

FREQUENCY RESPONSE OF MAGNETIC FLUX SHEATHS

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(Received 10 June; in revised form 11 December, 1978)

Abstract. When a sound wave is incident on a magnetic flux sheath, it causes fluctuations in the mean magnetic field of the sheath. We have calculated the space-average of the longitudinal component of these fluctuations and plotted this against the frequency of the incident sound wave. The main result is the presence of local maxima and minima in the response curve. If such maxima and minima could be detected in any actual observation then these would provide an estimate of the thickness of these magnetic structures.

1. Introduction

The coexistence of magnetic inhomogeneities and magneto-fluid waves in the solar atmosphere raises the question of their mutual interaction. Cram and Wilson (1975) provided some preliminary analysis on the basis of linear theory. From their results, it can be reasoned out that the forms of the perturbations of the various fluid-dynamical variables within the magnetic structures do depend on the frequency of the incident wave. In other words, for a given angle of incidence, there exists a definite frequency response of the magnetic structure, depending in turn upon the magnetic field intensity and the thickness of the structure. The aim of the present work is to (i) study how the shape of the response curve depends on a few of the above mentioned parameters, and (ii) suggest a possible way of estimating the thicknesses of very small magnetic structures. The particular fluid-dynamical variable chosen for this study was the mean value of the linearised magnetic fluctuations in the direction of the magnetic field. The space average of such fluctuations, when plotted against the frequency of the incident sound wave, showed local maxima and minima in the response curve. The separation between successive maxima contains information about the thickness of the magnetic structure.

At the centre of the solar disc, these fluctuations associated with the vertical structures in the solar atmosphere, will manifest themselves as changes in the longitudinal component of the magnetic field and can be detected with the present day magnetographs. The frequencies of maximum response, obtained from such observations, should enable us to estimate the thickness of the structures.

2. The Mean Magnetic Fluctuation in a Magnetic Flux Sheath

The zero order magnetic field in the structure is assumed to point in the y -direction and its intensity to vary along the x -direction. The profile of this variation is as represented in Figure 1. The details of the modes that can propagate in such a structure are given by Cram and Wilson. From their Equation (43), it follows that the

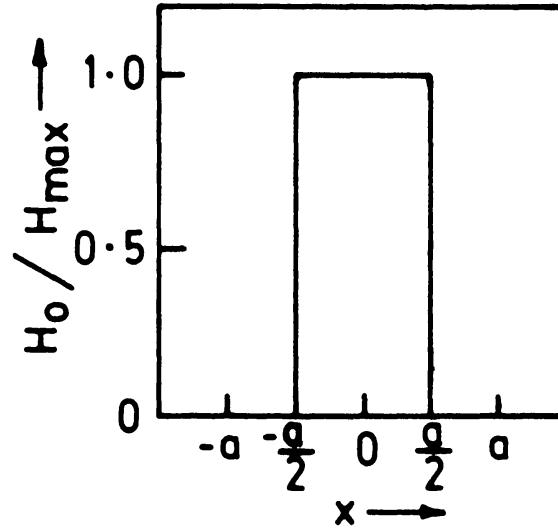


Fig. 1. Profile of the variation of magnetic field intensity in the structure. Here a is the thickness of the flux sheath.

amplitude of the velocity component along the magnetic field is given by

$$V = (Q_{r2} e^{ik_{x2}x} + Q_{t2} e^{-ik_{x2}x}) \left(\frac{C^2 - A^2}{C} \right) \cos \beta, \quad (1)$$

where Q_{r2} and Q_{t2} are the reflection and transmission coefficients, C is the phase velocity of the fast magnetosonic mode, A is the Alfvén velocity, β is the angle subtended by the transmitted ray to the interface and k_{x2} is the horizontal wave number. All these quantities are associated with the region of non-zero magnetic field and are calculated according to the formulae given by Cram and Wilson (1975). The amplitude of V is normalised to the incident wave amplitude, $I(\alpha, \nu)$ where ν is the frequency of incident sound wave. Thus V can be finally written as

$$V = (Q_{r2} e^{ik_{x2}x} + Q_{t2} e^{-ik_{x2}x}) \left(\frac{C^2 - A^2}{C} \right) \cos \beta I(\alpha, \nu). \quad (2)$$

The linearised equation for the magnetic fluctuation \mathbf{b} is

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}_0), \quad (3)$$

where \mathbf{V} is the fluid velocity perturbation and \mathbf{B}_0 is the zero order magnetic field. We know that

$$\mathbf{B}_0 = (0, B_0, 0). \quad (4)$$

For the perturbation of the field intensity we assume the form

$$\mathbf{b} = \hat{\mathbf{b}} \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t). \quad (5)$$

