A JOURNEY TO THE CENTRE OF A NEUTRON STAR

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INTRODUCTION:

The interest in the structure of neutron stars dates back to the early nineteen thirties when, shortly after the discovery of the neutron, it was realized that a gas composed of neutrons could form a stable astrophysical configuration. Around the same time, supernovae were discovered, and this prompted Zwicky to suggest that neutron stars may be formed as a result of supernova explosions. For quite a while thereafter the field remained dormant until the appearance of pulsars with their plausible identification with rotating neutron stars. This has led to an accelerated interest in the physics of neutron stars, especially from the viewpoint of the equation of state governing matter at high densities, while the recently discovered compact X-ray sources have focussed more sharply the role played by neutron stars in close binary systems.

We shall be primarily concerned in this communication with the internal structure of neutron stars. For this purpose let us undertake an imaginary journey from the surface to the centre of a neutron star, a distance typically of about 10 km over which there is a large variation of the density from approximately 10^7 gm/cm^3 at the surface ranging upto and beyond 10^{15} gm/cm^3 in its interior. The neutron star matter is as a result composed of a number of layers with different physical characteristics. What makes the study of the internal structure so fascinating is the confluence of various disciplines such as nuclear, particle, solid state and relativistic physics which goes to determine the march of the physical variables with depth below the surface. This has naturally stimulated a sizeable community of physicists to attempt to construct equations of state of matter at high density. The principal concern here is an understanding of the properties of matter at densities exceeding 10^{15} gm/cm^3 where the limiting mass for a stable neutron star is attained in model calculations. A knowledge of this limiting mass is highly desirable because recent attempts to discover black holes in our galaxy rely on some way to distinguish between a black hole and a neutron star on the basis of the mass. Current ideas regarding the compact X-ray source, Cygnus X-1, suggest a model, involving an accreting compact object with mass exceeding 6 $M_\odot$ orbiting around a normal star (Bolton 1972). Since the limiting mass for a stable neutron star is probably in the vicinity of 2 $M_\odot$, the object Cygnus X-1 presents itself as a strong candidate as a black hole. Apart from considerations regarding the limiting mass, the state of matter at high densities also plays a role in influencing the thermal history of a neutron star, as well as its detailed dynamics.

THE CRUST:

The outer layers of a neutron star having densities of the order of 10^7 gm/cm^3 are expected to be in the form of a solid crust. This region consists of bare nuclei immersed in a gas of relativistic electrons whose kinetic energies are so large compared to their interaction energy with the nuclei that the electrons are practically unperturbed by the presence of the nuclei. The nuclei are thus left with their charges unshielded from one another and the consequent strong Coulomb repulsion between the nuclei renders the system to achieve a minimum energy state by arranging themselves in a crystalline structure. The equilibrium distribution of nuclei in these layers can be determined by finding the nucleus that minimizes the total energy which includes electron energy and the lattice energy. This results in the nuclear species becoming progressively more neutron rich, beginning with ^{56}Fe and ending as ^{118}Kr at a density of about 4×10^{11} gm/cm^3 (Baym, Bethe and Pethick 1971). The extent of this crust below the surface varies from about 200 meters for heavy neutron stars ($M \gtrsim M_\odot$) to a few kilometers for lighter neutron stars ($M \gtrsim 0.2\, M_\odot$).

At this point in the interior the neutron drip sets in and deeper in the crust lies a lattice of nuclei coexisting with degenerate electrons and a degenerate sea of neutrons filling the interstitial space between the nuclei. As the density increases to about 10^{12} gm/cm^3, the neutron density in the interstitial region begins to rise making most of the contribution to the matter density and when the nuclear density is approached the nuclei are practically touching each other and eventually dissolve leaving a region composed of degenerate neutrons with Fermi energy $\sim$ 30 MeV.

Beyond the neutron drip point the nuclear physics becomes complicated and there exists a considerable body of work on the equation of state and the properties of nuclei in the region. Langer, Rosen, Cohen and Cameron (1969) employed a modified form of the semi-empirical mass formula to find that the nuclei dissolve into the background neutron sea at a density of 5×10^{11} gm/cm^3. Baym, Bethe and Pethick (1971) later undertook a microscopic description in terms of a compressible liquid-drop model to find that the crust survives upto a desnity of 2×10^{14} gm/cm^3. One rather curious feature of their computation was the size of the nuclei which were found to have mass number of several hundreds. This rapid increase in the mass number with the density was essentially due to an underestimate of the decrease caused in the nuclear surface energy by the presence of the surrounding neutrons—a situation which was later remedied by Ravenhall, Bennett and Pethick (1972) who did a Hartree-Fock calculation and found no tendency towards the build-up of giant nuclei. An independent computation by Buchler and Barkat (1971) based on a Thomas-Fermi model of the nuclei also indicated the dissolutoin of the crust around 2×10^{14} gm/cm^3 with no tendency towards giant nuclei. Recently Negele and Vautherin (1973) approached the problem via the density matrix approximation and found results essentially similar to those obtained by Buchler and Birkat and by Ravenhall et al. A novel feature of Negele and Vautherin calculation was a sharp jump in the mass number with increasing density due primarily to shell effects. It seems the structure and composition of the outer layers of a neutron star is now comparatively well understood and it is generally believed the crust extends below the surface upto a density of the order of 2×10^{14} gm/cm^3.

