# GRAVITATIONAL INSTABILITY OF INTERSTELLAR MAGNETIC CLOUDS

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**Abstract.** The gravitational instability of a nonrotating isothermal gaseous disk permeated by a uniform frozen-in magnetic field is investigated using a fourth-order perturbation technique. From the results it is found that the disk is stable when $n/B_0 < (\frac{4}{3}\pi^3 G)^{-1/2}$, where $n$ and $B$ are the column density of the disk and unperturbed magnetic field, respectively, and $G$ is the gravitational constant. The disk is gravitationally unstable only when $n/B_0 > (\frac{4}{3}\pi^3 G)^{-1/2}$.

## 1. Introduction

Elmegreen and Lada (1977) have discussed in detail the observational evidence for the occurrence of shock-induced star formation. Based on the observational results, which seem fairly convincing, they have discussed the qualitative features expected for such a process occurring in the shocked layer of gas preceding an H II region as the region expands into a molecular cloud. They extended the theory by modelling layers of shocked gas as self-gravitating, pressure-bound, isothermal, plane-parallel gas sheets, and then studied the dispersion relation for unstable perturbations of such sheets. Welter and Schmid-Burgk (1981) performed similar calculations for the case of curved sheet geometry. However, Elmegreen and Lada (1977) and Welter and Schmid-Burgk (1981) left out from the investigation the effect of a magnetic field on the fragmentation processes of the layer.

Nakano and Nakamura (1978) considered the effect of a magnetic field on the fragmentation processes of the layer but to solve the problem they used a first-order perturbation theory which gives an error of 13% (Nakano, 1981).

Thus the whole problem of gravitational instability of a gaseous layer threaded by a magnetic field is still interesting with respect to star formation. This paper presents the results of the above-mentioned problem solved by use of fourth-order perturbation technique of Krylov–Bogoliubov–Mitropolsky as developed by Kakutani and Sugimoto (1974).

## 2. Nonlinear Schrödinger Equation

In this section the nonlinear Schrödinger equation has been derived from hydromagnetic equations which presents the dynamical behaviour of interstellar magnetic clouds. The unperturbed equations are (Spitzer, 1977)

$$\rho \frac{\partial v}{\partial t} = \frac{1}{c}(j \times B) - \nabla p - \rho \nabla \phi, \tag{1}$$

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$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0 , \tag{2}$$

$$\nabla^2 \phi = 4\pi G \rho , \tag{3}$$

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) , \tag{4}$$

$$j = \frac{c}{4\pi} (\nabla \times B) ; \tag{5}$$

where $v$, $\rho$, $\phi$, and $B$ are fluid velocity of the gas, density, gravitational potential, and magnetic field, respectively. The Cartesian coordinates $(x, y, z)$ are used in this paper. The gaseous layer is assumed to be at rest in the unperturbed state, its density is uniform in the $x$ and $y$ directions, and threaded by a uniform magnetic field in the $z$ direction, $B = (0, 0, B)$, which is perpendicular to the galactic plane and the fluid velocity streaming also along the $z$ direction.

It is assumed that the gaseous layer will remain isothermal in the unperturbed and perturbed states. Now neglecting the radiation pressure, the gas pressure may be written as

$$p = C_s^2 \rho , \tag{6}$$

where $C_s$ is the isothermal sound speed. The density distribution in the unperturbed state is

$$\rho(z) = \rho_0 \operatorname{sech}(az) , \tag{7}$$

where $\rho_0$ is the density at the mid-plane $z = 0$ and

$$a = (2\pi G \rho_0)^{1/2} . \tag{8}$$

For weakly nonlinear systems, we can use the following expansion,

$$\begin{bmatrix} \rho \\ v \\ B \\ \phi \end{bmatrix} = \begin{bmatrix} \rho_0 \\ v_0 \\ B_0 \\ \phi_0 \end{bmatrix} + \varepsilon \begin{bmatrix} \rho^{(1)} \\ v^{(1)} \\ B^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} \rho^{(2)} \\ v^{(2)} \\ B^{(2)} \\ \phi^{(2)} \end{bmatrix} + \varepsilon^3 \begin{bmatrix} \rho^{(3)} \\ v^{(3)} \\ B^{(3)} \\ \phi^{(3)} \end{bmatrix} + \varepsilon^4 \begin{bmatrix} \rho^{(4)} \\ v^{(4)} \\ B^{(4)} \\ \phi^{(4)} \end{bmatrix} \tag{9}$$

where 0 indicates the unperturbed state values.

