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Photon Escape Probabilities in Expanding Atmospheres

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Abstract. A comparison of mean number of scatterings and escape probabilities has been made in isotropic scattering and dipole scattering by using the angle-averaged partial frequency redistribution function R_I. We have solved the equations of radiative transfer and statistical equilibrium simultaneously in a spherically symmetric expanding atmosphere. Two cases of atmospheric extension (*i.e.*) B/A=3 and 10 (where B and A are the outer and inner radii of the atmosphere) have been treated.

We find that the partial frequency redistribution gives a larger mean number of scatterings compared to that given by complete redistribution. Velocities tend to reduce the mean number of scatterings and in crease the mean escape probabilities.

Key words: mean number of scatterings—mean escape probability—partial redistribution function—isotropic scattering—dipole scattering

1. Introduction

It is well known that the radiative transfer effects in an optically thick resonance line are considerable and should be investigated by highly accurate methods. If the medium is optically thin, one can easily calculate, for example, the probability P_x that a photon, created at a given point in space and time, leaves the medium without being scattered or absorbed. If the optical depth at a normalized frequency x is given by τ_x then P_x = exp $(-\tau_x)$. This relation is true in the case of certain subordinate lines. For resonance lines, τ_x is much larger and a rigorous treatment of radiative transfer is required. Several people have studied the problem of mean number of scatterings and photon escape probability in a resonance line (Osterbrock 1962; Hummer 1964; Panagia and Ranieri 1973; Kunasz and Hummer 1974a, b) with various assumptions regarding geometry and nature of the media. However, in all these studies the main assumption was complete redistribution of photons in the line. A few have solved the radiative transfer equation correctly for the resonance line in a moving media relative to the rest frame. In these calculations the optical depths were assumed at line centre and mean number of scatterings and the mean escape probabilities were computed. In a general case one must reduce the number of free parameters such as optical depths, velocities etc. and obtain a consistent solution of line transfer in the resonance lines. There exist now fast and accurate techniques for obtaining simultaneous solutions of radiative transfer equation in a comoving frame and statistical equilibrium equation, for a resonance line (Peraiah 1980). The aim of this paper is to use such calculations to compute the mean number of scatterings and escape probabilities in a resonance line forming in a spherically symmetric expanding atmosphere. The mean number of scatterings and escape probabilities have been estimated with the partial redistribution function R_I . Isotropic and dipole scattering functions have been employed for the sake of comparison.

2. Brief description of procedure and discussion of the results

The procedure of obtaining a simultaneous solution of radiative transfer equation in a comoving frame and the statistical equilibrium equation for a non-LTE two-level atom with complete redistribution is described in Peraiah (1980 henceforth called Paper I). This procedure has been extended to include partial frequency redistribution (Peraiah 1981). We have made use of the redistribution functions to calculate the profile functions:

$$\phi(x) = \int_{-\infty}^{\infty} R(x, x')\, dx', \tag{1}$$

where

$$R_{I\text{-}AI}(x, x') = \frac{1}{\sqrt{\pi}} \int_{|\bar{x}|}^{\infty} \exp(-t^2)\, dt \tag{2}$$

for isotropic scattering (Unno 1952) and

$$R_{I\text{-}AD}(x, x') = \frac{3}{8} \left\{ \frac{1}{\sqrt{\pi}} \int_{|\bar{x}|}^{\infty} \exp(-t^2)\, dt\, [3 + 2(x^2 + x'^2) + 4x^2 x'^2] - (1/\sqrt{\pi}) \exp(-|\bar{x}|^2)\, |\bar{x}|\, (2|\bar{x}|^2 + 1) \right\}, \tag{3}$$

for dipole scattering in the angle-averaged redistribution function. The statistical equilibrium equation for a two-level atom that we have used is given by

$$N_1 \{B_{12} \int J_x \phi_x\, dx + C_{12}\} = N_2 \{B_{21} \int J_x \phi_x\, dx + C_{21} + A_{21}\}, \tag{4}$$

where N_1 and N_2 are the number densities in levels 1 and 2, B_{12}, B_{21} and A_{21} are Einstein coefficients, J_x is the mean intensity at the normalized frequency x and C_{12} and C_{21} are the collisional excitation and de-excitations rates (see Paper I). The quantity $\phi(x)$ is the absorption profile defined in equation (1). The probability that a photon is lost by collisional de-excitation is calculated by

$$\epsilon = \frac{C_{21}}{C_{21} + A_{21}\,[1 - \exp(-h\nu/kT)]^{-1}}. \tag{5}$$

The quantities C_{12} and C_{21} are given in Paper I. The quantity ϵ is given in Fig. 1. The iterative procedures are similar to those described in Paper I.

