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The chemical composition of Algol systems – II. The carbon and nitrogen abundances of the secondaries of U Cep and U Sge

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Summary. A spectrum synthesis analysis of Digicon spectra of CH and CN in the secondaries of U Cep and U Sge shows that they are carbon deficient, [C/Fe] ~ -0.5, and nitrogen rich, [N/Fe] ~ $+0.5$. The metal abundance of both secondaries is normal, [Fe/H] = 0.0 ± 0.3.

The C deficiency and N overabundance of the secondaries are the result of conversion of C to N by the CN cycle, while the secondaries were on the main sequence, followed by mixing to the surface. These abundances are therefore observational evidence in favour of the generally accepted idea that the secondaries of semi-detached systems are post-main-sequence objects.

The C deficiency and N overabundance of the secondaries are more marked than in the typical single G or K giant. We interpret this as a result of mass loss of the unprocessed surface material prior to the onset of mixing. At the time of mixing in the secondaries there was less unprocessed material and therefore less dilution of the CN cycle processed material than in a single G or K giant.

Mass loss subsequent to mixing does not alter the surface composition of the secondaries. This explains why secondaries that have suffered much more extensive mass loss than those of U Cep or U Sge do not show the extremes of composition that one would expect in the absence of mixing. An example is the secondary of S Cnc (K2), which has a mass of only $0.2 M_\odot$ and for which – from a preliminary analysis of CH – we find [C/Fe] ~ -0.5.

1 Introduction

In semi-detached close binary systems, mass transfer between the components and mass loss from the system may affect the chemical composition of both the more massive primary and the less massive secondary. The very low mass secondaries in a few Algol systems (e.g. AS Eri, S Cnc with $M_2 \sim 0.2 M_\odot$) are supposed to be in the last phase of mass transfer, and are expected to have a helium core surrounded by a thin hydrogen-burning envelope.

If an evolved component in an Algol system has lost mass down to layers previously processed by the CN-cycle, it should now show C and N abundance anomalies relative to normal red giants. The late-type subgiant and giant components of Algol systems have deep convective envelopes. This means that even if the mass loss did not remove all the unprocessed outer layer, C and N abundance anomalies relative to normal red giants are still to be expected, because in the mixed envelope there is less dilution of the CN cycle processed material than in a single red giant which has not undergone mass loss.

In Paper I (Parthasarathy, Lambert & Tomkin 1979), we showed that, contrary to published photometric and spectroscopic analyses, the secondary of U Cep possesses an essentially normal (i.e. solar) metal abundance. In this paper, we show that the secondary of U Sge is similarly normal and we extend the abundance analyses to carbon and nitrogen in the secondaries of both U Cep and U Sge.

2 Observations

Spectra of the secondaries of U Cep and U Sge were obtained during the total phase of primary eclipse using the McDonald Observatory 2.7-m telescope and the Tull coudé spectrometer equipped with either a 1024-element Reticon self-scanned silicon photodiode array detector (Vogt, Tull & Kelton 1978) or a self-scanned Digicon (Tull, Choisser & Snow 1975). The exposure time was always much shorter than the duration of totality so that the spectrum of the secondary was obtained without contamination by the much brighter primary.

A Reticon spectrum centred at 6320 Å and covering 210 Å was obtained to provide the metal abundance of the secondary of U Sge. This region (see Paper I) was chosen because it contains a selection of weak and moderately strong lines as well as several continuum points. A standard star – δ CrB, a giant of similar spectral type to that of the secondary of U Sge – was observed immediately following the U Sge observation. This standard star was observed at both the low (1.84 Å) resolution employed for the observation of the secondary of U Sge and also at higher resolution (0.23 Å). Note that the 1.84 Å resolution spectrum effectively fully resolves the lines in the spectrum of the secondary of U Sge because the stellar lines are rotationally broadened thanks to the synchronous rotation of the secondary; $v \sin i \simeq 80$ km s^{-1} or $\Delta\lambda_{\rm rot} \sim 1.7$ Å at 6320 Å.

