THE PULSES AS A DIAGNOSTIC TECHNIQUE IN THE SUN

G. C. DAS
Indian Institute of Astrophysics, Bangalore, India

(Received 17 December, 1979)

Abstract. In this paper we discuss a method of finding physical parameters by studying the pulses in the Sun. For the sake of a mathematical approach, we consider an ideal, highly relevant model which could exist in the Sun with the effects of ionization, due to which there will be a continuous formation of ionized particles. It is observed that the pulse originated at the centre of a dipole field propagates along the magnetic field. We derive a dispersion relation for these types of pulses, propagating from the centre to the solar surface. The time taken by the pulse from its source to the solar surface is also estimated, with due account of the ionization effects on the pulse. Without proper account of these effects, the technique employed in determing the physical parameters may lead to error. Temporal and spatial damping of the pulses lead to estimates of the velocity distribution of the ionized particles and of the amplitude of the magnetic field of the wave in pulse.

1. Introduction

The study of rotating stars, since first observed by Schlesinger (1911), has evoked much interest and has focused on the advantages of the dynamics of these stars. Several workers have studied rotating stars in different aspects to establish the close relation between mathematical theory and observations. Studies were based initially on observations, but some mathematical developments, on the basis of several observations, were later achieved especially through the study of magnetohydrodynamic waves in the Sun. Chandrasekhar (1953a, b, c) was first to show that, even for a slow rotation, magnetohydrodynamic waves in astrophysical problems are influenced by rotation. Later, Lehnert (1954) pointed out that the Coriolis force playing in a rotating medium is very much prone to change the various important phenomena of the Alfvén theory on sunspots (Alfvén, 1950). Lehnert has shown that the Coriolis force could play a more dominating role than magnetic force does in the solar system, and, as an example, he has shown that for a typical value of rotation $\Omega = 2 \times 10^{-6}\ \mathrm{s}^{-1}$ and Alfvén velocity $V = 2\ \mathrm{m\ s}^{-1}$, together with the assumption of a polar strength of less than 25 Gauss and wavelength larger than one-hundredth of a solar radius, the Coriolis force is fourteen times the magnetic force. Very recently Das (1979a) considered the model of a rotating ionized medium and estimated the group travel-time that the pulse travels along the magnetic field in the Sun. The group travel-time, from the source of the pulse to the solar surface, is given by the line integral

$$t(\omega) = \int_h^0 \frac{\mathrm{d}h}{v_g}, \tag{1}$$

Astrophysics and Space Science **71** (1980) 353–361. 0004–640X/80/0712–0353$01.35

where v_g is the group velocity of the wave along the path h. To use this integral, Das (1979) further modified the integral (1) on the assumption that the wave frequency is almost equal to twice the rotational frequency. This assumption leads to the integral being evaluated in linear form, from which the density and rotational frequency can be obtained. Although the model under consideration may be ideal for diagnosing the physical parameters, the present model is, to some extent, highly relevant and can be observed in the Sun. We consider the model as follows: As the pulse, which originated in the Sun, progresses along a certain magnetic field path, ionization occurs and there is consequently a continuous formation of ionized particles. It is assumed that the newly ionized particles observe the mean streaming velocity along the magnetic field and gyrate around the field lines with a mean transverse velocity. Ionization may cause instability of the waves or there may be some other effects, but we shall not consider them in this paper although the stability of the waves may be the subject of a subsequent paper.

In Section 2 we derive the group travel-time in the considered model. We also develop relations for the temporal and spatial damping rates for the pulses, and investigate the possibility of using these characteristics to estimate other physical parameters of the Sun.