The mean number of scatterings N is calculated as the ratio of the number of emittied photons to those generated in unit time (see Kunasz and Hummer 1974b) and this is given by

$$\langle N \rangle = \frac{\int_A^B K_L(r)\, S_L(r)\, r^2\, dr}{\int_A^B K_L(r)\, \epsilon(r)\, B(r)\, r^2\, dr}. \tag{6}$$

Here $K_L(r)$ is the line centre absorption coefficient per unit frequency band width given by

$$K_L(r) = \frac{h\nu}{4\pi\Delta\nu_D}\,[N_1(r)\, B_{12} - N_2(r)\, B_{21}] \tag{7}$$

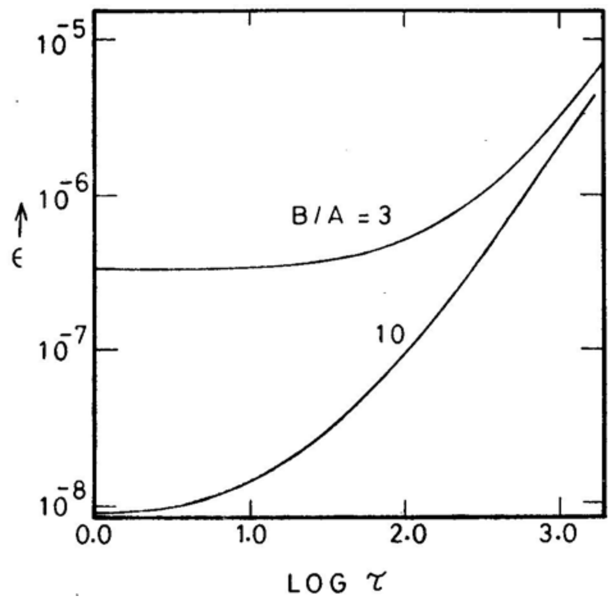

Figure 1. ϵ is given against τ.

$\Delta\nu_D$ being some standard frequency band width. We have taken $\Delta\ \nu_D$ here to be the Doppler width

$$\frac{\nu_0}{c}\left(\frac{2kT_0}{m}\right)^{1/2},$$

where T_0 is temperature and $S_L\ (r)$ is the line source function given in our case as

$$S_L\ (r) = \frac{N_2\ (r)\ A_{21}}{N_1\ (r)\ B_{12} - N_2(r)\ B_{21}} \tag{8}$$

Another important quantity we intend to study is the mean escape probability $\langle P \rangle$ This is given as the ratio of number of photons leaving the medium to those created (see Kunasz and Hummer 1974b) in the medium and is written as

$$\langle P \rangle = \frac{8\pi^2\ B^2 \int\limits_{-\infty}^{\infty} dx \int\limits_{-1}^{+1} I\ (x,\ \mu,\ r)\ \mu\ d\mu}{\int\limits_{A}^{B} 4\pi\ r^2\ K_L\ (r)\ \epsilon\ (r)\ B\ (r)\ dr}, \tag{9}$$

where $I(x,\ \mu.\ r)$ is the specific intensity making an angle $\cos^{-1}\ \mu$ with the radius vector at the radial point r.

Initially we have assumed a certain variation of number density of the ions and the velocity with which the gas is expanding in accordance with the equation of continuity. When there is motion, we have given a constant velocity gradient $dV/d\tau$ with the velocity increasing outwards starting with 0 mean thermal units (mtu) at $r=A$ and taking the values $V_B\ =\ V_{\max}$ at $r=B$. The maximum velocity V_B is set equal to 5, 10, 30 mtu in addition to the static case. We have calculated the mean number of scatterings and the mean escape probability for isotropic and dipole scattering cases for the sizes of the atmospheres $B/A=3$ and 10. In Table 1a the mean number of scatterings per unit velocity gradient is given for the hydrogen Lyman α line.