Digicon spectra of CH ($A^2\Delta - X^2\Pi$) and CN ($B^2\Sigma^+ - X^2\Sigma^+$) lines were obtained for the C and N abundance analyses. Exposures centred at 4310 Å were our primary source of CH lines. Exposures at 4215 and 3883 Å provided the CN $\Delta v = -1$ and 0 sequences respectively. Each of these Digicon spectra covered 120 Å at a resolution of 0.75 Å. Observations of δ CrB

Table 1. Details of the observed spectra of the secondaries of U Cep and U Sge.

	Date (UT) (of mid-exposure)	Phase (at mid-exposure)	Wavelength (Å)	Exposure time (min)	Resolution (Å)	Å/ Diode
U Cep	1980 July 21.355	0.988	4239–4364	35	0.75	0.134
U Cep	1980 July 21.387	0.0005	4130–4255	30	0.75	0.134
U Cep	1980 July 21.411	0.010	4350–4470	20	0.75	0.134
U Cep	1980 July 21.428	0.017	3939–3964	10	0.75	0.135
U Cep	1980 July 26.388	0.006	3807–3932	50	0.75	0.135
U Sge	1978 June 22.292	0.000	6211–6423	80	1.84	0.230
U Sge	1989 July 25.288	0.993	4247–4372	19	0.75	0.134
U Sge	1980 July 25.305	0.998	4137–4262	24	0.75	0.134
U Sge	1980 July 25.331	0.006	3836–3961	37	0.75	0.135
S Cnc	1982 Feb. 8.436	0.002	4247–4372	23	0.54	0.135

(G5 III) and κ Gem (G8 III), which is the standard for the secondary of U Cep, were made at the same wavelengths and resolution. Details of the observations are given in Table 1.

3 The abundance analysis

3.1 METHOD

In our search for chemical composition changes in the secondaries of Algol systems we first compare the observed spectra of the secondaries of U Cep and U Sge with the broadened spectra of the selected standard stars. The Hγ, Hζ and Hη absorption lines in the spectra of the secondaries of U Cep and U Sge are normal and free of emission and the atomic absorption features match very well with the corresponding atomic absorption features in the broadened spectra of κ Gem and δ CrB respectively. This comparison strongly suggests that (i) the secondaries of U Cep and U Sge have normal metal abundance and (ii) the C depletion is in excess of the evolutionary changes observed in normal giants. Spectrum synthesis is then applied to confirm that the spectroscopic changes are not simply the result of ill-chosen standard stars and to calibrate the line intensity differences in terms of abundance differences.

3.2 BASIC DATA FOR THE SECONDARIES OF U CEP AND U SGE

Application of spectrum synthesis requires the selection of model atmospheres with effective temperatures and surface gravities representative of the secondaries of U Cep and U Sge. Since reliable masses and radii of the secondaries of U Cep and U Sge are now available (Tomkin 1979, 1981; Popper 1980; Batten 1974), the surface gravities are known (see Table 2).

In Paper I, we showed that the observed spectrum of the secondary of U Cep corresponds to $T_{\mathrm{eff}} = 5000$ K and [Fe/H] = 0.0, which values are adopted here. We also showed that κ Gem (G8 III) is a close match to the secondary of U Cep. The basic parameters of κ Gem were discussed in Paper I, i.e. $T_{\mathrm{eff}} = 5000$ K, $\log g = 3.0$, [Fe/H] = 0.0 (see Williams 1971; Hansen & Kjaergaard 1971; Burbidge 1957).

Table 2. Physical parameters of U Cep and U Sge.

	U Cep		U Sge		Ref.
	Primary	Secondary	Primary	Secondary	
Period (days)	2.493		3.381		
V	6.96	9.11	6.61	9.13	
Spectral type	B7 V	G8 III–IV	B8.5 IV–V	G3 III–IV	1
Mass ($M_\odot$)	4.2	2.8	5.7	1.9	2, 3, 4, 5
q (mass ratio)	0.667		0.333		
Radius ($R_\odot$)	2.9	4.9	4.1	5.3	3
$\log g$	4.14	3.50	3.97	3.27	
T_{eff} (K)		5000		5600	
V_{rot} (km s^{-1})		100		80	
M_V	−0.9	2.5	−1.0	1.5	3
$\log L/L_\odot$	2.4	1.2	2.6	1.4	3