SUPRANUCLEAR DENSITY REGION :

Once the nuclei have dissolved the neutron star matter is composed largely of neutrons with a small admixture of protons and electrons. However, when the Fermi level of the electrons exceeds the muon rest mass, it becomes energetically convenient to introduce a negative muon with zero kinetic energy than to have an electron with its kinetic energy exceeding the muon rest mass energy . Langer et al. (1969), for example, find the muon threshold in the vicinity of 2.2×10^{14} gm/cm^3. At about this density the neutrons at the top of the Fermi sea interacting via an attractive potential are expected to be in a superfluid state (Clark and Chao 1969) which is maintained above the nuclear density, when the $^1S_0$ state tends to become repulsive, but the $^3P_2$ state still remaining attractive and thus allowing for an anisotropic superfluidity. In this region the protons at the top of the Fermi sea will very likely form a conventional superconducting fluid and it has been argued by Baym, Pethick and Pines (1969) that such a superconducting fluid of protons will not exclude the presence of a magnetic field as a result of the large electrical conductivity of the electrons making the characteristic time-scale for expulsion of the order of 10^{13} years. Greenstein (1970) has pointed out some interesting consequences of the interaction of the rotation field of a neutron star with its superfluid interior, with the former exhibiting itself by setting up a quantized vortex cores which are composed of normal matter.

It should be emphasized that a good equation of state is necessary for the computation of plausible neutron star models. The physics of the matter inside a neutron star does indeed appear to be reasonably well understood upto and a little above the nuclear density. But as we have noted earlier, it is the properties of matter in the neighbourhood of 10^{15} gm/cm^3 which are crucial in determining the limiting mass as well as the thermal history of a neutron star and its detailed dynamics of rotation. In what follows we shall discuss some of the possible states suggested in this density region.

The original calculations of Landau (1932) and Oppenheimer and Volkoff (1939) assumed a non-interacting Fermi gas of neutrons to yield a limit of $0.7M_\odot$ for a stable neutron star. The inclusion of the nuclear forces between neutrons naturally raises this limit, but the exact value of the upper limit is clearly subject to a considerable amount of uncertainty as can be seen from the results of Tsuruta and Cameron (1963) who obtained significantly different equations of state upon using Levinger and Simmons velocity-dependent potentials V_β and V_γ. This prompted Banerjee, Chitre and Garde (1970) to adopt an approach which assumes that when the nuclear forces become sufficiently repulsive, a possible minimum energy state can be achieved by keeping the neutrons as far away from one another as possible, i.e. by localizing them at lattice sites. Such a solid lattice of neutrons would be a nuclear analogue of the Coulomb lattice of nuclei which exists in the crust. The lattice calculation was performed with the harmonic approximation using the classical Debye model and employing the Reid Soft-core potential for the interaction between neutrons to find that neutron matter would crystallize in the vicinity of 8×10^{14} gm/cm^3.

Shortly afterwards, Pandharipande (1971) calculated the binding energy per particle of a neutron liquid again using the Reid soft-core potential. In his lowest-order variational formulation the trial wave function is expressed as a product of single-particle wave functions and the short range correlation a lá Jastrow. A differential equation for the correlation function is then derived by minimizing the energy with a constrained variation. The energy which Pandharipande obtained using clusters upto two-body was found to be lower approximately by a factor 2 compared with the energy computed by Banerjee et al in the density range $7.5 \times 10^{14} \leqslant \rho \leqslant 6 \times 10^{15}$ gm/cm^3. This immediately demonstrated the need for a quantum mechanical computation of the neutron lattice which is expected to lower the energy in relation to the classical lattice calculation by spreading the wave function around each lattice site thus collecting more attraction in the final energy values. Furthermore, the classical harmonic oscillator treatment is not quite adequate for a fully satisfactory description of a neutron lattice which is a highly quantum crystal in the sense that the zero point energy of the neutrons is comparable with the potential energy and the amplitude of oscillation becomes a sizeable fraction of the interparticle distance. A fully quantum mechanical solid-state computation was therefore set up by Canuto and Chitre (1973) based on the t-matrix formulation which has been successfully employed to study the physical properties of quantum crystals like He^3 and He^4. The variational method, though used extensively in the context of solid He^3 , has the unfortunate feature of not lending itself to a straightforward inclusion of spin and angular momentum, features which in fact dominate the nuclear interaction. Canuto and Chitre were able to show that energetically it is convenient to have neutrons in a crystalline structure rather than in a liquid state for values of the density exceeding 1.6×10^{15} gm/cm^3. A study of the mechanical properties also indicated that a neutron lattice can withstand shearing stresses at about the same density where energetically the solid phase is preferred to the fluid phase.

