# RADIO EMISSION FROM AN INTENSE ELECTROMAGNETIC BEAM–PLASMA SYSTEM

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(Received 18 January, 1979)

**Abstract.** Interaction of a high-intensity and low-frequency electromagnetic beam with a plasma is investigated. The dynamics of plasma particles under this strong electromagnetic field is calculated without using the perturbation procedures. The intensity of radio radiation from such particles is calculated and the results are applied to the extragalactic radio sources.

## 1. Introduction

The laser–plasma interaction has received much attention in the last few years due to its promising role in the controlled thermonuclear fusion research (Krishan *et al.*, 1976). A laser or, in other words, a monochromatic high-intensity electromagnetic beam impinges on a plasma and deposits a fraction of its energy on the particles. This system is capable of supporting various kinds of electrostatic and electromagnetic instabilities which may further heat the plasma (Brueckner and Jorna, 1974). In these circumstances, the plasma radiates in the form of X-rays or radio waves. The objective of this paper is to investigate the particle motion in the presence of such intense fields which forbid the use of the usual perturbation procedures and look for the collective competence of these particles in the radio emission. To study the emission mechanism, the simple theoretical picture of the system is where a high-intensity low-frequency electromagnetic beam impinges upon an ionized medium consisting of electrons and protons. The deposition of beam energy occurs at the surface where the plasma frequency is equal to the frequency of the electromagnetic beam. The general dispersion relation for the propagation of electromagnetic modes in an electromagnetic beam–plasma system has been derived in Section 2. Section 3 deals with the non-linear saturation of the electromagnetic instability near radio frequencies and thus the total energy content of the radio source is estimated.

## 2. Dispersion Relation

Let the vector potential associated with the low-frequency electromagnetic beam be given (in the dipole approximation) by

$$A(t) = A_0[\hat{e}_x \cos \omega t + \hat{e}_y \sin \omega t], \tag{1}$$

where $\hat{e}_x$ and $\hat{e}_y$ are the unit vectors and $\omega$ is the frequency of the electromagnetic

*Astrophysics and Space Science* **63** (1979) 209–214. 0004–640X/79/0631–0209$00.90

beam. The Hamiltonian for a particle in the presence of this field can be written as

$$H = \frac{1}{2m}\left[\mathbf{p} - \frac{e}{c}\,\mathbf{A}(t)\right]^2, \tag{2}$$

where $m$ is the mass of the electron, $e$ the electric charge, and $c$ the speed of light. The motion of the electron can be described by solving the Schrödinger equation for the Hamiltonian given by Equation (2). The wave function of the electron is found to be

$$\psi = \frac{1}{\sqrt{V}} \sum_{n=-\infty}^{\infty} J_n(\beta_{p_\perp}) \exp\left[-\frac{i}{\hbar} E_{n,p} t + \frac{i}{\hbar}\mathbf{p}\cdot\mathbf{x} - i\,\delta_{p_\perp} n + i\beta_{p_\perp} \sin \delta_{p_\perp}\right], \tag{3}$$

where $V$ is the volume,

$$E_{n,p} = \left[\frac{p^2}{2m} + \frac{e^2 A_0^2}{2mc^2} - \hbar n\omega\right], \qquad \beta_{p_\perp} = \frac{eA_0 p_\perp}{mc\hbar\omega}, \qquad p_\perp^2 = p_x^2 + p_y^2, \tag{4}$$

$J_n(\beta_{p\perp})$ being the Bessel function, and $\tan \delta_{p_\perp} = p_y/p_x$.

The transverse dielectric function can be calculated by using the many body techniques given by Harris (1969). One finds that

$$\varepsilon(q, \Omega) = 1 - \sum_{s,l} \frac{\omega_{psf}^2}{\Omega^2}\left[1 + \frac{q^2 V_{\text{ths}}^2}{(\Omega - l\omega + qu_s)^2} + \frac{q^2 V_{\text{ths}}^2}{(\Omega - l\omega - qu_s)^2} + \frac{l\omega}{2}\left\{\frac{1}{(\Omega - l\omega + qu_s)} + \frac{1}{(\Omega - l\omega - qu_s)}\right\}\right], \tag{5}$$

where $s = e, i$ particle species index; $\omega_{psf}^2 = \frac{1}{2}\omega_{ps}^2 J_{N_s - l}^2(z_s)$; $u_s = eA_0/\sqrt{2}\, m_s c$; $\omega_{ps}^2 = 4\pi n_s e^2/m_s$; $n_s$ is particle density; $\mathbf{q} = (q, 0, 0)$; $V_{\text{ths}}$ is thermal velocity; and $N_s \simeq z_s \equiv e^2 A_0^2/m_s c^2 \hbar\omega \gg 1$.