2. Basic Equations and the Derivation of Group Travel-Time for the Rotating Medium

We consider the ionized medium consisting of electrons (subscript e), and ions (subscript i) together with a newly born ion (subscript n) having a streaming velocity along the magnetic field and gyrating with a mean velocity around the field lines. We assume that the medium, in equilibrium state, is pervaded by a uniform magnetic field and is rotating with angular velocity Ω around the axis of the magnetic field. Since we are mainly interested in astrophysical problems, we can justifiably neglect the effect of centrifugal force from the dynamics of pulses in the Sun. The basic equations (with respect to a rotating frame of reference) are the Equation of Continuity

$$\frac{\partial n_\alpha}{\partial t} + \operatorname{div}(n_\alpha v_\alpha) = 0 \tag{2}$$

and the Equation of Motion

$$\frac{\partial v_a}{\partial t} + v_\alpha \nabla v_\alpha = \frac{q_\alpha}{m_\alpha}\left[E + \frac{v_\alpha \times H}{c}\right] + 2v_\alpha \times \Omega, \tag{3}$$

supplemented by Maxwell's equations

$$\nabla \times H = \frac{4\pi}{c}\sum q_\alpha n_\alpha v_\alpha + \frac{1}{c}\frac{\partial E}{\partial t}, \tag{4}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t}, \tag{5}$$

$$\nabla \cdot H = 0, \tag{6}$$

$$\nabla \cdot E = \sum 4\pi q_\alpha n_\alpha; \tag{7}$$

where $\alpha = i, e, n$; v_α is the velocity of α-type particles having mass m_α; and n_α is the number density. For ions, $q_\alpha = e$ when $\alpha = i, n$, and for electrons, $q_\alpha = -e$ when $\alpha = e$. Further, we assume that the newly born ionized particle observes the mean streaming velocity $v_\parallel$ along the magnetic field and gyrates around the field lines with a mean transverse velocity $v_\perp$.

Now we consider a plane wave propagating along the magnetic field in such a way that the first-order quantities are assumed to vary as $\exp[i(kr - \omega t)]$. Following Montgomery and Tidman (1964) and Uberoi and Das (1970), the dispersion relation can be written as

$$D(\omega, k) \equiv c^2k^2 - \omega^2 + \sum \frac{\omega\omega_{pi}^2}{\omega \pm \pi_i} + \frac{\omega\omega_{pe}^2}{\omega \mp \pi_e} + \\ + \frac{(\omega - kv_\parallel)\omega_{pn}^2}{(\omega - kv_\parallel \pm \pi_n)} + \frac{k^2v_\perp^2\omega_{pn}^2}{2(\omega - kv_\parallel \pm \pi_n)^2}. \tag{8}$$

where $\pi_i = \omega_{ci} + 2\Omega$, $\pi_n = \omega_{cn} + 2\Omega$, $\pi_e = \omega_{ce} - 2\Omega$ and all other conventional symbols have their usual meanings. The $\pm$ signs represent the right and left circularly polarized waves.

At present we have simplified the medium by freeing it from an applied magnetic field, but this has not been done for mathematical simplicity, as will be made clear later. Now, as the effect of the mean streaming velocity only accelerates the ionized particles along the axis of rotation, we consider only the influence of the mean gyrating velocity around the axis of rotation. Further, we consider the case where the wave frequency is almost twice the rotational frequency, and consequently, the group travel-time receives a major contribution from the left circularly polarized (LCP) wave, which is given by

$$n^2 = 1 + \frac{\omega_p^2}{2\Omega(2\Omega - \omega)} - \frac{k^2v_\perp^2\omega_{pn}^2}{2\omega^2(2\Omega - \omega)^2}, \tag{9}$$

where

$$n = \frac{ck}{\omega} \quad \text{and} \quad \omega_p^2 = \sum \omega_{p\alpha}^2,$$

