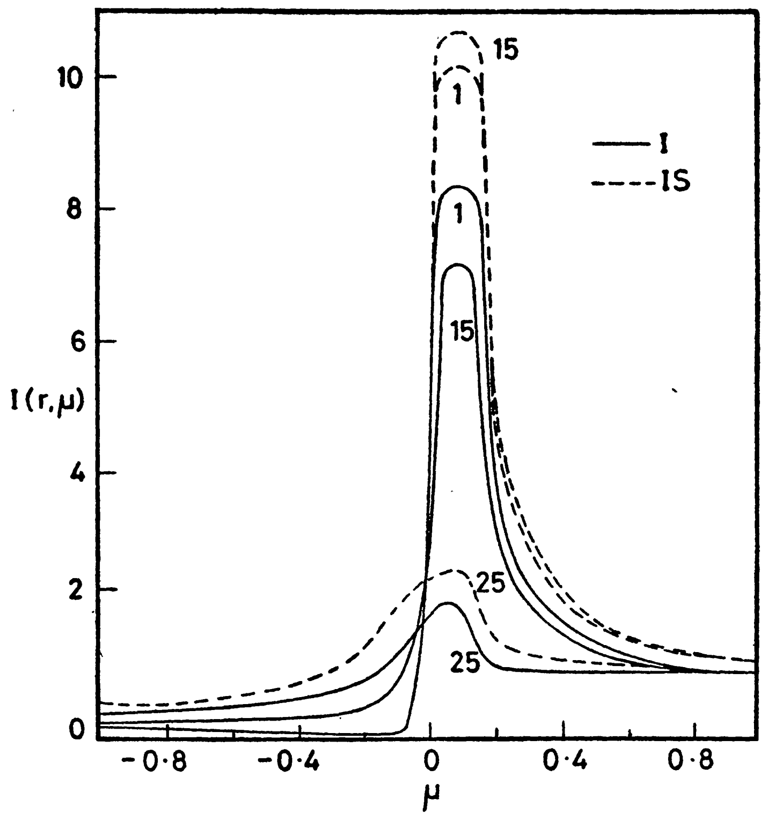

Figure 11. Distribution of radiation field at $\theta = 60°$, Case 3.

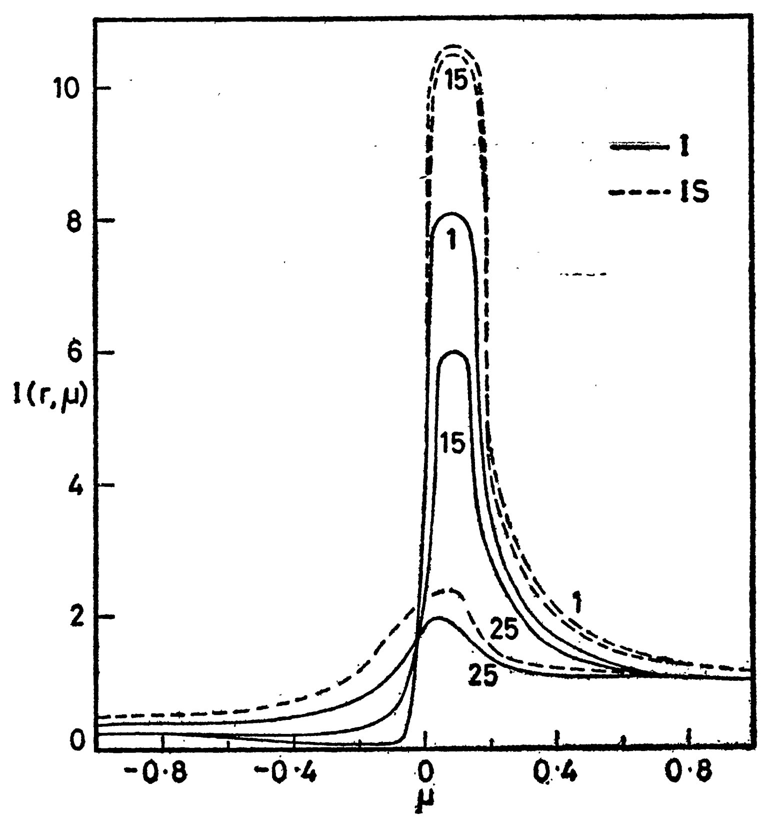

Figure 12. Distribution of radiation field at $\theta = 90°$, Case 3.