In order to arrive at Equation (5), the asymptotic behaviour of the Bessel functions for arguments much greater than unity has been utilized. The maximum value of the Bessel function is obtained when the order $N$ is approximately equal to the argument $z$. To obtain the solution of Equation (5), without falling into the drudgery of numerical computation, the method of an independent stream model is used. The details of this procedure are given in Krishan *et al.* (1978). The dispersion relation for the electromagnetic modes supported by the ionic component of the plasma system is given as

$$\varepsilon_i(q_i\Omega_i) = 1 - \sum_{l_i} \frac{\omega_{pif}^2}{\Omega_i^2}\left[1 + \frac{q_i^2 V_{\text{th}i}^2}{(\Omega_i - l_i\omega - q_i u_i)^2} + \frac{q_i^2 V_{\text{th}i}^2}{(\Omega_i - l_i\omega + q_i u_i)^2} + \frac{l_i\omega}{2(\Omega_i - l_i\omega + q_i u_i)} + \frac{l_i\omega}{2(\Omega_i - l_i\omega - q_i u_i)}\right] = \frac{q_i^2 c^2}{\Omega_i^2}. \tag{6}$$

For $qu_i \ll l_i\omega$ and $\Omega_i \simeq l_i\omega + qu_i$ the ions support a positive energy mode under the condition

$$F_i = \frac{1}{\Omega}\frac{\partial}{\partial\Omega}[\Omega^2\varepsilon_i(q_i\Omega)]\,|_{\Omega=\Omega_i} =$$

$$= \frac{2q_i^2c^2}{\Omega_i^2} + \frac{2\omega_{pif}^2}{\Omega_i^2}\left[1 + \frac{2q_i^2V_{\text{thi}}^2}{(q_iu_i)^2} + \frac{l_i\omega\Omega_i}{(q_iu_i)^2}\right] > 0. \tag{7}$$

Similarly, the electromagnetic modes of the electronic part of the plasma is given by

$$\varepsilon_e(q_e\Omega_e) = 1 - \sum_{l_e}\left[\frac{q^2V_{\text{the}}^2}{(\Omega_e - l_e\omega + q_eu_e)^2} + \frac{q^2V_{\text{the}}^2}{(\Omega_e - l_e\omega - q_eu_e)^2} + \right.$$

$$\left. + \frac{l_e\omega}{2(\Omega_e - l_e\omega + q_eu_e)} + \frac{l_e\omega}{2(\Omega_e - l_e\omega - q_eu_e)}\right] \times$$

$$\times \frac{\omega_{pef}^2}{\Omega_e^2} - \frac{\omega_{pef}^2}{\Omega_e^2} = \frac{q_e^2c^2}{\Omega_e^2}. \tag{8}$$

For $\Omega_e \simeq l_e\omega + q_eu_e - \delta_e$ one finds the value of $\delta_e$ to be

$$\delta_e^2 \simeq \frac{\omega_{pef}^2q_e^2V_{\text{the}}^2}{\left(1 - \dfrac{q_e^2c^2}{\Omega_e^2}\right)\Omega_e^2}. \tag{9}$$

Furthermore, $\Omega_e$ becomes a negative energy mode if

$$F_e = \frac{1}{\Omega}\frac{\partial}{\partial\Omega}\left[\Omega^2\varepsilon_e(q_e\Omega)\right]|_{\Omega=\Omega_e} =$$

$$= \frac{2q_e^2c^2}{\Omega_e^2} + \frac{2\omega_{pef}^2}{\Omega_e^2}\left[1 + \frac{q_e^2V_{\text{the}}^2}{\delta_e^2} + \frac{l_e\omega}{\delta_e}\frac{\Omega_e}{\delta_e} - \frac{l_e\omega}{\delta_e} - \frac{q^2V_{\text{the}}^2\Omega_e}{\delta_e^3}\right] < 0. \tag{10}$$