There have been semi-empirical attempts to examine the possibility of the crystallization of neutron matter especially from the point of view of determining the solidification pressure. Anderson and Palmer (1971) appealed to de Boer's quantum mechanical law of corresponding states; in this approach the solidification of neutron matter is estimated by scaling the known low temperature properties of quantum solids like He^3 to those characteristic of nuclear distances and energies. Anderson and Palmer obtained a solidification density for neutron matter in the vicinity of 5×10^{14} gm/cm^3, a result which was not significantly altered by a later refined version by Clark and Chao (1972) who adapted an averaged Reid potential to fit a Lennard-Jones shape. This naturally raises an important question whether a neutron star may have a crystalline structure throughout its interior, a consideration which is crucial from the point of view of the dynamics of a neutron star. The scaling argument based on the law of corresponding states cannot be taken too literally in so far as its numerical predictions regarding the solidification density are concerned. Nevertheless, it provides a positive answer to the question whether neutron matter will solidify under a sufficiently high pressure.

Recently there have been some more attempts to investigate the problem of the crystallization of dense neutron matter. Coldwell (1972) performed an Hartree-Fock calculation treating the degree of localization as a variational parameter to find that at a density of 7.8×10^{14} gm/cm^3 the neutron matter should crystallize. Coldwell makes use of an averaged two-body soft-core Reid potential for the interaction and his work is incomplete in the sense that the distortion of the wave function caused by the short-range repulsion which is incorporated by the introduction of a correlation function is completely neglected. Nosanow and Parish (1972) have treated the crystallization problem by using the Monte Carlo method and they find solidification of neutron matter at a somewhat low value of the density, $\sim 4.2 \times 10^{14}$ gm/cm^3. The chief merit of the Monte Carlo method is that it deals with the many-body clusters very satisfactorily, but the correlation function in this formulation has to be prescribed to have a form of the type, exp — $(b/r)^n$, where b and n are treated as variational parameters. Such a choice forces the correlation function to be practically zero until a distance of approximately 0.25 fermi and this results in reducing substantially the contribution of the repulsive part of the interaction. Thus a state independent choice of the correlation function is probably responsible for the rather low solidification density obtained by Nosanow and Parish. Schiff (1973) approached the solidification problem by arguing that at high desnity the dominance of the short-range repulsion is going to make the Pauli principle somewhat unimportant. Thus treating the Pauli principle as a perturbation the neutron matter is regarded as a Bose system to get a solidification density of $(2.9 \pm 0.5) \times 10^{15}$ gm/cm^3 with an approximate choice for the neutron-neutron interaction. This work emphasises that the main problem concerning the solidification of dense neutron matter is not so much the many-body techniques as the choice of the two-body interaction potential.

Pandharipande (1972) has attempted to study the solidification problem within the framework of his variational method and he finds no evidence for solidification of neutron matter upto a density of 3.3×10^{15} gm/cm^3. This could be a result of his formalism which takes no account of the dependence of the correlation function on angular momentum. Such an approximation is very unrealistic in the sense that the nuclear forces are known to be highly spin and angular momentum dependent and the behaviour of the interaction potentials near the origin is rather different from one wave to another, features which can hardly be incorporated in the choice of just one correlation function for all the states. In fact the final energy values are quite sensitive to the shape of the correlation functions near the origin. More seriously, Pandharipande uses the central part of the $^3P_2$ wave as the interaction uniformly for all the triplet states. There is no compelling physical reason for this choice in view of the fact that the full potential for $^3P_2$ wave consists of three components: central, tensor and LS. The LS-component in fact contributes a good deal of attraction which is completely ignored by retaining only the central part and this feature makes the triplet states mildy attractive ($\sim$10 MeV) instead of the full $^3P_2$ wave having an attraction of $\sim$55 MeV. It is therefore not surprising that the energy per particle for a neutron lattice is consistently overestimated in Pandharipande's work and he finds no evidence for crystallization upto a density of 3.3×10^{15} gm/cm^3. We hope that the results of various computations on the solidification of neutron matter will converge in not too distant future, and if the neutron matter does crystallize at some density which might typically prevail in the interior, then neutron stars would have a solid inner core in addition to the solid outer crust. Such a solid core would not affect so much the limiting mass which comes out to be 1.39 $M_\odot$ not altogether different from a maximum mass of 1.66 $M_\odot$ obtained by Baym et al with a fluid core. But the presence of a solid core inside a neutron star would significantly affect the dynamics of rotation and this has some observational consequences. The starquake theory of pulsar speed-ups (glitches) which has been so successful in accounting for the glitches observed in the Crab pulsar can be applied to the Vela pulsar only if it is assumed that the latter possesses a solid core. Pines, Shaham and Ruderman (1972) have put forward convincing arguments to explain the Vela glitches as arising from corequakes suddently releasing the elastic energy stored in the inner solid core, whose shear modulus is some five orders of magnitude larger than that of the crust and as a result the core has sufficient elastic energy to power the starquakes of the right magnitude and frequency. Furthermore, a rotating neutron star with an oblate solid core has a relatively rapid wobble and this could be the clock mechanism for the 35 day cycle observed in the accreting close X-ray binary, Hercules X-1 (Lamb et al 1972). Dyson (1972) has pointed out that in an elastic medium the transverse modes could get coupled with the gravity modes for a solid core with high density and sound velocity to be an efficient source of gravitational radiation.