1. Batten, Fletcher & Mann (1978).
2. Batten (1974).
3. Popper (1980).
4. Tomkin (1979).
5. Tomkin (1981).

From a comparison of the observed Strömgren $(b-y)_0$ colour of the secondary of U Sge (McNamara & Feltz 1976) with theoretical $(b-y)$ colours computed by Bell & Gustafsson (1978) we find a preliminary T_{eff} of 5420 K. The standard star δ CrB is a close match to the secondary of U Sge. For δ CrB, Williams (1971) and Hansen & Kjaergaard (1971) obtained $T_{\mathrm{eff}} = 5250$ K. Glebocki (1972) from a curve of growth analysis of the spectrum of δ CrB gave $T_{\mathrm{eff}} = 5250$ K. The effective temperature difference between the secondary of U Sge and δ CrB is 170 K according to $(b-y)$. The metal abundance of δ CrB is close to solar (Williams 1971; Glebocki 1972; Campbell 1978). As a check, the high (0.23 Å) and low (1.84 Å) resolution 6320 Å spectra of the standard star δ CrB were analysed and found to yield similar results. Equivalent widths of neutral iron lines were measured off the higher resolution (0.23 Å) spectra. The iron abundance of δ CrB was computed for each neutral unblended iron line using a standard line analysis computer program (Sneden 1974) and the $T_{\mathrm{eff}} = 5250$ K, $\log g = 3.0$, $[\mathrm{Fe/H}] = 0.0$ model atmosphere from the grid of Bell *et al.* (1976). If the criterion that the derived abundances show no correlation with the excitation potential of the line is applied, the best fit to the 6320 Å region spectrum of δ CrB is obtained for $T_{\mathrm{eff}} = 5400$ K, $\log g = 3.0$, $[\mathrm{Fe/H}] = 0.0$ and $\xi = 1.2\ \mathrm{km\,s^{-1}}$. Thus, we adopt $T_{\mathrm{eff}} = 5400$ K, for δ CrB. On the basis of the effective temperature difference between δ CrB and the secondary of U Sge, we adopt $T_{\mathrm{eff}} = 5600$ K for the secondary of U Sge. The Hγ, Hζ and Hη lines in the spectrum of the secondary of U Sge appear to be normal photospheric absorption lines; comparison of the Hγ line intensity with the standard star's Hγ line intensity suggests a spectral type of G2 which is in agreement with the adopted effective temperature. The adopted basic parameters of the secondaries of U Cep and U Sge are given in Table 2.

A microturbulent velocity of $\xi = 2.0\ \mathrm{km\,s^{-1}}$ which is an average value for G and K giants (Lambert & Ries 1981) is adopted in the analysis of the spectra of the secondaries of U Cep and U Sge. For the standard stars κ Gem and δ CrB the microturbulent velocity was set through the requirement that the neutral iron abundances show no trend with line equivalent widths.

In order to compare the spectra of the secondaries of U Cep and U Sge with either their standard stars or synthetic spectra, the latter must be convolved to the same effective resolution, i.e. the rotational broadening must be applied. The orbital periods and radii of the U Cep and U Sge secondaries demand equatorial rotational velocities of 100 and $79\ \mathrm{km\,s^{-1}}$ respectively on the assumption that they are rotating synchronously. To check this prediction, we matched the spectra of the secondaries of U Cep and U Sge to Gaussian broadened spectra of the standard stars. The broadening velocity, V_{G}, of the Gaussian velocity distribution $\exp[(-V/V_{\mathrm{G}})^2]$, is related to the rotational velocity by $V_{\mathrm{G}} = 2/3\, V_{\mathrm{rot}} \sin i$ (Unsöld & Struve 1949). The blue (4200, 4300 Å), red (6320 Å) and near-infrared spectra (8620 Å) of the standard stars were broadened and compared with the spectra of the secondaries of U Cep and U Sge. Strong atomic absorption features were used in this comparison because such lines are insensitive to moderate differences in abundance, effective temperature and surface gravities between the standard stars and the secondaries of U Cep and U Sge. The results are $V_{\mathrm{rot}} \simeq 100 \pm 10\ \mathrm{km\,s^{-1}}$ for the secondary of U Cep and $80 \pm 10\ \mathrm{km\,s^{-1}}$ for the secondary of U Sge which encompass the predicted synchronous velocities.

