PHASE TRANSITION TO QUARKS IN DENSE NEUTRON MATTER: EFFECT OF INSTANTON INTERACTIONS

(Letter to the Editor)

B. DATTA
Indian Institute of Astrophysics, Bangalore, India

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Abstract. Instanton effects are found to affect non-trivially the neutron matter to quark matter phase transition density. The relevance of the results for neutron stars is pointed out.

The picture of nucleons as composites of quarks suggests that matter compressed to extreme densities may exist in a (new) phase of quark gas. A number of physical and astrophysical situations exist where this can be of direct application – namely, heavy-ion collisions, the early Universe, neutron star interiors.

The factor that determines the transition density is the equation of state. For baryon number density n_B either large or small compared to $n_0 \simeq 1\ \mathrm{fm}^{-3}$, the use of perturbative QCD and conventional nuclear many-body techniques, respectively, are adequate for obtaining the equation of state. At intermediate densities $n_B \sim n_0$, where the transition to quark matter is likely to take place (based mainly on kinematic arguments), the effect of nonlinear fluctuations of the gauge field, the instantons, can tell more about the transition since instanton effects are partly responsible for hadronic properties (Shuryak, 1978, 1979; Gross *et al.*, 1981). In this Letter, we study the possible role of non-perturbative effects due to instantons on the neutron matter-quark matter phase transition, assumed to be of first order.

From perturbative QCD, the thermodynamic potential of neutron-like quark matter (that is, $n_d = 2n_u$, $n_s = 0$, where n_i is the number density of i-quark) to second order in the density-dependent coupling constant α is (Freedman and McLerran, 1977, 1978; Baluni, 1978a, b) with $\hbar = 1 = c$:

$$\Omega(\mu_u, \mu_d) = \frac{\mu_u^4 + \mu_d^4}{4\pi^2}\left\{-1 + \frac{2\alpha}{\pi} + \frac{\alpha^2}{\pi^2}\left(5.99 + 1.9 \ln \frac{\alpha}{\pi}\right)\right\}, \tag{1}$$

where

μ_i = chemical potential of the i-quark ,

$\alpha = [4\pi b \ln(\mu_d/\Lambda)]^{-1}$,

$b = (33 - 2N_f)/4\pi^2$, $N_f = 2$,

Λ = QCD scale parameter $\simeq 200$ MeV ,

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and the quarks are assumed to be massless, which is a fair assumption for densities under consideration here. So, the pressure (P_p) and energy density (ε_p) given by perturbative QCD are:

$$P_p = -\Omega\,, \tag{2}$$

$$\varepsilon_p = \frac{3(p_u^4 + p_d^4)}{4\pi^2}\left\{1 + \frac{2\alpha}{3\pi} + \frac{\alpha^2}{3\pi}\left(8.66 + 1.9\ln\frac{\alpha}{\pi}\right)\right\}, \tag{3}$$

where p_i is the i-quark Fermi momentum = $(\pi^2 n_i)^{1/3}$.

The effect of instantons is to alter the vacuum structure of QCD. This introduces an effective attraction which tends to make the equation of state relatively soft. The result is similar to the MIT bag model (Chodos *et al.*, 1974) but with a modified bag constant, and the pressure ($P_{\text{inst.}}$) and energy density $\varepsilon_{\text{inst.}}$ (Shuryak, 1979) are given by

$$P_{\text{inst.}} = -B_{\text{MIT}} + \tfrac{8}{3}B_{\text{MIT}}Cn_B^{-5/3}\,, \tag{4}$$

$$\begin{aligned} \varepsilon_{\text{inst.}} &= B_{\text{MIT}} - B_{\text{MIT}}Cn_B^{-5/3}\,, \\ B_{\text{MIT}} &= 0.0035\ (\text{GeV})^4\,; \end{aligned} \tag{5}$$

with $n_B = n_u = n_d/2$. The first terms in Equations (4) and (5) are subtracted pressure and energy density, and the second terms are due to instantons. The parameter C is the instanton suppression factor, and $\simeq 1\ \text{fm}^{-5}$.

Equations (4) and (5) are valid so long as the second terms are small compared to the first terms; otherwise one must incorporate higher-order instanton interactions, which are not clearly known. Since at lower densities one expects the neutron matter phase to be more favourable than the quark matter phase, the use of Equations (4) and (5) is a fair approximation to write the quark matter equation of state as

$$P = P_p + P_{\text{inst.}}\,, \tag{6}$$

$$\varepsilon = \varepsilon_p + \varepsilon_{\text{inst.}}\,. \tag{7}$$

For the neutron matter equation of state, we choose the model of Friedman and Pandharipande (1981) which is the most realistic microscopic calculation available, relevant to a wide density range of interest in astrophysics and heavy-ion collisions.