while the corresponding group travel-time reduces to

$$t(\omega) = \int_h^0 \frac{dh}{u_g}, \tag{10}$$

where u_g is the group velocity of the LCP wave and is obtained from

$$u_g = \frac{c}{n + \omega(\partial n/\partial \omega)}. \tag{11}$$

By use of (9) in (11), the group velocity can be obtained as

$$u_g = \frac{c\left[1 - \dfrac{\gamma k^2 v_\perp^2 \Omega}{\omega^2(2\Omega - \omega)}\right]^{1/2}}{\left[\dfrac{\omega_p}{[2\Omega(2\Omega-\omega)]^{1/2}}\left\{1 - \dfrac{\gamma k^2 v_\perp^2 \Omega}{\omega^2(2\Omega-\omega)}\right\} + \dfrac{1}{2}\dfrac{\omega\omega_p}{[2\Omega(2\Omega-\omega)]^{3/2}} \times \left\{1 - \dfrac{\gamma k^2 v_\perp^2 \Omega}{\omega^2(2\Omega-\omega)}\right\} - \dfrac{\gamma}{2}\dfrac{\omega_p v_\perp^2 (2\Omega)^{1/2}}{(2\Omega-\omega)^{1/2}} \times \left\{\dfrac{2k(\partial k/\partial\omega)}{2\omega(2\Omega-\omega)} + \dfrac{k^2(3\omega - 2\Omega)}{2\omega^2(2\Omega-\omega)^2}\right\}\right]}, \tag{12}$$

where $\gamma = (\omega_{pn}/\omega_p)^2$.

We assume that the propagation vector k can be represented as that obtained for a simplified ionized medium – i.e.,

$$k^2 = \frac{\omega^2}{c^2}\left[\frac{\omega_p^2}{2\Omega(2\Omega - \omega)}\right]; \tag{13}$$

and the corresponding value of $k(\partial k/\partial\omega)$ assumes the form

$$2k\frac{\partial k}{\partial \omega} = \frac{2\omega\omega_p^2(2\Omega - \omega/2)}{2c^2\Omega(2\Omega-\omega)^2}. \tag{14}$$

If we substitute (13) and (14) in (12), u_g is further modified in the case where – for the mode propagating with frequency 2Ω and for small $v_\perp$ – the contribution of $\gamma k^2 v_\perp^2 \Omega/\omega^2(2\Omega - \omega)$ is very small in comparison to unity, and u_g is then given by

$$u_g = \frac{c\left[\dfrac{\omega_p}{\{2\Omega(2\Omega-\omega)\}^{1/2}}\right]}{\dfrac{\omega_p^2(2\Omega - \omega/2)}{2\Omega(2\Omega-\omega)^2} - \dfrac{\gamma\omega_p^4 v_0^2(2\Omega - \omega/2)}{2\Omega(2\Omega-\omega)^4} - \dfrac{\gamma}{4}\omega_p^4\dfrac{v_0^2(3\omega - 4\Omega)}{2\Omega(2\Omega-\omega)^4}}, \tag{15}$$

where $v_0 = v_\perp/c$. The group travel-time (1), using (15), becomes

$$t(\omega) = \frac{1}{c}\int_h^0 \frac{\omega_p(2\Omega - \omega/2)}{\sqrt{2\Omega}(2\Omega-\omega)^{3/2}}\,dh - \frac{1}{c}\int_h^0 \frac{\gamma\omega_p^3 v_0^2(2\Omega - \omega/2)}{\sqrt{2\Omega}(2\Omega-\omega)^{7/2}}\,dh - \frac{1}{4c}\int_h^0 \frac{\gamma\omega_p^3 v_0^2(3\omega - 4\Omega)}{\sqrt{2\Omega}(2\Omega-\omega)^{7/2}}\,dh. \tag{16}$$

Integral (16) is the equation to use as a diagnostic technique through the observations of several pulses in the Sun travelling towards the solar surface. In order to evaluate the integral (16), we make the valid assumption that the density variation along the path of the pulses is negligibly small. Moreover, since we know that the rotation and presence of additional ions in the medium introduce the crossover frequencies at which there will be a wave change from left circularly polarized to right circularly polarized, and vice versa (Uberoi and Das, 1972) – and also since there is an abrupt change of group velocity at these crossover frequencies – we consider the path where the propagation of the pulse is completely an LCP wave. The rotation is assumed to vary linearly, and one of the best variations is probably

$$\Omega(h) = \Omega(0) + h\Omega'(0), \tag{17}$$

where $\Omega(0)$ is the rotation at the origin of the pulse and $\Omega'(0)$ is the gradient along the path h. Putting (17) in (16), the integral is evaluated approximately as