3.3 THE METAL ABUNDANCE OF THE SECONDARY OF U SGE

The 6320 Å Reticon spectrum is our primary source of the metal abundance for the secondary of U Sge. In Fig. 1, we show a portion of this spectrum together with the

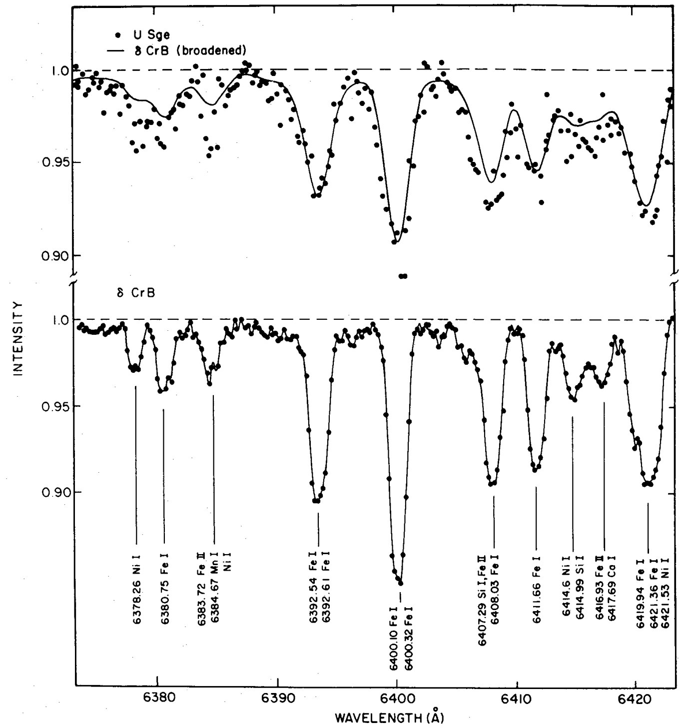

Figure 1. Spectra of the secondary of U Sge and the standard star δ CrB (G5 III). The lower panel shows the δ CrB spectrum at a resolution of 1.84 Å. In the upper panel, the δ CrB spectrum has been rotationally broadened (solid line) and superimposed on the spectrum of the secondary of U Sge (filled circles).

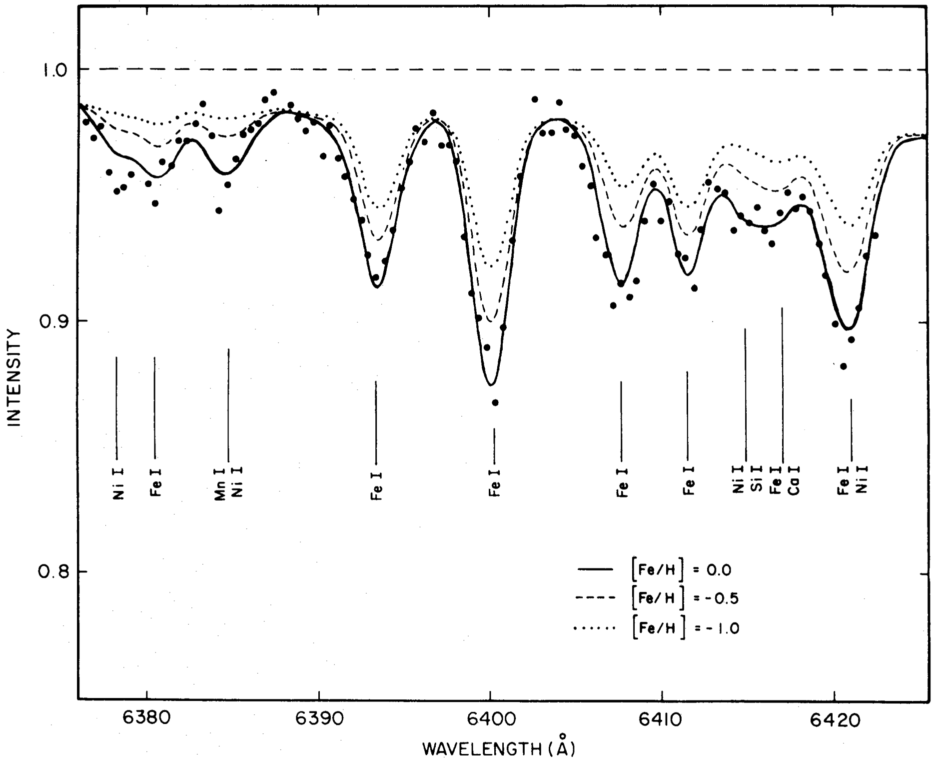

Figure 2. Observed (filled circles) and synthetic spectra of the secondary of U Sge. Synthetic spectra are shown corresponding to a model atmosphere with $T_{\rm eff} = 5600\,$K, $\log g = 3.0$ and ξ (microturbulence) = $2\,{\rm km\,s^{-1}}$ and different metal abundances. The observed spectrum has been smoothed by coadding counts in pairs of adjacent channels.

