LEPTON LOSS AND ENTROPY GENERATION IN STELLAR COLLAPSE

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(Received 26 August, 1982)

Abstract. The effects of uncertainties in determining the loss of neutrinos and the generation of entropy during the collapse of a stellar core are studied using a simple spherically-symmetric homologous collapse model with an improved treatment for the excited nuclei. It is found that the neutrino loss and entropy generation are not altogether insensitive to most variations of the assumptions regarding the input physics, collapse rate and initial conditions and in particular the need to determine accurately beta capture rates in stellar collapse is pointed out.

1. Introduction

In the gravitational collapse scenario of a star of mass $\simeq 10-25$ solar mass, an important physical feature is the trapping of neutrinos that are produced in the neutronization of the nuclei when the dense core becomes opaque to neutrino diffusion (at a density $\sim 10^{12}$ g cm^{-3}). The neutrino trapping affects the composition and the equation of state of the system until the stellar core bounces at a still higher density because of the stiff repulsion of the nuclear force at short ranges. Two physical quantities that characterize the collapse following the neutrino trapping are (a) loss in lepton (i.e., electron plus neutrino) number and (b) increase in the entropy. For fixed temperature and density, the lepton number per nucleon (Y_l) and the entropy per nucleon (s) completely determine the equation of state of the system. Hence, in order to develop realistic models for gravitational collapse and supernova theory, it is desirable that Y_l and s be determined by properly taking into account the initial stages of the collapse. An important constituent of the pre-supernova collapse matter is iron-peak nuclei, excited to temperatures $\sim (1-5)$ MeV. A realistic calculation that would incorporate all the details, namely, the properties of the finite nuclei, electron capture rates and neutrino transport is quite complicated. So in recent times simple models that aim to illustrate the basic physics involved have been suggested (Epstein and Pethick, 1981). While these computations do provide important insight, there remains considerable divergence in so far as the treatment of nuclear properties at finite temperatures is concerned. The purpose of this paper is to present improved calculations for Y_l and s by using better input physics to describe the excited nuclei. The chemical potentials of neutrons and protons in the nuclei-important factors in deciding how Y_l and s vary during the collapse – are

Astrophysics and Space Science **90** (1983) 109–115. 0004–640X/83/0901–0109$01.05.

determined using a temperature-dependent Hartree-Fock theory. Such a theory is an improvement over the commonly used semi-empirical mass formulae and Thomas–Fermi type approaches, and is known to provide a good description of the properties of excited nuclei, provided the excitation energies are not too large (i.e., $\lesssim 5$ MeV) as compared to the nuclear separation energies (Quentin and Flocard, 1980).

2. The Collapse Model

Following the usual approximation of spherical symmetry, we consider the stellar system to be a homogeneous sphere collapsing at a prescribed rate, and the effects of all the nuclei ascribed to one characteristic nucleus – the iron-nickel type. The assumption of spherically-symmetric collapse is based on simplicity considerations, and is not strictly valid. The extent to which rotation affects the collapse dynamics in so far as computions of Y_l and s are concerned is not well known. However, it may be noted that present observational evidence on masses of neutron stars, believed to be produced in supernova remnants, suggests that these lie near the Chandrasekhar limit, which is a natural unit of mass only for slowly rotating systems (Arnett, 1980). The following equations determine the time evolution of the electron number per nucleon (Y_e), the neutrino number per nucleon (Y_ν) and the specific entropy (s):

$$\frac{\mathrm{d}Y_e}{\mathrm{d}t} = -\beta_N Y_N , \tag{1}$$

$$\frac{\mathrm{d}Y_\nu}{\mathrm{d}t} = -\frac{\mathrm{d}Y_e}{\mathrm{d}t} - \frac{Y_\nu}{t_{\mathrm{esc}}} , \tag{2}$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = (kT)^{-1}(\mu_n + \mu_\nu - \mu_p - \mu_e)\frac{\mathrm{d}Y_e}{\mathrm{d}t} + (kT)^{-1}(\mu_\nu - \bar{\varepsilon}_\nu) Y_\nu / t_{\mathrm{esc}} ; \tag{3}$$

