ENERGY EXCHANGE DURING COLLISION OF A PAIR OF DISC-SPHERE GALAXIES

P. M. S. NAMBOODIRI and R. K. KOCHHAR
Indian Institute of Astrophysics, Bangalore, India

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Abstract. We have considered the collision of a pair of disc-sphere galaxies under the impulsive approximation to study the variation of the energy change of the disc due to the passage of a spherical galaxy. Various collision parameters like the impact parameter the orientation of the disc galaxy and the relative velocity of the perturber were considered. It is shown that for small impact parameters and high relative velocities, a slightly inclined position of the disc galaxy produces maximum energy exchange. The energy exchange is observed to be asymmetric with respect to the direction of motion of the perturber.

1. Introduction

During collision of two galaxies, the tidal force due to the perturber accelerates the particles in the test system which in turn increases the total internal energy of the test system. The fraction $\Delta U/|U|$ where U is the initial internal energy of the test galaxy and ΔU is its increment during a collision gives a measure of the damage done to the test system.

The impulse approximation has been used by several workers to study the tidal effects in stellar systems. (e.g., Spitzer, 1958; Alladin, 1965; Sastry and Alladin, 1970; Toomre, 1977; Ahmed, 1979; etc.). The important parameters in collisions of two galaxies are the impact parameter and the relative velocity of the encounter. The impulse approximation gives fairly accurate results in a collision with high relative velocity and small impact parameter. Miller and Smith (1980) pointed out that this approximation is not valid when the relative velocity of the galaxies is less than 1000 km s^{-1}. We consider collision of a disc-sphere pair under the impulse approximation where the relative velocities are greater than 1000 km s^{-1}. We give the necessary theoretical expressions in Section 2. The detailed numerical model is given in Section 3. In Section 4, we discuss our results and compare them with those of previous workers.

2. The Tidal Model

We consider the passage of spherical galaxy of mass M_s radius R_s about a disc galaxy of mass M_D and radius R_D. The centre of the disc galaxy is chosen as the origin of the coordinate system, the X-axis lying in the plane of the disc. The spherical galaxy is assumed to move along the YZ-plane with uniform velocity V along a straight line so that the Y-coordinate of the centre of M_s is always fixed say p. The coordinate of the centre of the spherical galaxy at any time is $(0, p, z)$. The z-axis makes an angle i with the plane of the disc.

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Let r' (x', y', z') be the distance of a representative star in the disc galaxy. The tidal force per unit mass $\mathbf{f}_T$ on a representative star due to the gravitational attraction of M_s is given by (Sastry and Alladin, 1970)

$$\mathbf{f}_T = \mathbf{f}_* - \mathbf{f}_G , \tag{1}$$

where $\mathbf{f}_*$ and $\mathbf{f}_G$ are the accelerations towards M_s of the representative star and the centre of mass of galaxy M_D, respectively, and they are given by

$$\mathbf{f}_* = -\nabla\left[-\frac{GM_s}{(r'^2 + \varepsilon_s^2)^{1/2}}\right], \tag{2}$$

$$\mathbf{f}_G = -\nabla\left[-\frac{GM_s}{(r^2 + \varepsilon_D^2)^{1/2}}\right], \tag{3}$$

where r is the distance of the centre of M_s, ε_s, and ε_D are softening parameters for M_s and M_D, respectively, and G is the gravitational constant. The introduction of the softening parameter takes into account the extended nature of the two configurations. The increment in the velocity of a representative star at any time t measured from the initial separation $Z_0 = -4$ is given by

$$\Delta\mathbf{V}(t) = \int_0^t \mathbf{f}_T \, \mathrm{d}t . \tag{4}$$

Integration of (4) gives the components of the velocity increments as

$$\Delta V(x) = -\frac{GM_s}{\rho^2 V} x'[\sin\theta_t - \sin\theta] , \tag{5}$$

$$\Delta V(y) = -\frac{GM_s}{V}\left[\frac{(y' - p)}{\rho^2}(\sin\theta_t - \sin\theta) + \frac{p^2}{\rho^2}(\sin\theta_2 - \sin\theta_1)\right], \tag{6}$$

$$\Delta V(z) = \frac{GM_s}{V}\left[\frac{\cos\theta_t - \cos\theta}{\rho} + \frac{\cos\theta_2 - \cos\theta_1}{\rho_1}\right]; \tag{7}$$

where

$$\rho^2 = x'^2 + (y' - p)^2 + \varepsilon_s^2 , \tag{8}$$

$$\rho_1^2 = p^2 + \varepsilon_D^2 , \tag{9}$$

$$\left.\begin{aligned} \theta_t &= \tan^{-1}\left[\frac{z' - z_0 - V_t}{\rho}\right], \\ \theta &= \tan^{-1}\left[\frac{z' - z_0}{\rho}\right], \\ \theta_1 &= \tan^{-1}\left[\frac{z_0}{\rho}\right], \\ \theta_2 &= \tan^{-1}\left[\frac{z_0 + V_t}{\rho_1}\right]. \end{aligned}\right\} \tag{10}$$