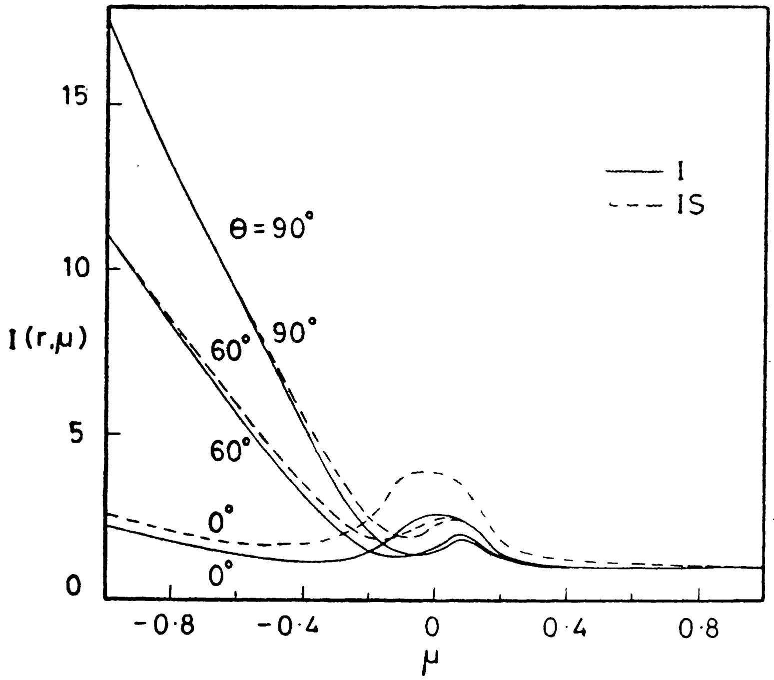

Figure 13. Distribution of the emergent radiation field at $\theta = 0°$, 60° and 90°, Case 1.

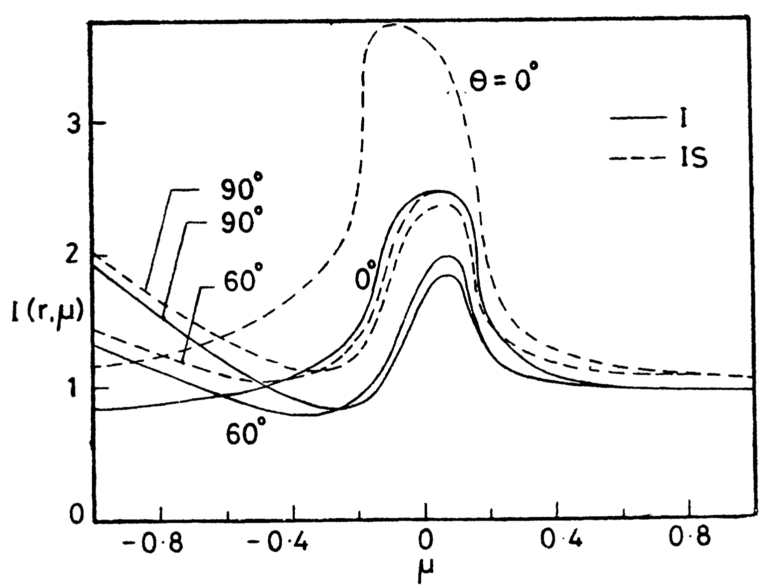

Figure 14. Distribution of the emergent radiation field at $\theta = 0°$, 60° and 90°, Case 2.