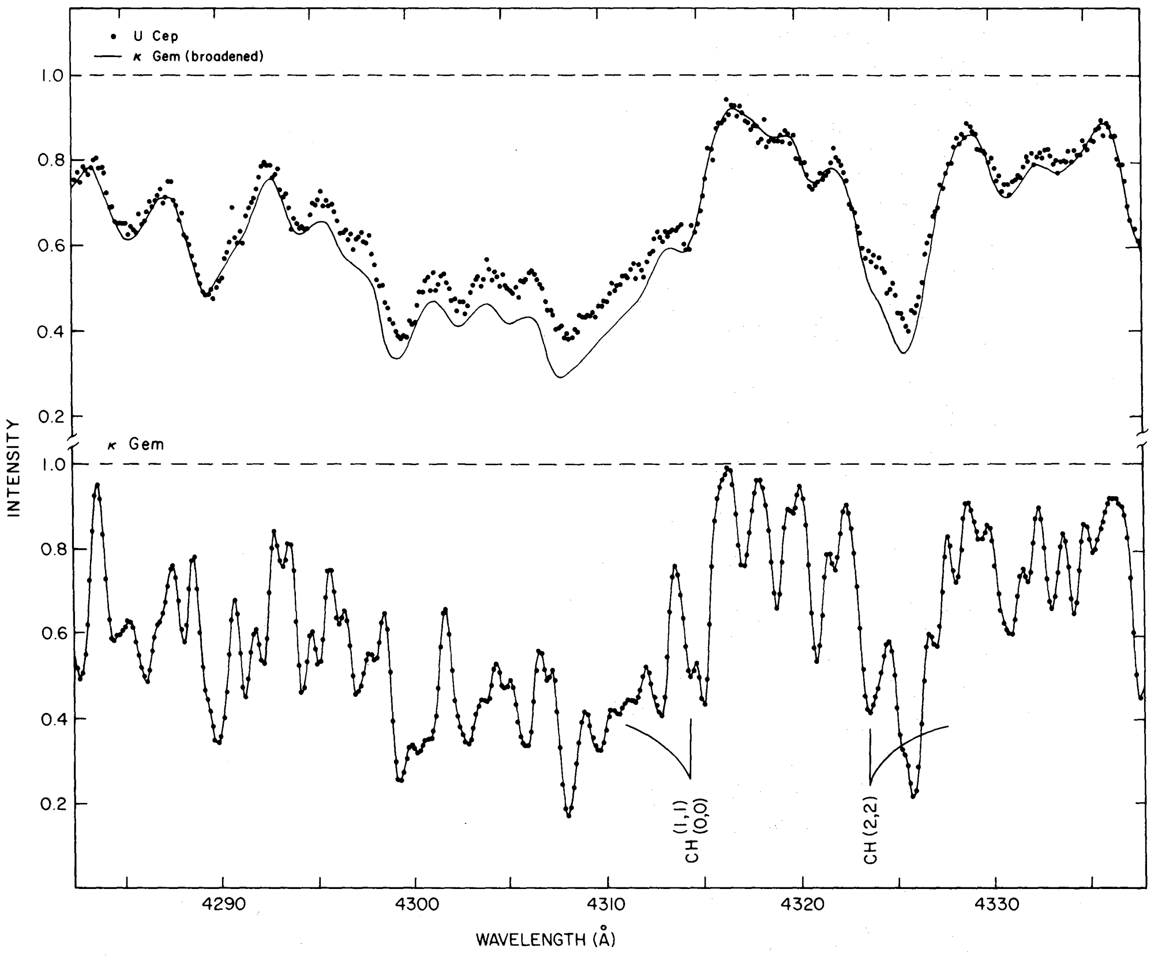

Figure 3. Spectra near 4310 Å of the secondary of U Cep and the standard star κ Gem (G8 III). The lower panel shows a 0.75 Å resolution spectrum of κ Gem. After broadening to account for the higher rotational velocity of the secondary, the κ Gem spectrum fits the spectrum of the secondary of U Cep except in regions where CH $A^2\Delta$–$X^2\Pi$ $\Delta v = 0$ sequence is a major contributor.

