EIGENFREQUENCIES OF RADIAL PULSATIONS OF STRANGE QUARK STARS

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ABSTRACT

We calculate the range of eigenfrequencies of radial pulsations of stable strange quark stars, using the general relativistic pulsation equation and adopting realistic equation of state for degenerate (u,d,s) quark matter.

The equation governing infinitesimal radial pulsations of a nonrotating star in general relativity was given by Chandrasekhar [1], and it has the following form :

$$F\frac{d^2\xi}{dr^2} + G\frac{d\xi}{dr} + H\xi = \sigma^2\xi, \tag{1}$$

where ξ(r) is the Lagrangian fluid displacement and $c\sigma$ is the characteristic eigenfrequency. The quantities F, G, H depend on the equilibrium profiles of the pressure(p) and density(ρ) and the radial distribution of the mass of the star, the details of which are given ref.[1] and so are not reproduced here.

The boundary conditions to solve the pulsation equation (1) are

$$\xi(r=0) = 0 \tag{2}$$

$$\delta p(r = R) = -\xi \frac{dp}{dr} - \Gamma p \frac{e^{\nu/2}}{r^2} \frac{\partial}{\partial r}(r^2 e^{-\nu/2}\xi)|_r = R = 0 \tag{3}$$

where Γ is the adiabitic index, defined in the general relativistic case as

$$\Gamma = (1 + \rho c^2/p)\frac{dp}{d(\rho c^2)} \tag{4}$$

Eq. (1) is of the Sturm - Liouville type and has real eigenvalues

$$\sigma_o^2 < \sigma_1^2 < \ldots < \sigma_n^2 < \ldots\ldots,$$

with the corresponding eigenfunctions ξ_o(r), ξ_1(r), ... ξ_n(r), ..., where ξ_n(r) has n nodes.

At high baryonic densities, bulk strange matter is in an overall colour singlet state, and can be treated as a relativistic fermi gas interacting perturbatively, the quark confinement property being simulated by the phenomenological bag model constant (B). Chemical equilibrium between the three quark flavours and electrical charge neutrality allow us to calculate the EOS from the thermodynamic potential of the system as a function of the quark masses, the bag pressure term (B) and the renormalization point μ_o. To second order in α_c, and assuming u and d quarks to be massless, the thermodynamic potential is given by [2] :

$$\Omega = \Omega_u + \Omega_d + \Omega_s + \Omega_{int.} + \Omega_e, \tag{5}$$

where Ω_i (i = u, d, s, e) represents the contributions of u, d, s quarks and electrons and Ω_{int} is the contribution due to interference between u and d quarks and is of order α_c^2:

$$\Omega_u = -\frac{1}{4\pi^2}\mu_u^4\left[1 - \frac{8\alpha_c}{\pi} - 16\left(\frac{\alpha_c}{\pi}\right)^2 ln\left(\frac{\alpha_c}{\pi}\right) - 31.3\left(\frac{\alpha_c}{\pi}\right)^2\right] \tag{6}$$

$$\Omega_d = \Omega_u(\mu_u \leftrightarrow \mu_d) \tag{7}$$

$$\Omega_s = -\frac{\mu_s^4}{4\pi^2}\Big\{(1-\lambda^2)^{1/2}(1-\frac{5}{2}\lambda^2)+ \\ \frac{3}{2}\lambda^4 ln\{[1+(1-\lambda^2)^{1/2}]/\lambda\} \\ -\frac{8\alpha_c}{\pi}\Big[3\Big((1-\lambda^2)^{1/2}-\lambda^2 ln\{[1+(1-\lambda^2)^{1/2}]/\lambda\}\Big)^2 \\ -2(1-\lambda^2)^2\Big]\Big\} \tag{8}$$