There are other states in which matter can exist at these high densitites. Sawyer and Scalapino (1973) and Migdal (1972) have argued that at a few times the nuclear density, the neutron star matter should make a transition to a phase which has approximately equal number of neutrons, protons and negative pions, via $n \rightarrow p + \pi^-$, once the neutron-proton chemical potential difference exceeds the negative pion rest mass. One of the astrophysical implications of pion condensation would be to affect the cooling rates of neutron stars. The appearance of pions would also effect a drastic reduction in the pressure which in turn would alter the limiting mass. It remains to be seen whether such a state is compatible with crystallization. We also expect the hyperons to appear in this density region, but the knowledge of the hyperon-nucleon and hyperon-hyperon interaction pontentials is so meagre that any conclusions that can be drawn regarding the hyperonic composition at these densities will necessarily be tentative. The equation of state of matter at densities very much beyond 10^{15} gm/cm^3 remains largely conjectural. There have been several discussions of Hagedorn-type equations of state at ultrahigh densities (Frautschi et al 1971) yielding very soft equations of state. The main problem in this region is not so much the lack of a reliable method, but rather whether the matter at these very high densities can be described on the basis of phenomenological forces between particles. It may well be sometime before various theoretical proposals pertaining to the behaviour of matter in the density range $2\times10^{14}\leqslant\rho\leqslant 10^{16}$ gm/cm^3 arrive at a convergent viewpoint.

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Brief Report on the 13th International Cosmic Ray Conference, Denver, Colorado, August 17-30, 1973

The invited lectures of astrophysical significance were by Pacini on Collapsed Objects, Reeves on Nucleosynthesis and Cosmic Rays, Gursky on New Developments in X-Ray astronomy, J. Geiss on Solar Wind Composition and History of Solar System, A. Hundhausen on Solar Magnetism, Corona and Solar Wind, Ostriker on Cosmic Acceleration Processes and Burbidge on Interstellar and Intergalactic Media. Pacini talked mainly about the oblique rotator model of pulsars. Gursky discussed the nature of galactic binary sources (speculating on whether we have detected a black hole), extragalactic sources and the present status of X-ray back-ground. Burbidge mentioned that the X-ray background does not present such an obstacle to the Gold-Hoyle Hot-Steady-State Universe, as it first seemed to do in the mid-sixties. Burbidge talked mainly about what constitutes the intergalactic medium—e.g. neutrinos, high energy cosmic rays, gravitational radiation, electromagnetic radiation at various wave-lengths, smoothed out distribution of matter and black holes. It seems if QSOs are not distant, we can fill up the medium with neutral hydrogen upto 10 percent of critical cosmological density required to close the universe. But it seems difficult to reach that value $\sim 4.7.\ 10^{-30}$ gms cm^{-3} on the present data. He mentioned another very recent unpublished observation on QSOs. Hazard's position measurements have led to identification of a 4C-source with a QSO of 17th mag at $z=0.45$. Within 5″ of that is another QSO with $z=1.4$. Also an emission line of Mg at $z=0.45$ coincides with an absorption line at $z=1.4$ of Mg. There are also emission lines of C IV common to both. This suggests they are related and their redshifts are not cosmological. The lectures on solar wind etc. mainly tried to build a self consistent picture of the solar magnetic field and charged particle motion.

There were contributed papers of various types grouped in a number of parallel sessions subjectwise—origin, airshowers, techniques, solar modulation, muons, high energy, etc. There was a Sarabhai Memorial session at which U.R. Rao (Chairman), M. G. K. Menon, B. Peters, B. Rossi spoke and a message from P.M.S. Blacket was read out.

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