$$\begin{aligned} t(\omega) = {} & \frac{1}{c}\frac{\omega_p(0)[2\Omega(0)]^{1/2}}{2\Omega'(0)}\left[\left(\frac{1}{\Delta\omega(0)}\right)^{1/2} - \left(\frac{1}{\Delta\omega(h)}\right)^{1/2}\right] - \\ & - \frac{\gamma}{5c}\frac{\omega_p^3(0)v_0^2(2\Omega(0))^{1/2}}{[2\Omega'(0)]}\left[\left(\frac{1}{\Delta\omega(0)}\right)^{5/2} - \left(\frac{1}{\Delta\omega(h)}\right)^{5/2}\right] - \\ & - \frac{\gamma}{10c}\frac{\omega_p^3(0)v_0^2[2\Omega(0)]^{1/2}}{[2\Omega'(0)]}\left[\left(\frac{1}{\Delta\omega(0)}\right)^{5/2} - \left(\frac{1}{\Delta\omega(h)}\right)^{5/2}\right]. \end{aligned} \tag{18}$$

Expanding to first order in terms of the small quantity $\Delta\omega(0)$ [i.e., $\Delta\omega(0) \ll \Delta\omega(\mathrm{h})$], Equation (18), after straightforward mathematical manipulation, can be obtained as

$$t(\omega) = \frac{1}{c}\frac{\omega_p(0)[2\Omega(0)]^{1/2}}{2\Omega'(0)[\Delta\omega(0)]^{1/2}} - \frac{3\gamma}{10c}\frac{\omega_p^3(0)v_0[2\Omega(0)]^{1/2}}{2n'(0)[\Delta\omega(0)]^{5/2}}. \tag{19}$$

Finally, the equation for the group travel-time (Equation (1)) is modified to Equation (19), which is very similar to Equation (11) obtained by Das (1979) except for the contribution of the newly born ions. The method of obtaining the rotational frequency and the density of the medium is now well defined and will not be discussed in detail here. When charged particles moving along an equivalent magnetic field are subjected to the oscillation from a perpendicular electric field, the particles absorb energy from the field. When an electromagnetic field is produced in the medium, the absorption causes the wave to be damped out with time or with distance. To estimate the damping rates of the pulses, we consider the Maxwellian velocity distribution for the ionized particles and, following Stix (1962) and Das (1979), the dispersion relation for an LCP wave can be written in the form

$$n^2 = 1 + \frac{\omega_p^2}{2\Omega(2\Omega - \omega)} - \frac{1}{2}\frac{k^2 v_\perp^2 \omega_{pn}^2}{\omega^2(2\Omega - \omega)^2} - \frac{i\sqrt{\pi}}{\omega k v_{\mathrm{th}}}\omega_p^2 \exp(-z^2), \tag{20}$$

where $z = \Delta\omega/kv_{\text{th}}$, $\Delta\omega = 2\Omega - \omega$ and v_{th} is the thermal velocity of ionized particles. To find the temporal and spatial damping rates, we consider that either ω or k is complex. We consider first that k is real and ω is expressed as $\omega = \omega_r + i\omega_i$ with the condition $|\omega_i| \ll |\omega_r|$. Substituting the expression for ω into Equation (20), we obtain approximately the damping factor, ω_i, for the wave frequency equal to 2Ω, as

$$\omega_i = \frac{c\sqrt{\pi}[\Delta\omega(0)]^{5/2}}{\omega_p(0)[2\Omega(0)]^{1/2}} \frac{1}{v_{\text{th}}} \exp\left[-\eta(0)\right], \tag{21}$$

where

$$\eta(s) = \frac{c^2(\Delta\omega(s))^3}{2\Omega(s)\omega_p^2(s)v_{\text{th}}^2}.$$

We now define the fraction contribution, t_f of the newly born ion to the group travel-time $t(\omega)$ as

$$\theta \cdot t = \frac{3\gamma}{10c} \frac{\omega_p^3(0)v_0^2[2\Omega(0)]^{1/2}}{2\Omega'(0)[\Delta\omega(0)]^{5/2}}, \tag{22}$$

where $\theta = t_f/t$, and thus $\Delta\omega(0)$ is obtained as

$$\Delta\omega(0) = \left(\frac{3\gamma}{10\theta}\right)^{2/5} \frac{\left(\dfrac{\omega_p(0)}{c}\right)^{6/5} v_\perp^{4/5}[2\Omega(0)]^{1/5}}{[2\Omega'(0)]^{2/5}t^{2/5}}. \tag{23}$$