3. The Numerical Model

We represent the spherical galaxy by the Plummer model whose density distribution is given by

$$\rho(r) = \frac{3M_s\varepsilon_s^2}{4\pi}\,(r^2 + \varepsilon_s^2)^{-5/2}. \tag{11}$$

The disc galaxy is assumed to be of exponential type with surface density

$$\sigma(r') = \sigma_0\, e^{-4(r'/R_D)}, \tag{12}$$

where σ_0 is the central density of the disc and is given by

$$\sigma_0 = \frac{M_D}{2\pi R_D^2 f}, \tag{13}$$

where f is a constant depending upon the density distribution. The value of f for this particular model turns out to be 0.05575.

We distribute 400 stars in the disc galaxy in 20 circular rings of width 0.05 each according to the density distribution (12). There are 20 sets of stars, each set being characterised by its common distance r' from the centre of the disc. The initial internal energy U of the disc is given (cf. Ballabh, 1973) by

$$U = -\frac{GM_D^2}{2R_D}\,U_{\text{het}}, \tag{14}$$

where

$$U_{\text{het}} = \frac{1}{2f^2}\int_0^1 \left\{\sum_{j=0}^{5} a_j\alpha^{j+1}\right\}\{-\textstyle\sum a_j H_j(\alpha, \pi/2)\}\,\mathrm{d}\alpha \tag{15}$$

and $\alpha = r'/R_D$. The coefficients a_j are given by Wyse and Mayall (1942) and H_j are tabulated by Ballabh (1973).

The geometry of collision is given in Figure 1. The plane of the disc galaxy cuts the YZ-plane in the line OP so that $\angle POZ = i$ being the inclination of the disc with z-axis. $S(x', y', z')$ is a representative star in the disc at a distance r' from its centre.

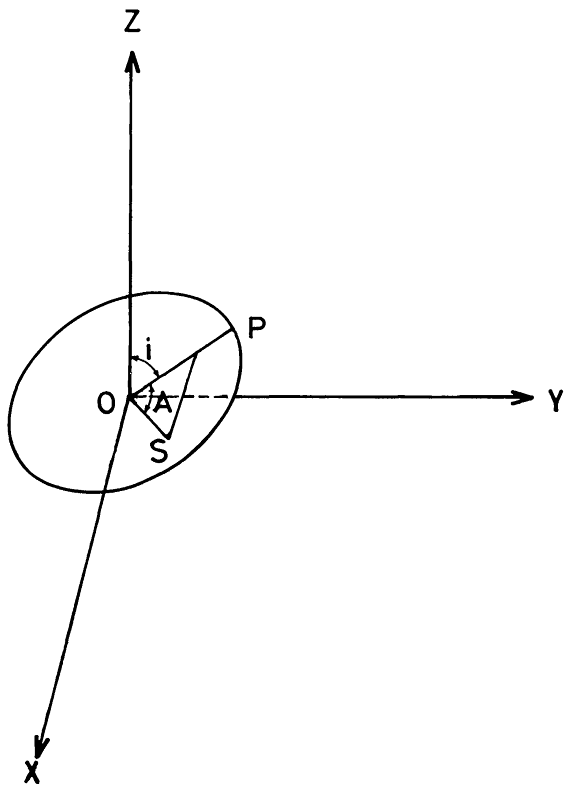

Fig. 1. The geometry of the collision.