From the condition of phase-matching at the interface (Cram and Wilson, 1975), the

component of \mathbf{k} along the interface must be the same on either side of the boundary. If k is the total magnitude of the incident wave vector, then its component along the interface is given by $k \cos \alpha$, where α is the angle between the incident ray and the interface. Hence the component of \mathbf{k} normal to the interface within the sheath can be given by

$$k_{x2}/k_{y2} = \tan \beta . \tag{6}$$

Thus

$$k_{x2} = k_{y2} \tan \beta = k \cos \alpha \tan \beta . \tag{7}$$

Therefore, within the sheath, we have

$$\mathbf{k} = (k \cos \alpha \tan \beta) k \cos \alpha, 0) . \tag{8}$$

Substituting Equations (4), (5), and (8) in Equation (3), we have, after some algebra,

$$\hat{b}_y = B_0(\sin^2 \alpha / \cos \alpha), (V/S) , \tag{9}$$

where V is given by Equation (2).

We are interested in those structures which are below the limits of telescopic resolution of present day standards (like, for example, the structures mentioned in Stenflo (1976) and Tarbell and Title (1977)). We, therefore, integrate over x to get the net fluctuation in magnetic flux, ϕ , as

$$\phi = 2B_0(Q_{t2} + Q_{r2})(1 - M \cos^2 \alpha) \sin^2 \alpha I(\alpha, \nu) \sin(k_{x2}a/2)/k_{x2} . \tag{10}$$

Since we can expect the sound waves to be incident at different angles, we integrate (10) over α to obtain

$$\langle \phi \rangle = \frac{2}{\pi} \int_0^{\pi/2} \phi \, d\alpha . \tag{11}$$

For sake of illustration let us now choose two different forms for $I(\alpha, \nu)$:

Case (i):

$$I(\alpha, \nu) = Kf(\nu) , \tag{12}$$

where K is some constant and $f(\nu)$ is the frequency spectrum of the incident sound waves. Equation (11) corresponds to isotropic propagation. Such a choice was prompted by the work of Stein (1967) where he obtains isotropic emission of sound waves at frequencies far above the acoustic cut-off frequency of the stratified atmosphere. Figure 2 shows the result of including (12) in (11) *sans* $f(\nu)$ for three different values of M . The means of eliminating $f(\nu)$ will be discussed later. In the figure, the abscissae are the dimensionless frequencies ν/ν_c where ν_c is a characteristic acoustic frequency obtained by dividing the sound speed S by the thickness a of the flux sheath. For a typical value of S (7 km s^{-1}) at the photosphere ν_c will be $\approx 0.07 \text{ Hz}$ for $a = 100 \text{ km}$ and will be $\approx 0.007 \text{ Hz}$ for $a = 1000 \text{ km}$. The ordinates are