The monochromatic plane wave is given by

$$\rho^{(1)} = q \exp(i\psi) + \bar{q} \exp(-\psi i) , \tag{10}$$

where $q$ is the amplitude, $\bar{q}$ is its complex conjugate, $\psi = (kx - \omega t)$ is the phase, $k$ is the wavenumber, and $\omega$ the frequency. The amplitude $q$ is a slowly-varying function of $x$ and $t$, which is

$$\frac{\partial q}{\partial t} = \varepsilon A_1(q, \bar{q}) + \varepsilon^2 A_2(q, \bar{q}) + \varepsilon^3 A_3(q, \bar{q}) + \varepsilon^4 A_q(q, \bar{q}) \,,$$
$$\frac{\partial q}{\partial x} = \varepsilon C_1(q, \bar{q}) + \varepsilon^2 C_2(q, \bar{q}) + \varepsilon^3 C_3(q, \bar{q}) + \varepsilon^4 C_4(q, \bar{q}) \,; \tag{11}$$

and their complex conjugates. The quantities $A_1$, $C_1$, $A_2$, $C_2, \ldots$ can be determined from the conditions that the perturbation envisaged by equations are from secularities. Substituting Equations (9)–(11) into Equations (1)–(4) and using Equations (5)–(8), the equations to different orders in $\varepsilon$ are obtained. Then using the solution of $\varepsilon$-order equations, the solutions of $\varepsilon^2$-order equations are obtained and $\varepsilon^2$-order solutions help to get the solutions of $\varepsilon^3$-order equations. From the condition for the removal of resonant secularity in the $\varepsilon^3$-order equation for $\rho^{(3)}$, the nonlinear Schrödinger equation is obtained. The values of the constants of integration have been determined from the conditions of removal of resonant secularity in the equation for $\rho^{(4)}$. All these operations have been discussed in detail in a previous paper by Ghosh *et al.* (1983). Using coordinate transformations as

$$\xi = \varepsilon(x - V_g t) \quad \text{and} \quad \tau = \varepsilon^2 t \,, \tag{12}$$

the nonlinear Schrödinger equation for the hydromagnetic fluid in an interstellar cloud may be written as

$$i\,\frac{\partial q}{\partial \tau} + P\,\frac{\partial^2 q}{\partial \xi^2} = Q[q^2]q \,, \tag{13}$$

where

$$P = \frac{1}{2}\,\frac{\mathrm{d}V_g}{\mathrm{d}k} = \omega\,\frac{a^2 k + a k^2 + k^3}{a^3} \,,$$
$$Q = 1 - \frac{(\frac{4}{3}\pi^3 G n^2/B_0^2)\,(a^2 - k^2)\,(a^2 - ka + k^2)}{a^4 + k a^3 + k^2 a^2} \,, \tag{14}$$
$$n = \int_{-\infty}^{+\infty} \rho_0 \,\mathrm{sech}\,(az)\,\mathrm{d}z \,;$$

and $V_g$ is the group velocity of the hydromagnetic fluid in the gaseous layer.

## 3. Discussion

Gravitational stability and instability may be studied from the nonlinear Schrödinger equation (13). The criterion for a nontrivial marginal stability is represented from Equations (13) and (14) as (cf. Hasegawa, 1972)

$$PQ = 0 \tag{15}$$

or

$$\frac{4\pi^3 G n^2}{3B_0^2} = \frac{a^4 + ka^3 + k^2a^2}{(a^2 - k^2)(a^2 - ka + k^2)} = f(k, a)\,. \tag{16}$$

The variation of $f(k, a)$ with $k/a$ is presented in Figure 1. Since $f(k, a)$ is negative for $k/a > 1$, no marginal modes are present in this region. From Figure 1 it is found that the gaseous disk becomes unstable only when