where Y_N is the number of nuclei per baryon, β_N is the electron capture rate on the nuclei, t_{esc} is the characteristic time for a neutrino to escape from the stellar interior, the various μ's are the chemical potentials and $\bar{\varepsilon}_\nu$ is the average energy of the escaping neutrinos. Equation (1) corresponds to the loss of electrons due to beta capture process, Equation (2) to lepton loss due to neutrino diffusion, and Equation (3) follows from the first law of thermodynamics. In Equation (3) the term involving Y_ν is the so-called neutrino down-scatter term corresponding to the lowering of energy of the escaping neutrinos, and the term involving $\mathrm{d}Y_e/\mathrm{d}t$ corresponds to the departure from beta equilibrium.

The electron capture on the nuclei is characterized mainly by the transition $^{56}\mathrm{Fe} \to {}^{56}\mathrm{Mn}$, in which an $^1f_{7/2}$ proton gets converted to an $^1f_{5/2}$ neutron via a single Gamow–Teller transition with a rate given by (Bethe *et al.*, 1979)

$$\beta_N = -\frac{6G^2 g_A^2 (Z-20)}{7\pi^3 \hbar c^8} \int_W^\infty \mathrm{d}\varepsilon_e \, \varepsilon_e^2 \varepsilon_\nu^2 n\left(\frac{\varepsilon_e - \mu_e}{kT}\right) \times \left\{1 - n\left(\frac{\varepsilon_\nu - \mu_\nu}{kT}\right)\right\} , \tag{4}$$

where G is the weak interaction coupling constant ($\simeq 3 \times 10^{-12} m_e^{-2}$), ε_e and ε_ν are the energies of the initial electron and final neutrino, and $n(x)$ is the Fermi distribution function $(e^x + 1)^{-1}$. The threshold energy W for the electron capture is

$$W = \mu_n - \mu_p + \Delta ,$$

where Δ corresponds to the excitation of the daughter nucleus, and $\simeq 3$ MeV.

The following physical quantities must be specified in order to determine the changes $\Delta Y_l\,(=\Delta Y_e + \Delta Y_\nu)$ and Δs: Y_N, $t_{\rm esc}$, $\bar{\varepsilon}_\nu$, μ_e, μ_ν, μ_n, and μ_p. In addition the initial values of Y_e, Y_ν, and s, the collapse time and the temperature of the system must also be supplied. Since the uncertainty in calculating the equation of state lies more in the treatment of the excited nuclei rather than the degenerate gas of leptons, so, for the leptons we have taken the relevant physical quantities as from the existing literature (Bethe *et al.*, 1979; Epstein and Pethick, 1981) and the excited nuclei are treated using a temperature-dependent Hartree-Fock theory. This facilitates a proper comparison of our results with those that do not have temperature effects in describing the excited nuclei.

The chemical potentials of neutrons and protons in the excited nuclei have been determined using the relation

$$\sum_i g_i n_i \equiv \sum_i g_i \{1 + e^{(e_i - \mu)/kT}\}^{-1} = N , \tag{5}$$

where g_i is the multiplicity of the ith single-particle energy level, characterized by e_i, and N is the fixed total number of nucleons (Banerjee *et al.*, 1981). The e_i's are determined by solving the Hartree–Fock equations

$$\left\{ -\nabla \cdot \frac{\hbar^2}{2m_q^*(\mathbf{r})} \nabla + V_q(\mathbf{r}) \right\} \phi_i = e_i \phi_i , \tag{6}$$

where $m_q^*(\mathbf{r})$ is the effective mass of the nucleon ($q = n$ or p) and $V_q(\mathbf{r})$ is the single-particle Hartree-Fock potential which depends on the (local) number, kinetic energy and spin density functions, respectively:

$$\rho_q(\mathbf{r}) = \sum_i n_{q,i} |\phi_{q,i}(\mathbf{r})|^2 ,$$

$$\tau_n(\mathbf{r}) = \sum_i n_{q,i} |\nabla \phi_{q,i}(\mathbf{r})|^2 ,$$

$$\mathbf{J}_n(\mathbf{r}) = -i \sum_i n_{q,i} \phi_{q,i}^*(\mathbf{r}) (\nabla \times \boldsymbol{\sigma}) \phi_{q,i}(\mathbf{r}) ;$$

where

$$n_{q,i} = \{1 + \exp((e_{q,i} - \mu_q)/kT)\}^{-1} ,$$

i refers to the particular energy level and $\phi_{q,i}(\mathbf{r})$ the corresponding single-particle orbital. In addition, $m_q^*(\mathbf{r})$ and $V_q(\mathbf{r})$ depend on a set of numerical parameters called Skyrme parameters. For our computations we have used the (zero temperature) Skyrme parameter set I as given by Vautherin and Brink (1975); the temperature dependence of the effective interaction having been found to be negligible (Buchler and Datta, 1979). The resulting chemical potentials for neutrons and protons, μ_n and μ_p are listed in Table I.

TABLE I

Chemical potentials of neutrons and protons, μ_n, μ_p, for the iron-peak nucleus as a function of temperature using Hartree-Fock method with Skyrme interaction (see text)

kT (MeV)	μ_n (MeV)	μ_p (MeV)
1	−13.45	3.65
2	−13.66	−3.96
3	−14.10	−4.52
4	−14.60	−5.28
5	−15.15	−6.07

At the high density and temperature characteristic of the collapsing stellar system, the effective electron rate gets considerably reduced from its value given by Equation (4) due to the blocking of the final neutrino phase space and also the neutron shell-blocking effect (Fuller, 1982). The reduction factor is somewhat uncertain, and depends on the details of the equation of state. So we have introduced a multiplicative parameter β in the electron capture rate, as in Epstein and Pethick (1981), and have varied it from 1.0 to 0.01. Three more numerical parameters (σ, η, χ) are introduced (see Epstein and Pethick, 1981) to account for the uncertainties in the neutrino diffusion time, the neutrino energy relaxation and the collapse time, so that

$$t_{\mathrm{diff}} = \sigma(3R^2/\lambda c\pi^2)\,,$$

$$\bar{\varepsilon}_\nu = \left(\varepsilon_\nu^i + \eta\,\frac{t_{\mathrm{esc}}}{t_e}\,\mu_\nu\right)\Big/\left(1 + \eta\,\frac{t_{\mathrm{esc}}}{t_e}\right),$$

$$t_{\mathrm{collapse}} = \chi t_{ff}\,,$$

where R is the core radius, λ is the neutrino mean free path, ε_ν^i is the average energy with which the neutrinos are produced, t_{ff} is the free-fall time and t_e $(=\lambda/c)$ is the electron-neutrino scattering time (see Epstein and Pethick, 1981; or Brown *et al.*, 1982; and the references given therein). The escape time t_{esc} is taken to be the larger of the two times: t_{diff} and t_{ff}.

3. Results and Discussion

Equations (1)–(3) have been integrated for the following choice of the initial conditions

$$Y_e^{(0)} = 0.41\text{–}0.43\,,$$

$$Y_\nu^{(0)} = 0\,,$$

$$s^{(0)}/k = 0.9\,,$$

$$Y_N = 0.43\,,$$

which are believed to characterize the onset of the collapse (Brown *et al.*, 1982). The chemical potentials for the leptons are taken as (cf. Bethe *et al.*, 1979):

$$\mu_e = 11.1(\rho_{10} Y_e)^{1/3}\ \text{MeV}\,,$$

$$\mu_\nu = 0\ \text{MeV}\,,$$

where ρ_{10} is the matter density in units of 10^{10} g cm^{-3}.