$$\Omega_{int.} = \frac{1}{\pi^2}\Big(\frac{\alpha_c}{\pi}\Big)^2\Big\{8\mu_u^2\mu_d^2 ln\Big(\frac{\alpha_c}{\pi}\Big) - 1.9\mu_u^2\mu_d^2 \\ -19.3\Big\{\mu_u^4 ln\left[\mu_u^2/(\mu_u^2+\mu_d^2)\right]+ \\ \mu_d^4 ln\left[\mu_d^2/(\mu_u^2+\mu_d^2)\right]\Big\} \\ -4(\mu_u^2+\mu_d^2)\Big\{\mu_u^2 ln\left[\mu_u^2/(\mu_u^2+\mu_d^2)\right] \\ +\mu_d^2 ln\left[\mu_d^2/(\mu_u^2+\mu_d^2)\right]\Big\} \\ +\frac{4}{3}(\mu_u-\mu_d)^4 ln\Big[|\mu_u^2-\mu_d^2|/(\mu_u\mu_d)\Big] \\ +\frac{16}{3}\mu_u\mu_d(\mu_u^2+\mu_d^2) ln\Big[(\mu_u+\mu_d)^2/(\mu_u\mu_d)\Big] \\ -\frac{4}{3}(\mu_u^4-\mu_d^4) ln\Big(\frac{\mu_u}{\mu_d}\Big)\Big\} \tag{9}$$

$$\Omega_e = -\frac{\mu_e^4}{12\pi^2}. \tag{10}$$

Here μ_i is the chemical potential of the ith particle species and $\lambda = m_s/\mu_s$. We neglect the strange-quark contribution to order α_c^2 and higher in the thermodynamic potential Ω_s. The screened charge α_c is obtained by solving the Gell-Mann–Low equation [2] :

$$\mu\frac{d\alpha_c(\mu)}{d\mu} = \Big\{-\frac{58}{3\pi} - 8\pi\mu\frac{d}{d\mu}\pi_s(\mu)\Big\}\alpha_c^2 - \frac{460}{3\pi^2}\alpha_c^3(\mu), \tag{11}$$

which includes the effects of the strange-quark mass in the lowest order. The higher order contribution to the Gell-Mann–Low equation due to strange quarks may be ignored because these are important only at low densities where the coupling is strong but the pair production of massive strange quarks is unimportant (see ref. [2] for further discussions).

The vacuum polarization tensor $\pi_s(\mu)$ for the strange quarks is given by

$$\pi_s(\mu) = \frac{1}{4\pi^2}\Big\{\frac{5}{9} - \frac{4m_s^2}{3\mu^2} - \frac{2}{3}\left[(1-2m_s^2)/\mu^2\right]\times \quad (12)$$
$$\left(1+4m_s^2/\mu^2\right)^{1/2} \text{ arctanh } \left(1+4m_s^2/\mu^2\right)\Big\}^{-1/2}$$

In Eq. (15), $\alpha_c(\mu_o)$ is the value of α_c at the renormalization point μ_o, where it is taken to be equal to 1.

The total energy density and the external pressure of the system are given by

$$\epsilon = \Omega + B + \sum_i \mu_i n_i \quad (13)$$

$$p = -\Omega - B \quad (14)$$

where n_i is the number density of the ith particle species. For specific choices of the parameters of the theory (namely, m_s, B and μ_o), the EOS is now obtained by calculating ϵ and p for a given value of μ :

$$\mu \equiv \mu_d = \mu_s = \mu_u + \mu_e \quad (15)$$

by solving for μ_e from the condition that the total electric charge of the system is zero.

There is an unphysical dependence of the EOS on the renormalization point μ_o, which, in principle, should not affect the calculations of physical observables if the calculations are performed to all orders in α_c [3,4]. In practice, the calculations are done perturbatively and, therefore, in order to minimize the dependence on μ_o the renormalization point should be chosen to be close to the natural energy scale, which could be either $\mu_o \simeq B^{1/4}$ or the average kinetic energy of quarks in the bag, in which case, $\mu_o \simeq$ 313 MeV. In the present study, our choise of μ_o is dictated by the requirement that stable strange matter obtains at zero temperature and pressure with a positive baryon electric charge [5]. This leads to the following representative choice of the parameter values : EOS model 1 : B = 56 MeV fm^{-3}; m_s = 150 MeV; μ_o = 150 MeV. EOS model 2 : B = 67 MeV fm^{-3}' m_s = 150 MeV; $\alpha_c = 0$. Model 2 corresponds to no quark interactions, but a non-zero mass for the strange quark.