$$\frac{n}{B_0} > (\tfrac{4}{3}\pi^3 Q)^{-1/2}\,, \tag{17}$$

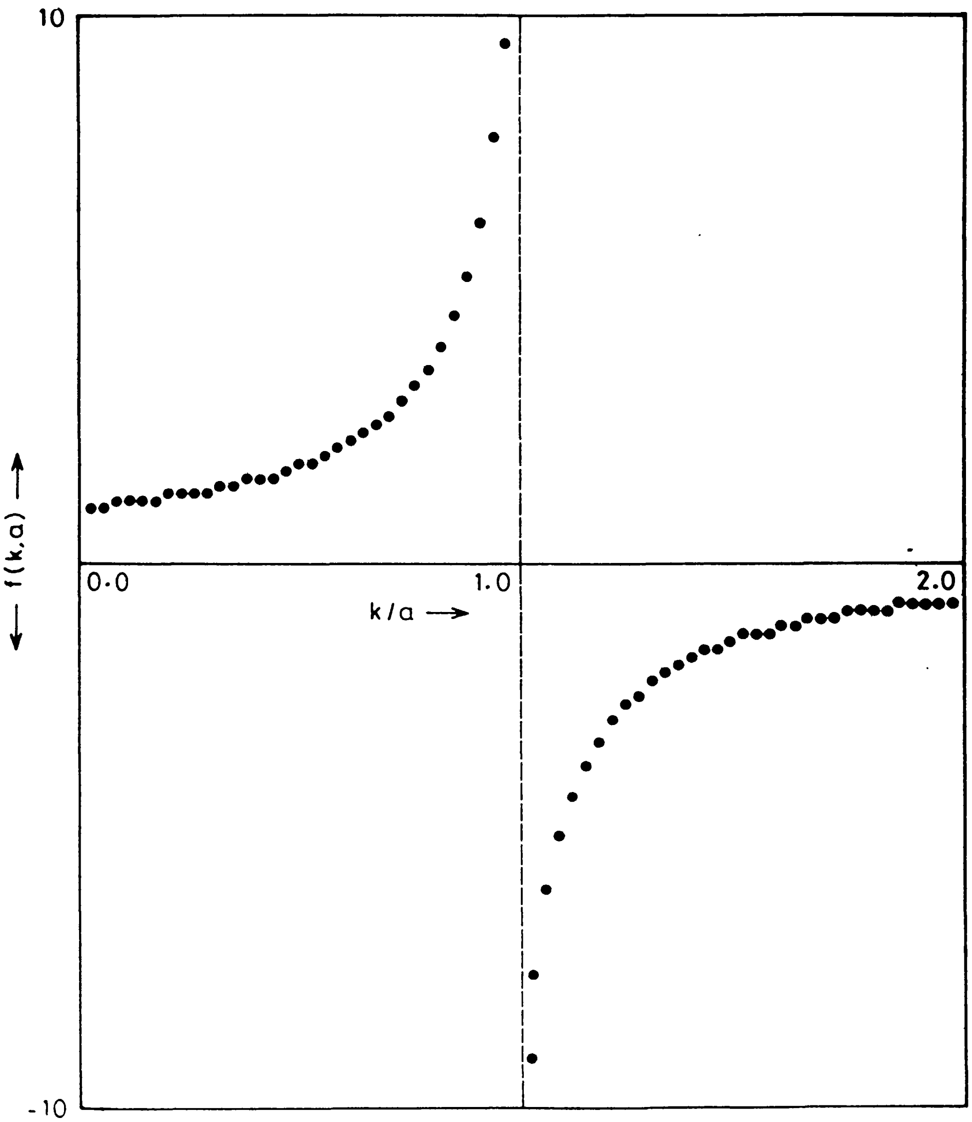


Fig. 1. Stability–instability curve of the interstellar magnetic clouds.

and it will be stable when

$$\frac{n}{B_0} < (\tfrac{4}{3}\pi^3 Q)^{-1/2} . \quad (18)$$

From Equation (17) it is found that when this condition is satisfied, there will be a critical wavelength

$$\lambda_c = \frac{2\pi}{a(1 - 3^{1/2} B_0/2\pi^{3/2} G^{1/2} n)} , \quad (19)$$

above which waves are unstable. Again from Equation (19) it is seen that the critical wavelength will decrease as $n/B_0$ increases and it never becomes less than $2\pi/a$.

Using virial theorem, Strittmatter (1966) obtained the condition for the contraction of a spherical magnetic cloud as

$$M/\pi R B_0 > (\tfrac{12}{5}\pi^2 G)^{-1/2} , \quad (20)$$

and for an oblate spheroidal magnetic cloud as

$$M/\pi R B_0 > (\tfrac{9}{10}\pi^4 G)^{-1/2} , \quad (21)$$

where $M/R$ is the mean column density of the cloud. However, his results differ only by a factor of about 1.9. The criteria for a spheroidal cloud of $0 < e < 1$ are in between conditions (20) and (21), which is insensitive to the flatness of the cloud.

Nakano and Nakamura (1978) also obtained the condition for the contraction of a magnetic cloud using a first-order perturbation theory as

$$\frac{\sigma_0}{B_0} > (4\pi^2 G)^{-1/2} , \quad (22)$$

which gives an error of 13% (Nakano, 1981). The result obtained from Equation (17) is equal to the average result obtained from conditions (20), (21), and (22). Thus the nonlinear perturbation technique gives a more accurate result than linear perturbation analysis to obtain the condition for gravitational instability of interstellar magnetic clouds.

The critical mass for contraction, i.e. for gravitational instability, may be obtained from condition (17) as

$$M_c = n(\lambda_c)^2 = (2C_s/G)^{3/2} (\pi/\rho_0)^{1/2} \{1 - \sqrt{3}\, B_0/(2\pi^{3/2} G^{1/2} n)\} . \quad (23)$$

The critical mass $M_c$ given by Equation (23) may be used even for a flattened cloud though this condition has been obtained by assuming that the cloud is spherical.

The solutions of some differential equations obtained for different order of $\varepsilon$ and values of certain integrals have been obtained by numerical methods using a VAX-11/780 computer at Kavalur Observatory and Figure 1 was obtained by use of the same computer.

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