Combining (23) and (21), we find the damping rate ω_i to be given by

$$\omega_i = At^{-1} \exp\left[-Bt^{-6/5}\right], \tag{24}$$

where

$$A = \frac{3\gamma}{10\theta} \frac{\sqrt{\pi}}{v_{\text{th}}} \frac{v_0^2\omega_p^2(0)}{2\Omega'(0)}$$

and

$$B = \frac{\left(\dfrac{3\gamma}{10\theta}\right)^{6/5}\left(\dfrac{\omega_p(0)}{c}\right)^{8/5} v_\perp^{12/5}}{v_{\text{th}}^2[2\Omega(0)]^{2/5}[2\Omega'(0)]^{6/5}}.$$

Again, the damping rate of the particle is related to its velocity distribution function, and can be represented as

$$\omega_i \simeq -\frac{2c\pi^2[\Delta\omega(0)]^{5/2}}{\omega_p(0)[2\Omega(0)]} F(v), \tag{25}$$

where $F(v)$ is the velocity distribution of the particles. By use of (22) and (24) in (25), the velocity distribution function $F(v)$ can be obtained as

$$F(v) = \bar{c} \exp\left[-Bt^{-6/5}\right], \tag{26}$$

where

$$\bar{c} = \frac{1}{2c\pi^{3/2}} \frac{1}{v_{\text{th}}}.$$

Alternatively, we assume ω is real and $k = k_r + ik_i$ with $|k_i| \ll |k_r|$. As before, a separation of the real and imaginary parts of k in (20) gives

$$k_r = \frac{\omega}{c}\left[\frac{\omega_p^2}{2\Omega(2\Omega - \omega)} - \frac{k^2 v_\perp^2 \omega_{pn}^2}{2\omega^2(2\Omega - \omega)}\right]^{1/2}$$

and

$$k_i = \frac{\sqrt{\pi}\Delta\omega}{2\omega_p^2} \frac{1}{v_{\text{th}}} \exp\left[-\eta(s)\right], \tag{27}$$

where $\eta(s)$ is defined by (21). The total attenuation for the damping pulses is obtained by integrating k_i over the path of propagation, and is given by

$$\beta = 2\int k_i \,\mathrm{d}h = \int_{\Delta\omega(h)}^{\Delta\omega(0)} \frac{\sqrt{\pi}\,\Delta\omega}{2v_{\text{th}}2\Omega'(0)} \frac{1}{\omega_p^2} \exp\left[-\eta(s)\right] \mathrm{d}(\Delta\omega). \tag{28}$$

The integral is evaluated by expanding to the first order in terms of the small quantity $\Delta\omega(0)$ [i.e., $\Delta\omega(0) \ll \Delta\omega(h)$], and we have

$$\beta = \frac{\sqrt{\pi}}{3c\omega_p^2(0)} \left(\frac{\Delta\omega(0)}{2\Omega(0)}\right)^{1/2} \left(\frac{2\Omega(0)}{2\Omega'(0)}\right) \frac{\exp\left[-\eta(0)\right]}{\sqrt{\eta(0)}}. \tag{29}$$

Incorporating the expression for $\Delta\omega(0)$ obtained in Equation (23), the total attenuation takes the form

$$\beta = A' t^{2/5} \exp\left[-B' t^{-6/5}\right] \tag{30}$$

where

$$A' = \frac{v_{\text{th}}\sqrt{\pi}[2\Omega(0)]^{4/5}\left[\dfrac{10c\theta}{3\gamma}\right]^{2/5}}{3[\omega_p(0)]^{11/5}[2\Omega'(0)]^{3/5}[v_0]^{4/5}}$$

and

$$B' = B \text{ [given in (24)]}.$$

From (30), the variation of the wave magnetic field B_1 with time can be expressed as

$$|B_1| \propto \exp\left[-\beta\right]. \tag{31}$$

The total attenuation of the wave can be related with the electric field and the magnetic field on the assumption that the wave-energy of the pulses along the equivalent magnetic field should be constant per unit bandwidth of the pulses. This leads to an expression of the Poynting flux, P, in the form