spectrum of δ CrB after broadening to 80 km s^{-1} rotational velocity. Although δ CrB and the secondary of U Sge may not be a perfect match in effective temperature and gravity, this direct comparison of their spectra surely eliminates the possibility that it is significantly metal deficient. Naftilan (1976), who analysed 15 Å mm^{-1} spectra of the 3500–4800 Å region, suggested a metal deficiency of [Fe/H] = − 0.7.

A more precise evaluation of the metal abundance of the secondary of U Sge was attempted through spectrum synthesis. The line list was compiled from the solar spectrum and the extensive tables provided by Kurucz & Peytremann (1975). Oscillator strengths for these lines were obtained primarily from inversion of the solar equivalent width using the Holweger–Müller (1974) model solar atmosphere. We synthesized the spectrum of the secondary of U Sge. The interval 6323 to 6426 Å was synthesized using interpolated model atmospheres ($T_{\rm eff}$ = 5600 K, log g = 3.0, [Fe/H] = 0.0) from the grid of Bell *et al.* (1976). A microturbulent velocity ξ = 2.0 km s^{-1} is adopted. The synthetic spectra were convolved with the instrumental profile and broadened to account for the rotational velocity of the secondary of U Sge. Synthetic spectra were also computed using metal-deficient model atmospheres ($T_{\rm eff}$ = 5600 K, log g = 3.0, [Fe/H] = − 0.5 and − 1.0). Observed and synthetic spectra in the wavelength interval 6377 to 6426 Å are compared in Fig. 2. The synthetic