$\angle POS = A$. The coordinates of the star S are given by

$$x' = r' \sin A, \qquad y' = r' \cos A \sin i, \qquad z' = r' \cos A \cos i. \tag{16}$$

The average increase in the kinetic energy per unit mass of the disc is given by

$$\langle \Delta U(r') \rangle = \tfrac{1}{2} \langle \Delta V(r') \rangle = = \frac{1}{2N} \sum_{i=1}^{N} [(\Delta V x_i)^2 + (\Delta V y_i)^2 + (\Delta V z_i)^2], \tag{17}$$

where N is the number of stars in the disc galaxy. The change in the internal energy ΔU of the disc galaxy which is the same as the change in the kinetic energy under the impulse approximation is obtained by interrating $\langle \Delta U(r') \rangle$ over the entire mass of the disc galaxy. Therefore, we have

$$\Delta U = \int_0^{R_D} \langle \Delta U(r') \rangle \frac{\mathrm{d}M_D}{\mathrm{d}r'} \, \mathrm{d}r'. \tag{18}$$

Substituting for dM_D/dr', we find that Equation (18) becomes

$$\Delta U = \int_0^{R_D} \langle \Delta U(r') \rangle \frac{2\pi\sigma_0}{M_D} r' e^{-4(r'/R_D)} dr' =$$

$$= \frac{1}{R_D^2 f} \int_0^{R_D} \langle \Delta U(r') \rangle r' e^{-4(r'/R_D)} dr' . \tag{19}$$

4. Numerical Results and Discussion

We choose a system of units in which $M_s = M_D = 10^{11} M_\odot, R_s = R_D = 10$ kpc and time $t = 10^7$ year. In this system, the gravitational constant $G \simeq 0.045$ and the unit of velocity is approximately 1000 km s^{-1}. Time was reckoned from the instant $z_0 = -4.0$. We take $\varepsilon_s = 0.107 R_D$ and $\varepsilon_D = 0.4 R_D$ following Zafarullah and Sastry (1987).

The energy change of the disc galaxy due to the passage of the spherical galaxy was computed using (19) for $p = 0$, 0.5, 1.0, and 2.0 with initial velocities $V = 1.0$, 1.2, 1.5, and 2.0. Figure 2 shows the change in energy of the disc galaxy as a function of the

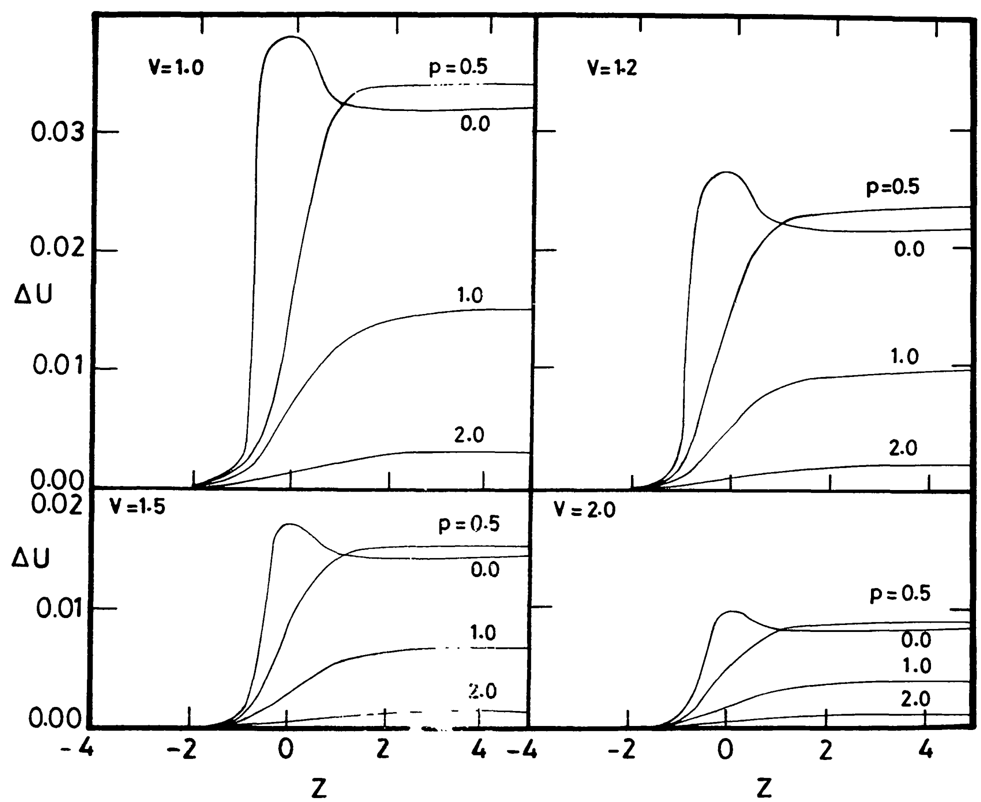

Fig. 2. The variation of ΔU as a function of Z for various values of velocity and impact parameter. $i = 90°$.

separation of the two galaxies for $i = 90°$ (i.e., head-on collision) and for different values of p and V. It can be seen that the energy change is maximum for $p = 0$ during the collision and it reaches a constant value after the collision. This agrees with the result of Ahmed and Alladin (1981) in which they considered head-on collision of two spherical galaxies. The values of ΔU for $p = 0$ are systematically below their values for $p = 0.5$ after the collision has taken place. This is exhibited in all the four cases that have been considered. This shows that a head-on collision produces slightly lesser damage to the disc galaxy than off-centre collision.