Equilibrium configurations of strange quark stars, calculated for the above EOS, are presented in Table 1, which lists the gravitational mass (M), radius (R), obtained by integrating the relativistic stellar structure equations the surface redshift (z) given by

$$z = (1 - 2GM/c^2R)^{-1/2} - 1 \quad (16)$$

and the period (P_o) corresponding to the fundamental frequency Ω_o defined as [6] :

$$\Omega_o = \left(3GM/4R^3\right)^{1/2} \quad (17)$$

as functions of the central density (ρ_c) of the star.

We solved Eq. (1) for the eigenvalue σ by writing the differential equation as a set of difference equations. The equations were cast in tridiagonal form and the eigenvalue found by using the EISPACK routine. This routine finds the eigenvalues of a symmetric tridiagonal matrix by the implicit QL method.

Results for the oscillations of quark stars corresponding to EOS models 1 and 2 are illustrated in Fig. 1. For purpose of comparison, we have included in Fig.1 the results for quark stars corresponding to (a) the simple MIT bag EOS (non-interacting, massless quarks and B = 56 MeV fm^{-3}) and also (b) neutron stars corresponding to a recently given neutron matter EOS [7]. The plots in Fig.1 are for the oscillation time period ($= 2\pi/c\sigma$) versus the gravitational mass (M). The fundamental mode and the first four harmonics are considered. The period is an increasing function of M, the rate of increase being progressively less for higher oscillation modes. The fundamental mode oscillation periods for quark stars are found to have the following range of values :

MIT bag model	:	(0.14 - 0.32) milliseconds
EOS model 1	:	(0.10 - 0.27) "
EOS model 2	:	(0.06 - 0.30) "

For neutron stars, we find that the range of periods for the $l = 0$ mode is $\simeq 0.3$ milliseconds. For the higher modes, the periods are ≤ 0.2 milliseconds, similar to the case of quark stars.

Inclusion of strange quark mass and the quark interactions make the EOS a little 'softer' as compared to the simple MIT bag EOS. This is reflected in the value of the maximum mass of the strange quark star (see Table 1). For the pulsation of quark stars this gives, for l=0 mode eigenfrequencies, values as low as 0.06 milliseconds. The main conclusion that emerges from our study, therefore, is that use of realistic EOS can be important in deciding the range of eigenfrequencies, at least for the fundamental mode of radial pulsation. The results presented here thus form an improved first step of calculations on radial oscillations of neutron stars with a quark matter core presented by Haensel et al. [8], whose numerical conclusions are expected to get altered.

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Table 1

EQUILIBRIUM STRANGE QUARK STAR MODELS

Equation of State	ρ_c (10^{14} g cm^{-3})	$M/M_\odot$	R (km)	Surface redshift (z)	P_o (milliseconds)
Model 1	24.0	1.958	10.55	0.487	0.488
	20.0	1.967	10.78	0.472	0.503
	16.0	1.951	11.02	0.448	0.522
	12.0	1.864	11.22	0.401	0.548
	8.0	1.521	11.02	0.299	0.591
	6.0	0.997	9.93	0.192	0.624
	5.0	0.485	7.99	0.104	0.646
Model 2	24.0	1.863	10.09	0.483	0.468
	20.0	1.862	10.29	0.465	0.482
	16.0	1.829	10.49	0.435	0.500
	12.0	1.710	10.62	0.381	0.527
	8.0	1.281	10.14	0.263	0.568
	6.0	0.645	8.37	0.138	0.600
	5.0	0.092	4.48	0.032	0.622
MIT Bag	24.0	2.021	10.81	0.493	0.500
model	20.0	2.033	11.04	0.480	0.514
(B=56	16.0	2.023	11.29	0.450	0.533
MeV fm^{-3})	12.0	1.947	11.52	0.410	0.558
	8.0	1.635	11.41	0.310	0.604
	6.0	1.150	10.52	0.210	0.636
	5.0	0.666	8.98	0.130	0.659