The standard Maxwell construction for a first-order phase transition from neutron matter to quark matter is performed using Gibbs's chemical potential, given by $\mu = (P + \varepsilon)/n_B$, of each phase as a function of the pressure. The thermodynamically favoured state corresponds to the minimum value of μ. The transition pressure (and, hence, the transition density) is determined by the intersection point of the μ-curves of the two phases.

The result of our calculation is shown in Figure 1 (for $\Lambda = 200$ MeV and $C = 1\ \text{fm}^{-5}$). It shows that the effect of instantons is to shift the transition pressure from 5.6×10^{36} dynes cm^{-2} to 5.4×10^{36} dynes cm^{-2}. The corresponding shift in the transition (Baryon number) density n_B is from 2 fm^{-3} to 1.9 fm^{-3}, or in terms of the total

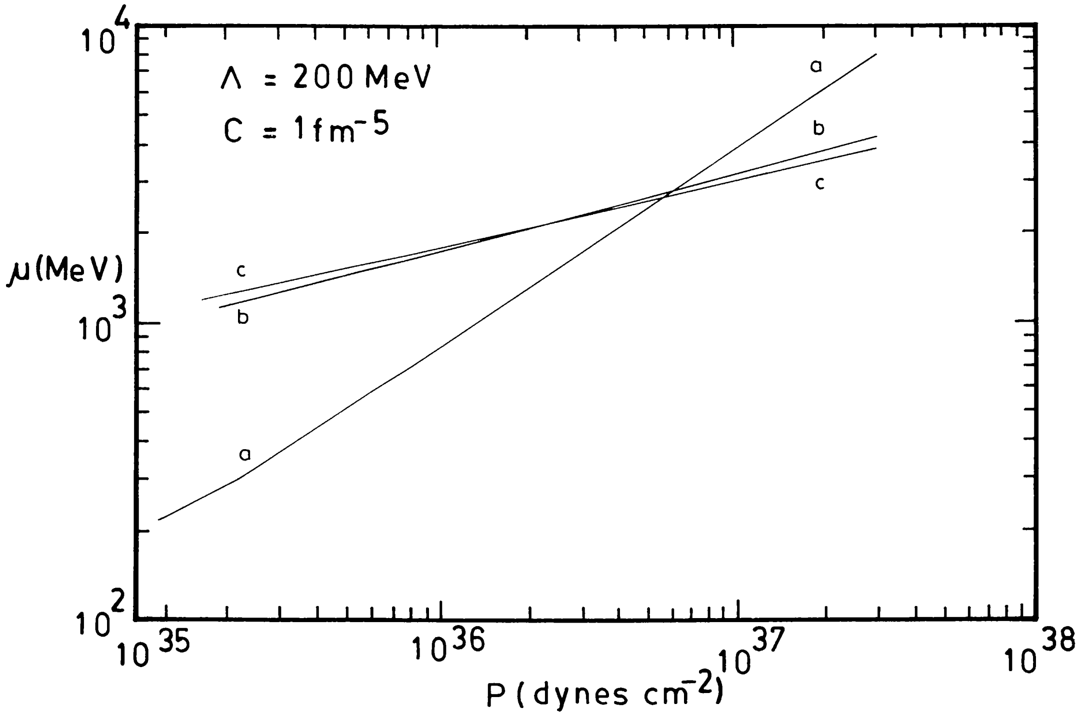

Fig. 1. Chemical potential versus pressure. Curve (a) is for neutron matter (Friedman and Pandharipande, 1981), curve (b) for quark phase of neutron matter corresponding to perturbative QCD, and curve (c) is for the quark phase corresponding to perturbative QCD plus instanton effects.

mass energy density, from 6.76×10^{15} g cm^{-3} to 6.25×10^{15} g cm^{-3}. This result is found to be insensitive to a variation in the parameter C; for C equal to 0.1 fm^{-5} or 10 fm^{-5}, Figure 1 remains essentially unaltered.

In reality the instanton interactions are more complex than has been considered here, and corrections due to zero modes, dynamical masses of light quarks (Callan *et al.*, 1978) and possible vacuum instabilities of the gluon field (Migdal, 1978) must be properly taken into account to get the equation of state of quark gas. However, our calculation shows that even the lowest-order effects of instantons in high-density matter is non-trivial. Since $\rho = 6.35 \times 10^{15}$ g cm^{-3} is not too far from the maximum value of density of neutron star cores as given by present theoretical neutron star models (for a recent review, see Datta, 1988), the composite interaction due to instantons may alter the theoretically believed maximum stable mass for neutron stars ($\simeq 2\,M_\odot$). It has been suggested (Witten, 1984), that an equilibrium number density of s quarks will be present even though the neutron matter is made up of only u and d quarks. Because the s quark mass is non-negligible and is related to the renormalization point, the conclusions reported above may get altered. The dynamical importance of the s quark mass and its relationship to its abundance and the equation of state will be reported in a future paper.

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