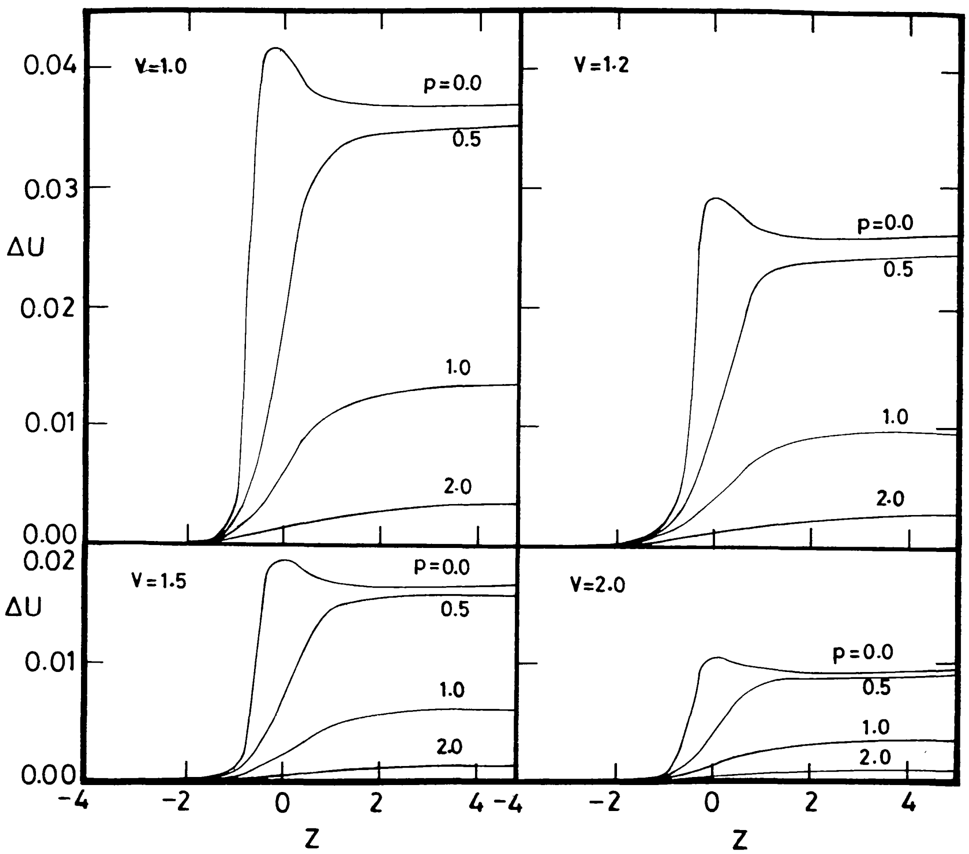

Fig. 3. Same as Figure 2. $i = 45°$.

Figures 3 and 4 present the same results as Figure 2, but for $i = 45°$ and $i = 0°$, respectively. The general trend in both these figures are the same except for a slight enhancement in ΔU for $i = 0°$ case (i.e., face-on collision). Most of the changes in ΔU occur before the encounter. After the encounter, ΔU remains almost constant. An increase of the relative velocity of the galaxies and the impact parameter reduces the value of ΔU significantly.