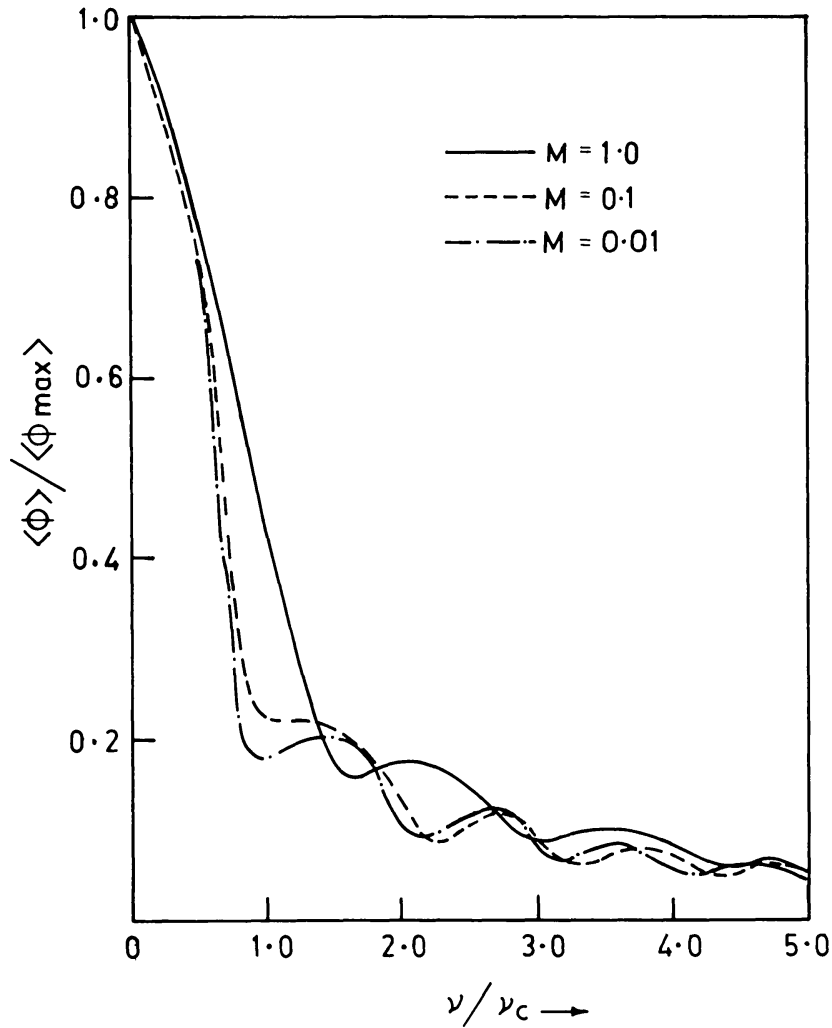


Fig. 2. Frequency response with $I(\alpha, \nu)$ given by Equation (12) for three different values of the field intensity; $M = 1.0, 0.1,$ and 0.01 .

values of $\langle \phi \rangle$ normalised to $\langle \phi_{\max} \rangle$ the maximum amplitude encountered in a particular situation.

Case (ii):

$$I(\alpha, \nu) = \begin{cases} 1, & \alpha \leq 30^\circ \\ 0, & \alpha > 30^\circ \end{cases} \quad \text{for } \nu \leq \nu_1,$$

and

$$I(\alpha, \nu) = \begin{cases} 0, & \alpha < 30^\circ \\ 1, & \alpha \geq 30^\circ \end{cases} \quad \text{for } \nu > \nu_1.$$

(13)

This choice of $I(\alpha, \nu)$ as a two-dimensional π -function was prompted by the work of Deubner (1976) where higher frequencies were inferred to be preferentially horizontal. Figure 3 shows the result of including (13) in Equation (11) with ν_1/ν_c chosen arbitrarily as 0.5.

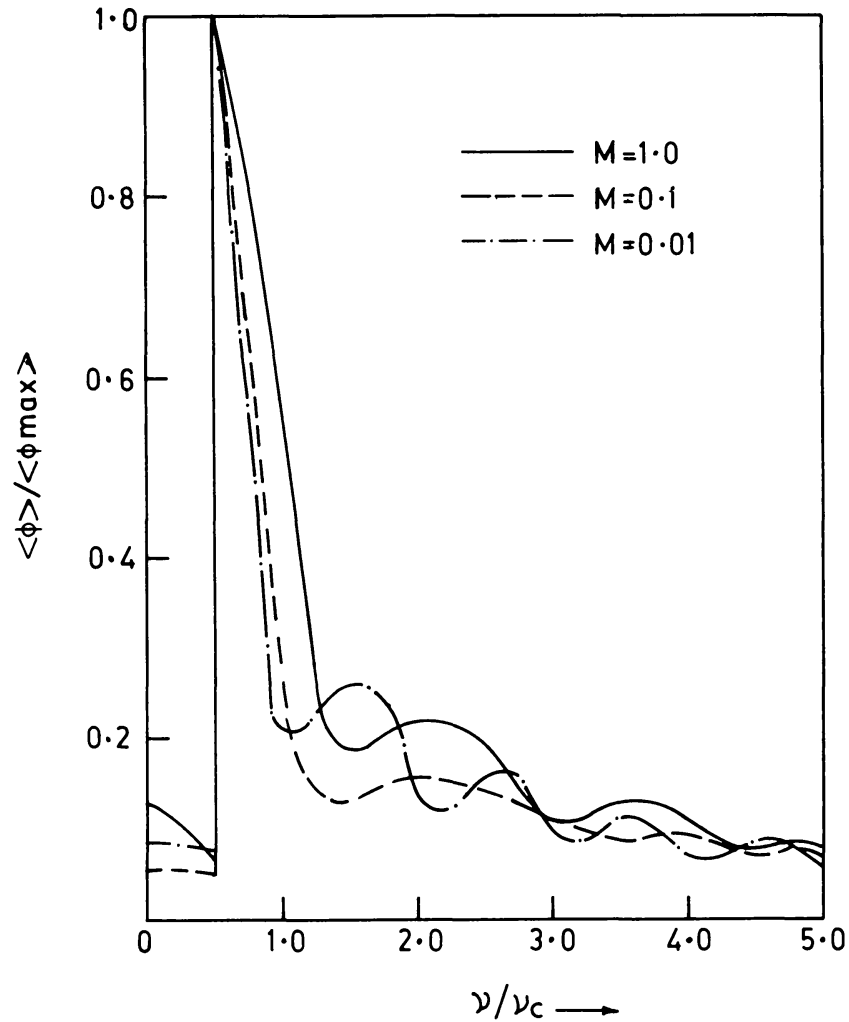


Fig. 3. Frequency response with $I(\alpha, \nu)$ given by Equation (13) for three different values of the field intensity; $M = 1.0, 0.1,$ and 0.01 .