To see the effects of uncertainties in determining ΔY_l and Δs, calculations were done corresponding to $kT = 1$ MeV, and for different choices of the parameters σ, η, χ, β.

TABLE II

Variations in ΔY_l and Δs with the parameters of the collapse model for initial electron fraction = 0.41. ΔY_l and Δs are insensitive to η, which was varied from 0.1 to 10

σ	β	χ	ΔY_l	$\Delta s/k$
0.5	0.01	7	0.0086	0.0013
		14	0.0155	0.0028
	0.1	7	0.0774	0.0189
		14	0.1251	0.0462
	1.0	7	0.3704	0.2882
		14	0.3867	0.4843
1.0	0.01	7	0.0091	0.0013
		14	0.0170	0.0028
	0.1	7	0.0821	0.0189
		14	0.1389	0.0462
	1.0	7	0.4052	0.2881
		14	–	–
2.0	0.01	7	0.0094	0.0013
		14	0.0180	0.0028
	0.1	7	0.0848	0.0189
		14	0.1479	0.0462
	1.0	7	–	–
		14	–	–

The results are presented in Tables II and III for two different choices of the initial lepton number fraction (0.41 and 0.43). The final values of Y_l and s are found to be insensitive to variations of the neutrino energy relaxation parameter η. This means that the energy with which the neutrinos are produced is more or less same as the neutrino chemical potential. The variation is more pronounced with respect to the parameter β, with ΔY_l and Δs being small for small β and conversely. Present theoretical models of supernova explosions invoke the mechanism of shock waves formed in the stellar collapse (Sack *et al.*, 1980; Brown *et al.*, 1982; Hillebrandt, 1982), with the shock energy depending critically on the final value of the electron fraction. In view of this and the result ΔY_l and Δs depend on β, an accurate determination of the beta capture rate in stellar collapse assumes importance, for it will then help decide the viability of the shock-wave supernova models. The variations with respect to the remaining parameters, χ, which is $\sim 7-14$ (Arnett, 1977), and σ, which is $\sim 0.5-2$ (Epstein and Pethick, 1981), are found to be moderate, with Δs independent of variations in σ. The above variations can be attributed to the fact that our input value for $|\mu_n - \mu_p|$ which is $\lesssim 10$ MeV, and which decides the rate of beta capture reaction for a given μ_e, is small compared to that in other computations (Epstein and Pethick, 1981; Bethe *et al.*, 1979) where it ranges from 13–40 MeV. The final values of Y_l are not found to be functions of the ratio σ/x

TABLE III

Variations in ΔY_l and Δs with the parameters of the collapse model for initial electron fraction $= 0.43$. ΔY_l and Δs are insensitive to η, which was varied from 0.1 to 10

σ	β	χ	ΔY_l	$\Delta s/k$
0.5	0.01	7	0.0095	0.0006
		14	0.0171	0.0015
	0.1	7	0.0848	0.0137
		14	0.1367	0.0384
	1.0	7	0.3974	0.2813
		14	0.4113	0.4813
1.0	0.01	7	0.0100	0.0006
		14	0.0187	0.0015
	0.1	7	0.0898	0.0137
		14	0.1516	0.0384
	1.0	7	–	–
		14	–	–
2.0	0.01	7	0.0103	0.0006
		14	0.0197	0.0015
	0.1	7	0.0927	0.0137
		14	0.1612	0.0384
	1.0	7	–	–
		14	–	–

alone. Since σ/x is proportional to the ratio of the diffusion time to the collapse time, this suggests that beta capture rates in stellar collapse could be important, and merit a careful study. Finally, a comparison of the results in Tables II and III shows that neither ΔY_l nor Δs is altogether independent of the initial electron fraction, indicating that the early stages of the collapse do affect the subsequent stages of the core implosion.

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