Figure 5 compares the changes in ΔU as a function of p, the impact parameter, for three orientations of the disc galaxy, i.e., $i = 90°$, $i = 45°$, and $i = 0°$ when the galaxies

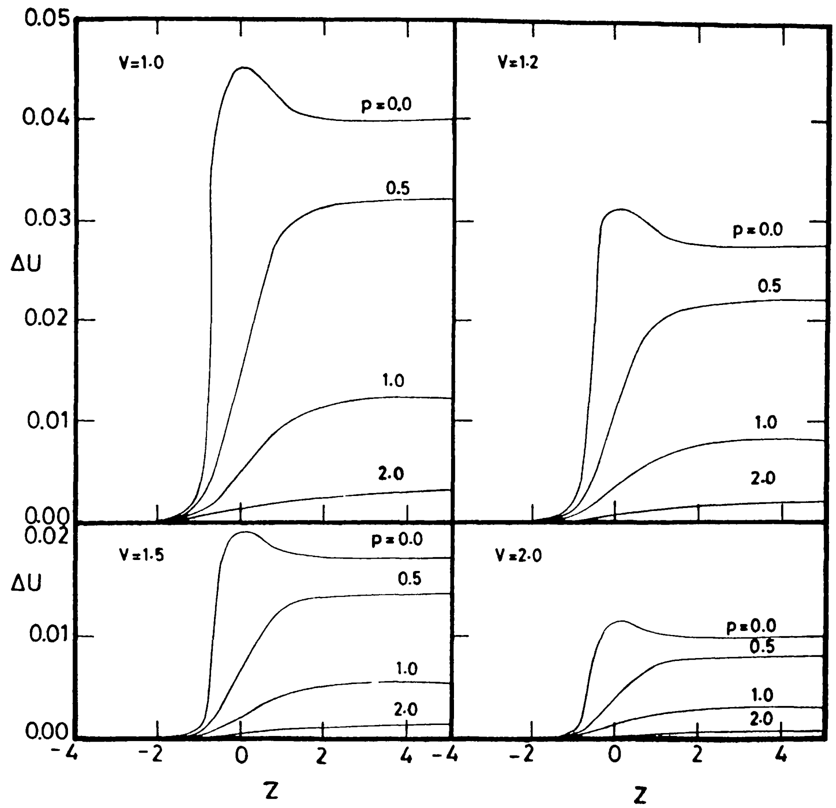

Fig. 4. Same as Figure 2: $i = 0°$.

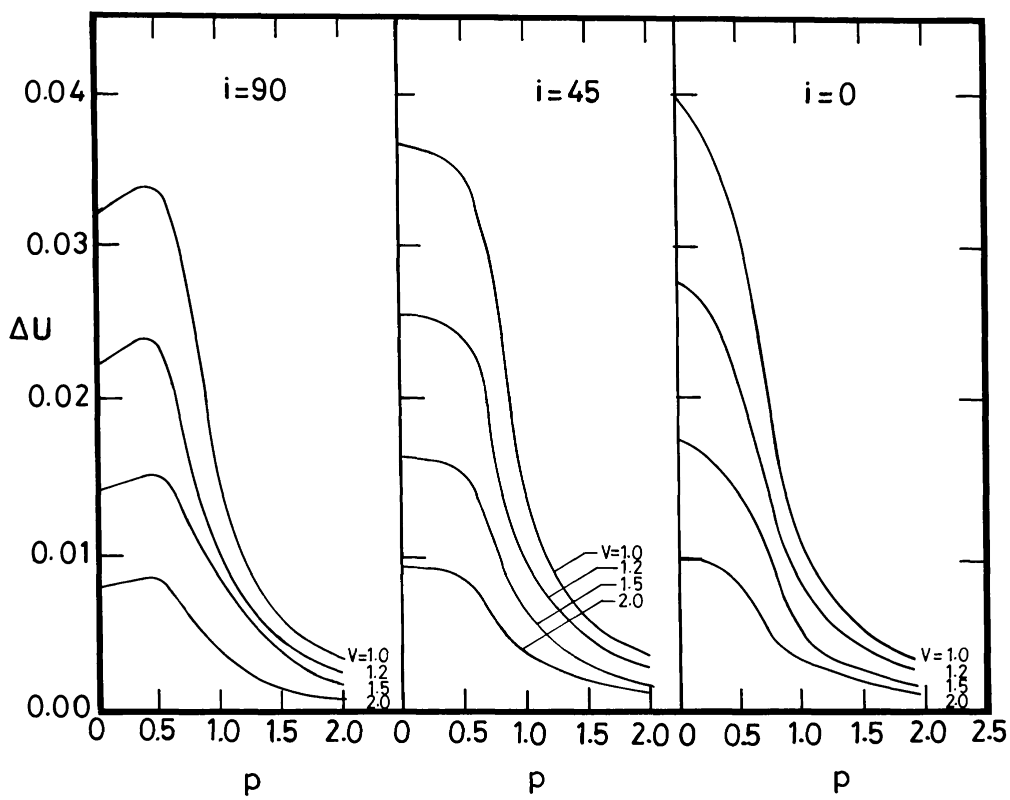

Fig. 5. The variation of ΔU as a function of the impact parameter p for $i = 90°$, $45°$, and $0°$.

are sufficiently apart – i.e., $z = 5.0$. The tidal effects are maximum for $p = 0.5$ in $i = 90°$ case where as in the other two cases this occurs at $p = 0$. At large-impact parameters, the energy change is seen to be negligible for any inclination of the disc galaxy.