The conservation laws for the interaction of the positive energy mode $\Omega_i$ and the negative energy mode $\Omega_e$ are

$$\text{Sgn } F_e|\Omega_e| + \text{Sgn } F_i|\Omega_i| = 0 \tag{11}$$

and

$$\mathbf{q}_e + \mathbf{q}_i = 0.$$

The linear growth rate $\gamma_0$ of the electromagnetic instability at $|\Omega_e| = |\Omega_i| = \omega_0 \simeq l_i\omega$ can be calculated by the mode–mode coupling. One finds that

$$\gamma_0 = \frac{\omega_{pe}^2}{2\omega_0}\frac{1}{\sqrt{|F_i||F_e|}} = \frac{\omega_{pe}^2}{2\omega_0}\sqrt{\frac{\delta_e}{2\omega_0}}\frac{1}{\sqrt{2x}}, \tag{12}$$

where

$$2x = \frac{2q^2c^2}{\omega_0^2} < 1.$$

### 3. Nonlinear Saturation

As the electromagnetic wave $\omega_0$ grows with growth rate $\gamma_0$, the trajectory of the plasma particles begins to get affected in the field of the instability. As a result, the modification in the energy of the particles inhibits the growth rate and thus saturates the instability. More formally this is known as the orbit perturbation theory.

The process contributing to the energy renormalization of the particles consists of two wave-particle vertices and is shown in Figure 1. The total energy of the particle in the presence of the electromagnetic instability $\omega_0$ is found to be

$$E'_{np} = E_{n,p} + \Delta E_{n,p},$$

$$\Delta E_{n,p} = \sum_{l_s,\mathbf{q}'} \frac{p_y^2 J^2_{N_s - l_s}(z_s) J^2_{N_s}(z_s) \omega^4_{Bs}(q')}{4m\omega_0^2(q')} \times$$

$$\times \left[\frac{1}{(\omega_0(q') - Vq' - l_s\omega)^2} + \frac{1}{(\omega_0(q') - Vq' + l_s\omega)^2}\right]. \tag{13}$$

If we use this modified value of the particle energy, the nonlinear dispersion relation turns out to be

$$1 - \sum_{l_s, s} \frac{\omega^2_{psf}}{\Omega_q^2}\left[1 + \frac{q^2 V^2_{\text{ths}}}{(\Omega_q - l_s\omega - qu_s - A_s)^2} + \frac{q^2 V^2_{\text{ths}}}{(\Omega_q - l_s\omega + qu_s - A'_s)^2} + \right.$$

$$\left. + \frac{l_s\omega}{2(\Omega_q - l_s\omega - qu_s - A_s)} + \frac{l_s\omega}{2(\Omega_q - l_s\omega + qu_s - A'_s)}\right] =$$

$$= \frac{q^2c^2}{\Omega_q^2}, \tag{14}$$

where

$$A_s = \sum_{l_s, q'} \frac{\omega^2_{Bs}(q')}{\omega_0^2(q')} \frac{q'^2 u_s^2 J^2_{N_s}(z_s) J^2_{N_s - l_s}(z_s)}{(\omega_0(q') - q'u_s - l_s\omega)^3}$$

$$A'_s = A_s(-u_s),$$

and

$$\omega_{Bs}(q') = \left[\frac{e^2E^2(q')q'^2}{m_s^2}\right]^{1/4}$$

is the bounce frequency; while $E(q')$ is the amplitude of the field associated with the instability.

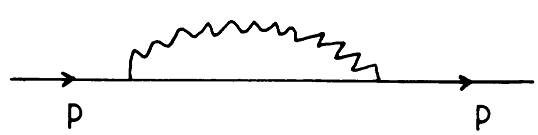


Fig. 1. Non-resonant wave–particle interactioncontributing to the self-energy correction.

The solution of Equation (14) can be found in terms of the following expansions:

$$\begin{aligned}\Omega_q &= \Omega_0 + \Omega_1, \\ \Omega_0 &= \omega_0 + i\gamma_0, \\ \Omega_1 &= \omega_1 + i\gamma_1.\end{aligned} \tag{15}$$

If $\Omega_1 \ll \Omega_0$, one finds the expression for the nonlinear growth rate $\gamma_1$ to be

$$\gamma_1 = -3\,\frac{\omega_{Be}^4(q)}{\delta_e^4}\frac{q^2 V_{\text{the}}^2}{\omega_0^2}\,\gamma_0 J_{N_e - l_e}^2(z_e) J_{N_e}^2(z_e) \times \times \left(1 + \frac{9}{2}\frac{\gamma_0^2}{\delta_e^2}\right) \Big/ \left(1 + \frac{9\gamma_0^2}{\delta_e^2}\,x\right). \tag{16}$$