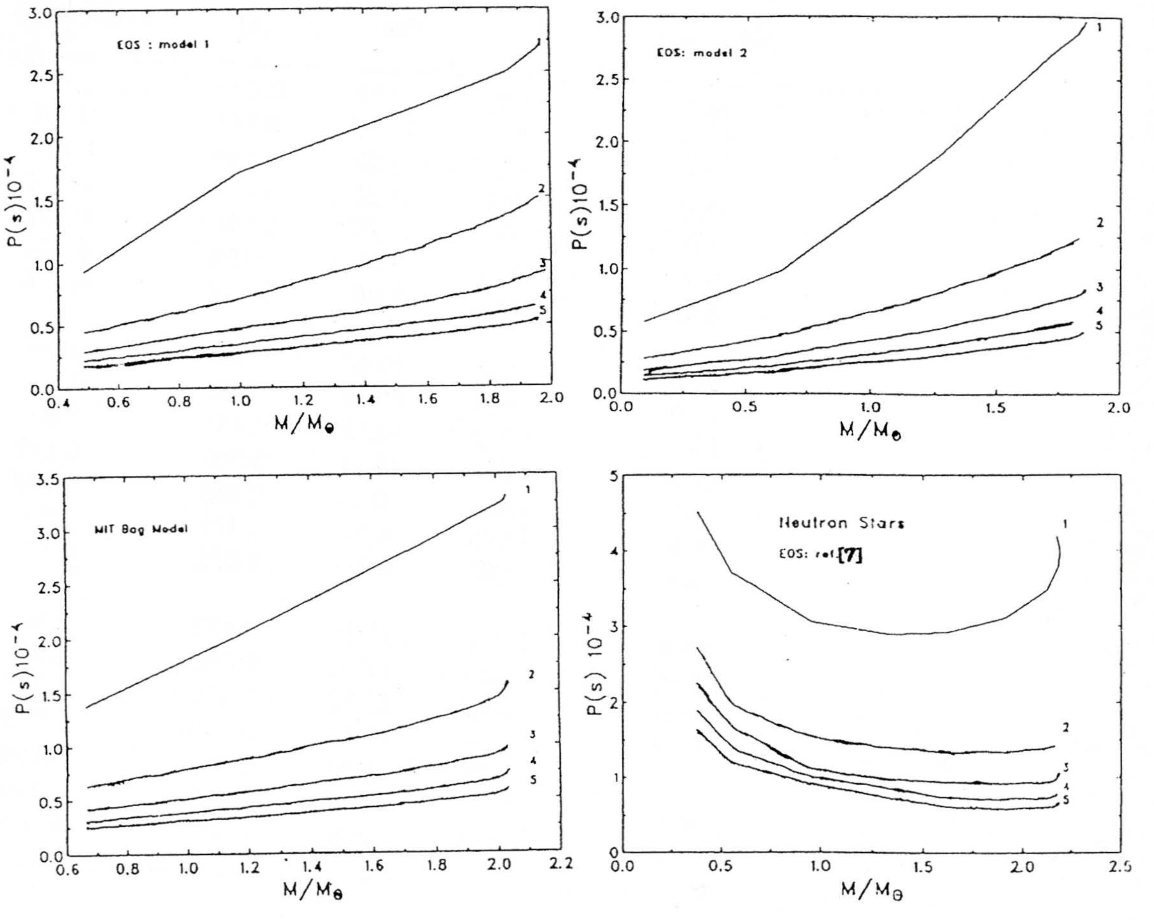

Figure 1. Periods of radial pulsations as functions of the gravitational mass. The top two and bottom left boxes correspond to strange quark stars. The bottom right box is for stable neutron stars corresponding to beta-stable neutron matter, model UV14 + UVII, ref. [7]. The labels 1, 2, 3, 4, 5 correspond respectively to the fundamental and the first four harmonics.