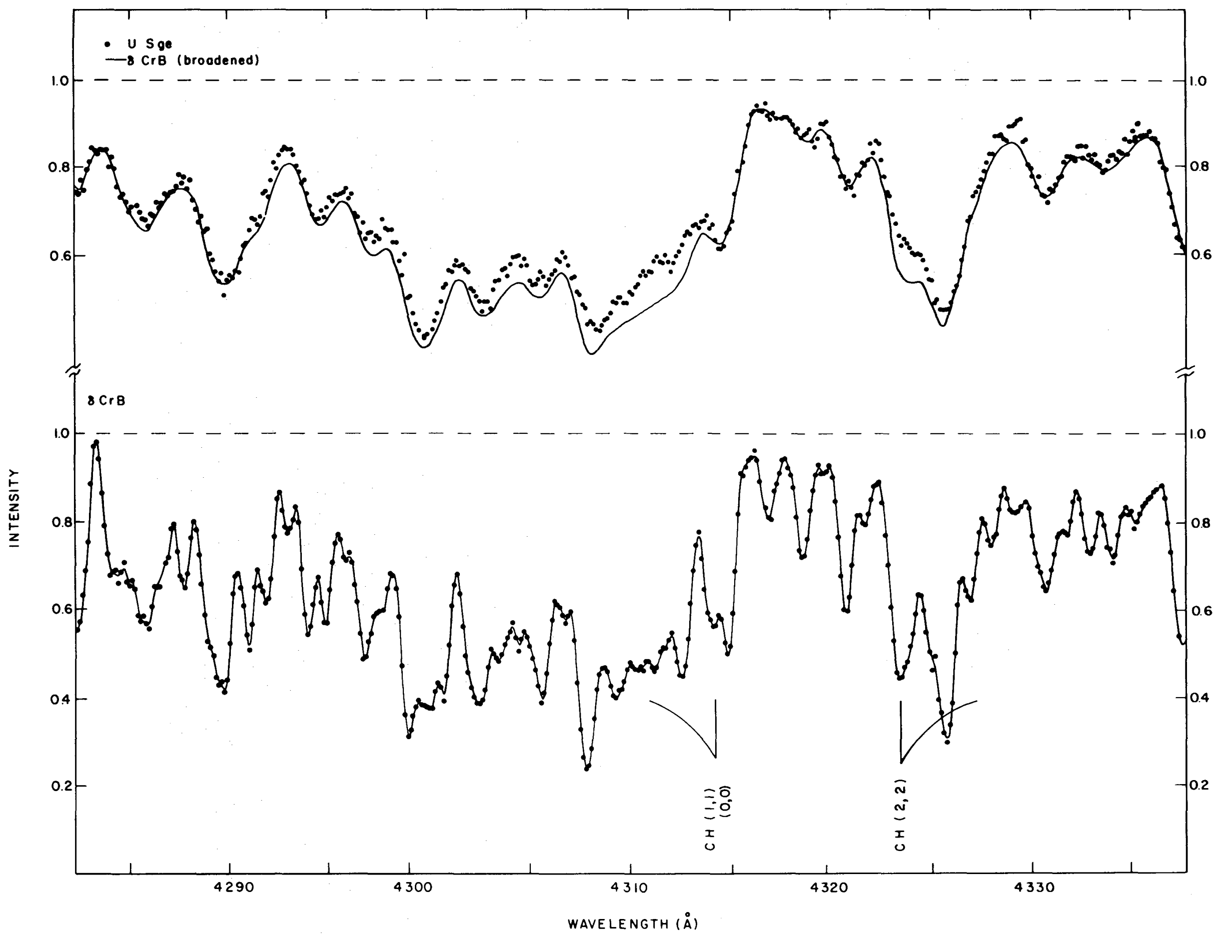

Figure 4. Spectra near 4310 Å of the secondary of U Sge and the standard star δ CrB. See caption to Fig. 3.

spectrum computed using the model atmosphere $T_{\text{eff}} = 5600\,\text{K}$, $\log g = 3.0$, $[\text{Fe/H}] = 0.0$, is a rather good fit to the observed spectrum of the secondary of U Sge. Our analysis shows that the iron-peak elements have normal abundances of $[\text{Fe/H}] = 0.0 \pm 0.3$. We do not confirm the metal deficiency ($[\text{Fe/H}] = -0.7$) claimed by Naftilan (1976). The fact that the secondary of U Sge has a normal metal abundance is most clearly shown by the direct comparison with δ CrB (Fig. 1); spectrum synthesis is here employed to calibrate the visual impression supplied by this comparison.

3.4 CARBON AND NITROGEN

The numerous CH lines in the 4290–4328 Å interval are the source of the C abundance. In Figs 3 and 4 we compare directly a portion of the observed spectra of the secondaries of U Cep and U Sge with the spectrum of the respective standard stars. The spectra of the standard stars are broadened to match the rotational broadening of the spectra of the secondaries. A small C deficiency for the secondaries of U Cep and U Sge relative to the standard stars κ Gem and δ CrB is found by this direct comparison (Figs 3 and 4). Note that the broadened spectra of the standard stars and the spectra of the secondaries are quite similar in the intervals to which CH is a minor contributor, e.g. λλ 4283 to 4290 Å, λλ 4315 to 4322 Å and 4330 to 4337 Å. This correspondence is additional evidence that neither secondary is significantly deficient in metals.