The saturation amplitude is determined by a requirement that the nonlinear damping rate $\gamma_1$ compensates for the linear growth rate $\gamma_0$ or $|\gamma_1| = |\gamma_0|$, which gives

$$\frac{E^2}{8\pi} = \frac{m^2 V_{\text{the}}^2}{24\pi e^2}\frac{\omega_{pe}^4}{\omega_0^2}\frac{1}{(1-x)^2}\frac{J_{N_e - l_e}^2(z_e)}{J_{N_e}^2(z_e)}. \tag{17}$$

## APPLICATION TO EXTRAGALACTIC RADIO SOURCES

It is well known that the extended extragalactic double radio sources derive their energy from the centrally placed nuclear regions which may contain a large number of galaxies, pulsars or quasars. The origin of energy in galactic nuclei in turn is believed to be gravitational in nature. The carriers of this energy emanating from the nuclei may be relativistic particles or low-frequency electromagnetic waves, or both. Rees (1971) assumed this flow of energy to be mainly in the form of high-intensity and low-frequency electromagnetic waves which would be beamed due to the self-focusing effects. The details of the formation of two oppositely directed beams of relativistic fluid and of the conversion of the bulk energy contained in the fluid into random particle energy has been considered by Blandford and Rees (1974). The radiation mechanism of these radio sources is believed to be a combination of the synchrotron and the Compton processes. Although this model has been successful in explaining many of the observed features of the extended extragalactic radio sources, the details of the transfer of energy from the low-frequency electromagnetic beam to the plasma particles which then emit radio waves, have not been formulated. As is obvious, the theoretical system considered here can be identified with that for extragalactic radio sources. To obtain an estimate of the radio energy content of these sources, let us try to put some numbers in Equation (17).

For $\omega \sim \omega_{pe} \sim 10^3\ \text{s}^{-1}$, $u/c \sim 0.1$, $V_{\text{the}} \sim 10^8\ \text{cm s}^{-1}$,

$$x = \frac{q^2c^2}{\omega_0^2} = \left(\frac{l_i}{l_e} - 1\right)^2 \frac{c^2}{u^2} \qquad \text{from Equation (11)}$$

$$l_i = 5\,499\,998, \qquad l_e = 5\,000\,000,$$

one finds that

$$\omega_0 \sim l_i\omega \sim 5 \times 10^9 \text{ s}^{-1}$$

and

$$\frac{E^2}{8\pi} = 10^{-17} \frac{J^2_{N_e - l_e}(z_e)}{J^2_{N_e}(z_e)} \text{ erg cm}^{-3}.$$

The energy density associated with the low-frequency electromagnetic beam is $E_0^2/8\pi \sim 10^{-10}$ erg cm$^{-3}$. One observes that the fraction of the energy density of the beam converted into radio intensity is about $10^{-7}$. If the volume of the radio source is $10^{75}$ cm$^3$, then the total energy content of the radio source is $10^{58}$ erg since the factor $J^2_{N_e - l_e}(z_e)/J^2_{N_e}(z_e) \sim 1$ for $l_e \ll N_e$.

The choice of $l_i$ and $l_e$, in the particular way they have been written, emphasizes their integral nature and the phenomenon of resonance excitation.

## 4. Conclusions

The model requiring a continuous energy supply from the nucleus of the parent object, of extragalactic extended radio sources – given first by Rees (1971) – has been reinvestigated. In particular, the dynamical effects of a high-intensity electromagnetic beam on the particles has been worked out in detail. The condition $\Omega_{ce}/\omega \geqslant 1$ emphasized by Rees (1971) (where $\Omega_{ce}$ is the gyrofrequency of an electron in the equivalent magnetic field given by $B^2_{eq}/8\pi = E_0^2/8\pi$) is precluded in the treatment given here in the form of the argument of the Bessel function being very much greater than unity. Furthermore, a collective mechanism which may be comparable to the multiphoton excitation process for the production of radio waves is suggested, which seems to furnish a fairly good estimate of the energy content of the extragalactic radio sources.

## References


Blandford, R. D. and Rees, M. J.: 1974, *Monthly Notices Roy. Astron. Soc.* **169**, 395.
Brueckner, K. A. and Jorna, S.: 1974, *Rev. Mod. Phys.* **46**, 325.
Harris, E. G.: 1969 in A. Simon and W. B. Thompson (eds.), *Advances in Plasma Physics*, Vol. 3, Interscience, New York.
Krishan, V., Krishan, S., Sinha, K. P. and Ganguli, A.: 1975, *App. Phys. Letters* **29**, 90.
Krishan, V., Sinha, K. P. and Krishan, S.: 1978, *Astrophys. Space Sci.* **53**, 13.
Rees, M. J.: 1971, *Nature* **229**, 312.