3. Estimation of Structure Thickness

Figures 2 and 3 show local maxima and minima in the response curve. The abrupt behaviour at $\nu/\nu_c = 0.5$ in Figure 3 is solely due to the abrupt nature of $I(\alpha, \nu)$. The difference between successive maxima $\Delta\nu_m/\nu_c$ is given in Table I. It is seen that, on the average,

$$\Delta\nu_m/\nu_c = 1.2. \quad (14)$$

These results lead us to believe that it may be worthwhile to attempt the detection of local maxima in the magnetic fluctuations from observations. If the experiment yields a consistent difference between successive maxima, then we could estimate the thickness from Equation (14) as

$$a \approx S/\Delta\nu_m, \quad (15)$$

where we have dropped the numerical factor of order unity.

TABLE I
Dimensionless frequencies of local maxima and the differences
between successive maxima

$I(\nu, \alpha)$	M	ν_m/ν_c	$\Delta\nu_m/\nu_c$	
From Equation (12)	1.0	2.1	1.5	
		3.6	1.2	
		4.8		
	0.1		2.8	1.0
			3.8	1.0
			4.8	
	0.01		1.4	1.3
			2.7	0.8
			3.5	1.1
		4.6		
From Equation (13)	1.0	2.1	1.5	
		3.6	1.2	
		4.8		
	0.1		2.0	1.9
			3.9	1.0
			4.9	
	0.01		1.6	1.0
			2.6	0.9
			3.5	1.1
		4.6		

Mean $\Delta\nu_m/\nu_c = 1.2$.

4. Limitations

One should, however, bear in mind the following limitations while attempting to use the above treatment to estimate the sizes of tiny magnetic structures.

(1) If many structures with a rather continuous distribution of sizes fall within the slit of the magnetograph, the maxima will be smoothed out. On the other hand, if there is a dominant size in the distribution, this will show up in the observations.

(2) The incident wave amplitude may itself be a function of frequency (for example, $f(\nu)$ in Equation (12)) and may contribute its own shape to the response curve. This effect can be seen in the observations of Severny (1971) and Tanenbaum *et al.* (1971) where they have observed five minute oscillations in the magnetic fluctuations. This is due to the large power present in the incident beam at the period of five minutes. The way out would be to have simultaneous velocity measurements of the nearly non-magnetic regions. Even here complications can arise. The line of sight velocity oscillations sometimes yield a structure due to the inhomogeneity of the line contributing region (Deubner, 1976). This effect will not be seen, however, for the magnetic fluctuations since they are weighted by a $\sin^2 \alpha$ -factor in favour of

horizontally propagating waves. Hence one should be extremely careful while comparing the magnetic and velocity observations.

(3) The present work considers only two dimensional structures with a magnetic field that does not vary in the line of sight. It may be necessary to include variation in this direction to make the treatment less removed from reality. However, any horizontal structuring of the field will certainly introduce a factor behaving like $\sin(k_{x2}a/2)/k_{x2}$ because of the integration over this direction. Hence one can expect maxima and minima in the response curve even for more realistic models of the zero order magnetic field.

(4) It has already been shown how two different forms of $I(\alpha, \nu)$ can yield very different response curves. A particular form of $I(\alpha, \nu)$ might sometimes drastically reduce the detectability of the maxima.

In spite of all the above limitations one could attempt to look for magnetic fluctuations yielding local maxima and minima in the frequency spectrum. If indeed such a structure is obtained, and if we can eliminate the afore-mentioned sources for the structure, then it could be reasonably interpreted as due to the presence of magnetic elements, with a size approximately equal to $S/\Delta\nu_m$.

5. Summary

We have seen that when sound waves are incident on a magnetic flux sheath, they cause fluctuations in the zero order magnetic field. If these sheaths are thinner than the slit of the instrument used to observe them, then the fluctuations will be averaged out in space. Such an integrated amplitude of the fluctuations would show maxima and minima in the frequency response. This behaviour can be used to estimate the sizes of thin magnetic structures by looking for the fluctuations in their field intensities and locating the frequencies of maximum response.

Acknowledgements

The author gratefully acknowledges the many valuable suggestions given by Dr K. R. Sivaraman and Dr M. H. Gokhale. The author also thanks Dr L. E. Cram for his comments on the manuscript.

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