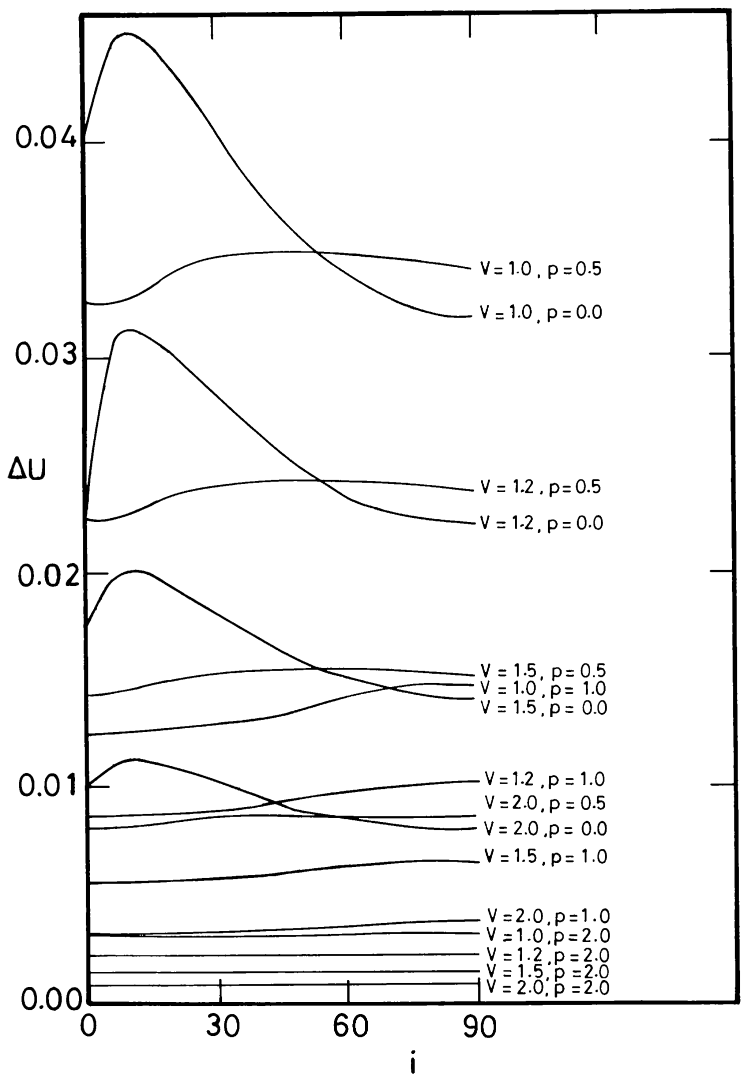

Fig. 6. The variation of ΔU as a function of the inclination angle i for various values for velocity and impact parameter.

Figure 6 shows the variation of ΔU as a function of the inclination of the disc galaxy with the z-axis. It can be seen that for $p = 0$, the value of ΔU increases with i, reaches a maximum for $i = 10°$ and then slowly decreases and reaches a constant value (Chatterjee, 1980). A slight enhancement in the value of ΔU is observed in cases where $p = 0.5$ and 1.0. The change in energy is negligible for the case $p = 2.0$.

We have considered the collision of a disc-sphere galaxy under the impulse approximation to study its energy change with respect to the collision parameters, viz., the impact parameter, the orientation and the relative velocity of the perturber. The initial energy of the unperturbed disc in terms of the units used is $U = 0.03089$. We have used sufficiently high velocities for the perturber to make use of the impulsive approximation. The fast encounters keep the stellar system intact without causing much damage. The disc galaxy gains considerable energy for small impact parameters during the collision at the expense of the orbital energy of the perturber.

The energy change is optimum for slightly inclined position of the disc galaxy – i.e., for $i = 10°$. For $p = 0$, most of the increment in the binding energy occurs before the closest approach distance and for other cases this occurs after the closest approach. This shows that the tidal effects of the perturber is asymmetric with direction of motion of the perturber.

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