$$P \propto \left|\frac{\mathrm{d}\omega}{\mathrm{d}t}\right|;$$

and, using (19), we find that

$$P \propto \frac{\mathrm{d}}{\mathrm{d}t}(\Delta\omega)$$

or

$$|E_1||B_1| \propto (\Delta\omega)^{3/2} \propto t^{-3}, \tag{32}$$

where t is defined by Equation (19); B_1 is the equivalent magnetic field introduced by rotation; and E_1 is the electric field. Again, for the constant Poynting flux, the ratio between the magnetic field and the electric field is equal to the refractive index, and thus we have

$$\frac{|E_1|}{|B_1|} \propto (\Delta\omega)^{1/2} \propto t^{-1}. \tag{33}$$

Combining (32) and (33), we find the magnetic field B_1 to be of the form

$$|B_1| \propto (\Delta\omega)^{1/2} \propto t^{-1} \tag{34}$$

and

$$|E_1| \propto (\Delta\omega) \propto t^{-2}.$$

Using (23) and (31), we find the magnetic field to vary as

$$|B_1| \propto \frac{1}{t^{1/5}} \exp\left[-A' t^{2/5} \exp\left(-B' t^{-6/5}\right)\right], \tag{35}$$

where A' and B' are as previously stated in (30). Thus, Equation (35) gives the relative wave amplitude of the equivalent magnetic field as a function of group travel-time, rotational frequency and density of the medium. The determination of the group travel-time for the different pulses determines the rotational frequency and the density. The inclusion of damping effects gives the temporal and spatial damping from which the velocity distribution of ionized particles and the magnetic field can be obtained.

So far, we have discussed a method for determining the physical parameters of the Sun by observing the pulse propagating from its source to the solar surface. Dispersion effects play some part in the considered model by an equivalent magnetic field, but the presence of an applied magnetic field does not change the mathematical development as the treatment can run in parallel if we simply change the rotational frequency 2Ω by an equivalent rotation frequency $2\bar{\Omega} = \omega_{c\alpha} + 2\Omega$, where $\omega_{c\alpha}$ is the cyclotron frequency of the α-type particles. In the case of a non-uniform magnetic field or non-uniform rotation, it is necessary to have an expansion similar to that for rotational frequency, and the technique needs some mathematical manipulation if we are to determine the group travel-time and thus to diagnose the physical parameters.

3. Conclusion

In this paper, we have developed a mathematical model for diagnosing the physical parameters of an ionized medium rotating with uniform angular velocity. However, the chosen model is idealistic, and needs further analysis.

Acknowledgements

The author gratefully acknowledges the kind hospitality of Dr M. K. V. Bappu, Director, Indian Institute of Astrophysics, Bangalore, India. The work has been supported by C.S.I.R. Government of India.

References

Alfvén, H.: 1950, *Cosmical Electrodynamics*, Chap. V, Clarendon Press, Oxford.
Chandrasekhar, S.: 1953a, George Darwin Lecture, *Monthly Notices Roy. Astron. Soc.* **116**, 667.
Chandrasekhar, S.: 1953b, *Proc. Roy. Soc.* **A217**, 306.
Chandrasekhar, S.: 1953c, *Astrophys. J.* **118**, 7.
Das, G. C.: 1979, Under Communication.
Das, G. C.: 1979a, *Astrophys. Space Sci.* **66**, 505.
Lehnert, B.: 1954, *Astrophys. J.* **119**, 647.
Montgomery, D. and Tidman, D.: 1964, *Plasma Kinetic Theory*, McGraw-Hill, New York.
Schlesinger, F.: 1911, *Monthly Notices Roy. Astron. Soc.* **71**, 719.
Stix, T. H.: 1962, *The Theory of Plasma Waves*, McGraw-Hill, New York.
Uberoi, C. and Das, G. C.: 1970, *Plasma Phys.* **12**, 661.
Uberoi, C. and Das, G. C.: 1972, *Plasma Phys.* **14**, 201.