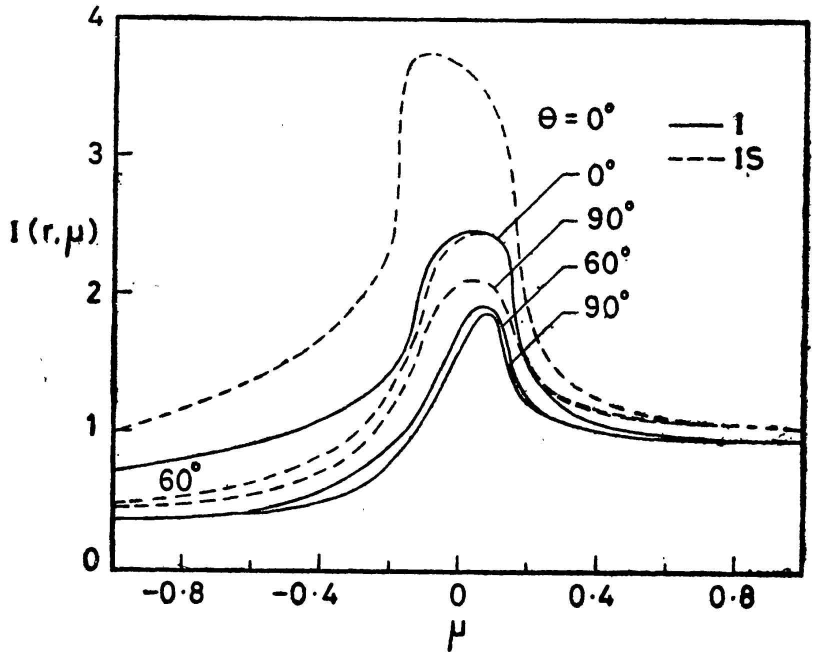

Figure 15. Distribution of the emergent radiation field at $\theta = 0°$, 60° and 90°, Case 3.

the star and the intensity coming from point S be I_S. The incident radiation at P will be $I_S\ \mu \cos \gamma$. We have considered three cases.

Case 1: $I_Q/I_S = 0{\cdot}1$

Case 2: $I_Q/I_S = 1$

Case 3: $I_Q/I_S = 10$

We have assumed scattering due to electrons in the atmosphere of the component. Calculations have been done with several electron densities N_e(at $r=r_{in}$). N_e is set equal to 10^{12} cm^{-3} to 10^{15} cm^{-3}. We have presented results for $N_e = 10^{14}$ cm^{-3}. With this density, the radial optical depth becomes 1.1. The results are given in the form of distribution of radiation at different points along the radii such as Q_1, Q_2 *etc.* by solving the Equations (11) and (12). These are presented in Figs 4–12 for the three cases. We have divided the medium into 25 shells. The radiation field I (θ, r, μ) are plotted against μ for a specified θ and r (in terms of the shell number where the shell numbers are counted from outside to inside, *i.e.* $n=1$ at $r=r_{out}$ and $n=25$ at $r=r_{in}$). In Figs 4–6 we have plotted $I(r, \mu)$ at $\theta=0°$, 60° and 90° respectively for Case 1. The continuous curves denote the external radiation and the dotted curves indicate the resultant radiation field due to both external and self radiation. In Fig. 4, the co-latitude $\theta=0$, corresponds to the radiation along OY. At the bottom of the atmosphere ($n=25$ or $r=r_{in}$) the effect of external radiation is not much but increases when the self radiation is added to it. At the shell $n=15$, we see that the combined radiation field is maximum while at the outermost layer ($n=1$) it is not as large as that at $n=15$. This is not difficult to understand on physical grounds. At $n=1$

we have the diluted self radiation and at $n=25$, the external radiation becomes weak. In the middle we have the combined radiation field of both, although partly diluted. In Fig. 5, we have plotted $I(r,\mu)$ for $\theta=60°$. We can see that the same effects seen in Fig. 4, are also there in this figure, although the radiation going towards the centre of the star is enhanced. In Fig. 6, $I(r,\mu)$ is given for $\theta=90°$. The distribution of radiation resembles that given in Fig. 5; again the radiation towards the star (along OX) increases rapidly.

The results of Case 2 are given in Figs 7, 8 and 9 for $\theta=0°$, 60° and 90° respectively. Generally, the radiation field in Case 2, shows characteristics which are similar to those in Case 1. The main difference between these two cases is that more radiation comes out in Case 2 than in Case 1. The amount of radiation that is coming out remains almost the same whether $\theta=0°$, 60° or 90°. Figs 10–12 describe the radiation field for Case 3. At $\theta=0°$, $I(r,\mu)$ is similar to those given in Fig. 4. In this case also, we see more radiation coming out than going inside.

In Figs 13–15, we have plotted $I(n=1, \mu)$ for Cases 1, 2 and 3 respectively. From Fig. 13, one can see that the intensities at $\theta=0$ are quite small and increase considerably at $\theta=90°$. In case 2, we have again the same phenomenon (Fig. 14) although at $\theta=90°$ (along OS) more radiation goes into the star than comes out. $I(n=1,\mu)$ for Case 3 (Fig. 15) shows again the same features of Fig. 14, with much less radiation going into the star both at $\theta=0°$ and 90°.

4. Conclusions

We have developed a method of obtaining radiation field along the spherical surface irradiated by an external point source of radiation, as a preliminary step towards full understanding of reflection effect in close binary systems. Here, we have assumed electron scattering and we need to investigate the temperature changes in the atmosphere due to the irradiation. Furthermore, when the components are very close, one should calculate the effects of irradiation from an extended non-spherical surface.

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