Table la shows that, in general, dipole scattering gives greater mean number of scatterings than isotropic scattering. As the velocity increases the mean number of scatterings decreases. We also notice that media with higher optical depths have greater mean number of scatterings. We can make a fair comparison between our results (for $V=0$) and those of Panagia and Ranieri (1973, Table 1) which we have reproduced in Table lb. They have calculated the mean number of scatterings using a Voigt profile, assuming $V=0$ and complete redistribution. By comparing the results of Tables la and lb we find that the assumption of partial frequency redistribution gives a larger mean number of scatterings than that given by complete redistribution. This seems to be true in moving media also. For example, if we compare our results with those of Kunasz and Hummer (1974b, Table 1) we find that the mean number of scatterings in our case is greater (even at much higher velocities) than what they have obtained with complete redistribution at smaller velocities ($V=1$ or 2). These differences can be explained on the basis of the fact that we have considered an almost purely scattering medium ($\epsilon \sim 10^{-5} - 10^{-8}$) whereas Kunasz and Hummer have investigated a medium with $\epsilon = 10^{-2}$.Inanycase, it is important to note that partial frequency redistribution gives higher mean number of scatterings than complete redistribution and this is further accentuated in the case of dipole scattering. The effects of high velocities are to reduce the mean number of scatterings both in the case of isotropic and dipole scattering. However, when the geometrical depth is increased from $B/A=3$ to $B/A=10$ the mean number of scatterings do not show an increase as shown in Panagia and Ranieri (1973, Fig. 1). This difference arises because our optical depth $T(B/A=10)<T(B/A=3)$ and this brings out the fact that the optical depth at the centre of the line plays a more important role than the greater geometrical thickness of the atmosphere.

In Table 2, the mean escape probabilities $\langle P \rangle$ is given for the same parametric values of *V, B*/*A* and *T* the total optical depth as given in Table 1.

There are three ways that a photon can escape the medium: (1) by diffusion of the photon through space, (2) by a line centre photon reaching into the wings after several scatterings and (3) by the translational motion of the gases towards the outward surface of the atmosphere. The solution of the radiative transfer in our calculations

Table 1a. Mean number of scatterings of a Lyman α photon for the total optical depth *T*.

	V	$R_{I\text{-iso}}$	$R_{I\text{-dip}}$
	0	6·68 10^4	1·12 10^5
B/A = 3	5	1·4 10^4	2·2 10^4
	10	6·97 10^3	1·2 10^4
T = 1986	30	2·13 10^3	3·37 10^3
	0	5·66 10^4	1·43 10^5
B/A = 10	5	1·327 10^4	2·86 10^4
	10	5·65 10^3	1·6 10^4
T = 1763	30	1·87 10^3	4·73 10^3

Table.1b. Mean number of scatterings: Results of Panagia and Ranieri (1973, Table 1)

T \ B/A	3	12
10^3	2·055 10^3	1·732 10^3
10^4	9·555 10^3	8·789 10^3

Table 2. Mean escape probabilities $\langle P \rangle$

	V	$R_{I\text{-iso}}$	$R_{I\text{-dip}}$
	0	0·028	0·013
B/A = 3	5	0·136	0·06
	10	0·278	0·1303
T = 1986	30	0·86	0·401
	0	0·0198	0·008
B/A = 10	5	0·10	0·042
	10	0·198	0·083
T = 1763	30	0·59	0·25

takes into account all these effects, together with the partial frequency redistribution of photons in the line. Here, our main concern is to study the difference that arises by using the partial redistribution with isotropic and dipole scattering functions.

Let us consider how photons diffuse through space. We have studied two cases of B/A=3 and 10, for the same set of velocities. We notice that when the photon has to diffuse through large distances, its probability of escape is reduced. This is true in both the cases of scattering functions. Therefore, the mean escape probabilities for all V's, for B/A=3 are consistently larger than those for B/A=10 for all V's. We have seen in Table 1 that the mean number of scatterings is smaller for T=1763 compared to those for T=1986. This means that a photon has lesser chance of reaching the wings and hence a small escape probability. However, these two effects are highly compensated by the radially moving matter. When the gases which emit and scatter radiation move rapidly towards the outer surface of the atmosphere, the photons are enabled to escape the medium more easily. At V= 30, the escape probability is nearly 0·9 in the case of B/A=3 and isotropic scattering. However, this is reduced to 0·6, when the geometry is extended by three times. A similar analysis holds good for the escape probability with dipole scattering. It is interesting to note that only twice as many photons escape by isotropic scattering compared to dipole scattering. More photons are back-scattered in the case of dipole scattering than in the case of isotropic scattering.

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