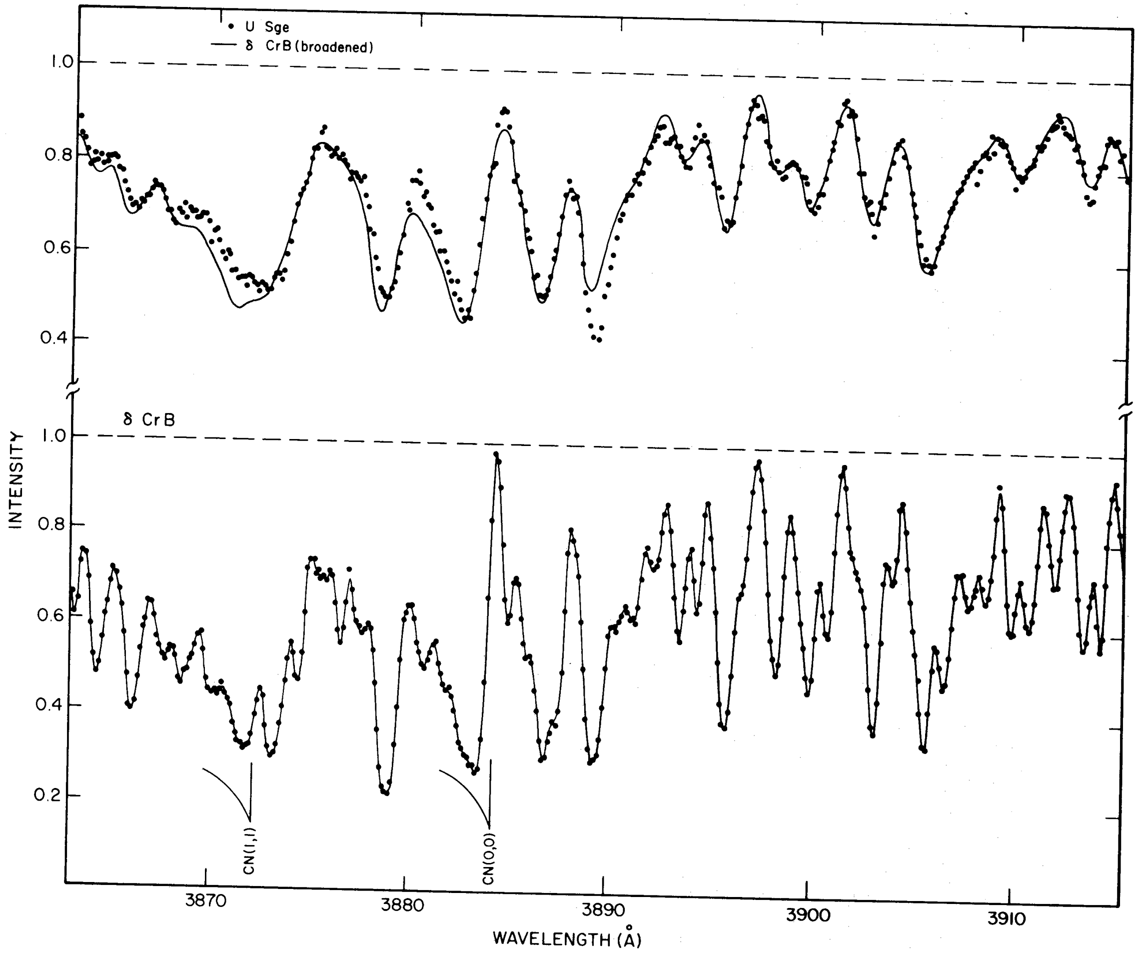

Figure 5. Spectra near 3980 Å of the secondary of U Sge and the standard star δ CrB. The lower panel shows a 0.75 Å resolution spectrum of δ CrB. After rotational broadening, the δ CrB spectrum (solid line in upper panel) fits the spectrum of the secondary of U Sge except for a slight mismatch within the blue-degraded bands of the CN $B^2\Sigma^+–A^2\Sigma^+$ $\Delta v = 0$ sequence.

The CN violet system lines in the secondaries of U Sge and U Cep are slightly weaker than in the standard stars. Figs 5 and 6 show the comparison of U Sge with its standard star for the two CN regions. Because the strength of CN depends on the product of the C and N abundances and C is deficient the weakness of CN does not necessarily mean that N is deficient; in fact we will find that it is overabundant. These suggestions are given a quantitative form by an application of spectrum synthesis.

CH lines from the (0, 0), (1, 1) and (2, 2) bands of the $A^2\Delta–X^2\Pi$ system dominate the selected 4290–4328 Å interval. These lines are predicted to be rather insensitive to $T_{\rm eff}$ and $\log g$ errors. Two regions – 4190 to 4223 Å and 3860 to 3887 Å – were selected as containing a useful number of CN lines. Our line lists were compiled from Moore & Broida (1959), Grevesse & Sauval (1973), Sneden (1974) and Kurucz & Peytremann (1975). Basic data for CH and CN and the solar CNO abundances were taken from Lambert (1978). Oscillator strengths for atomic lines were computed from the solar equivalent widths with elemental abundances from Ross & Aller (1976). We checked that the synthetic solar spectrum matched the observed solar spectrum (Delbouille, Neven & Roland 1973). After establishing the atomic and molecular line list, synthetic spectra were computed and

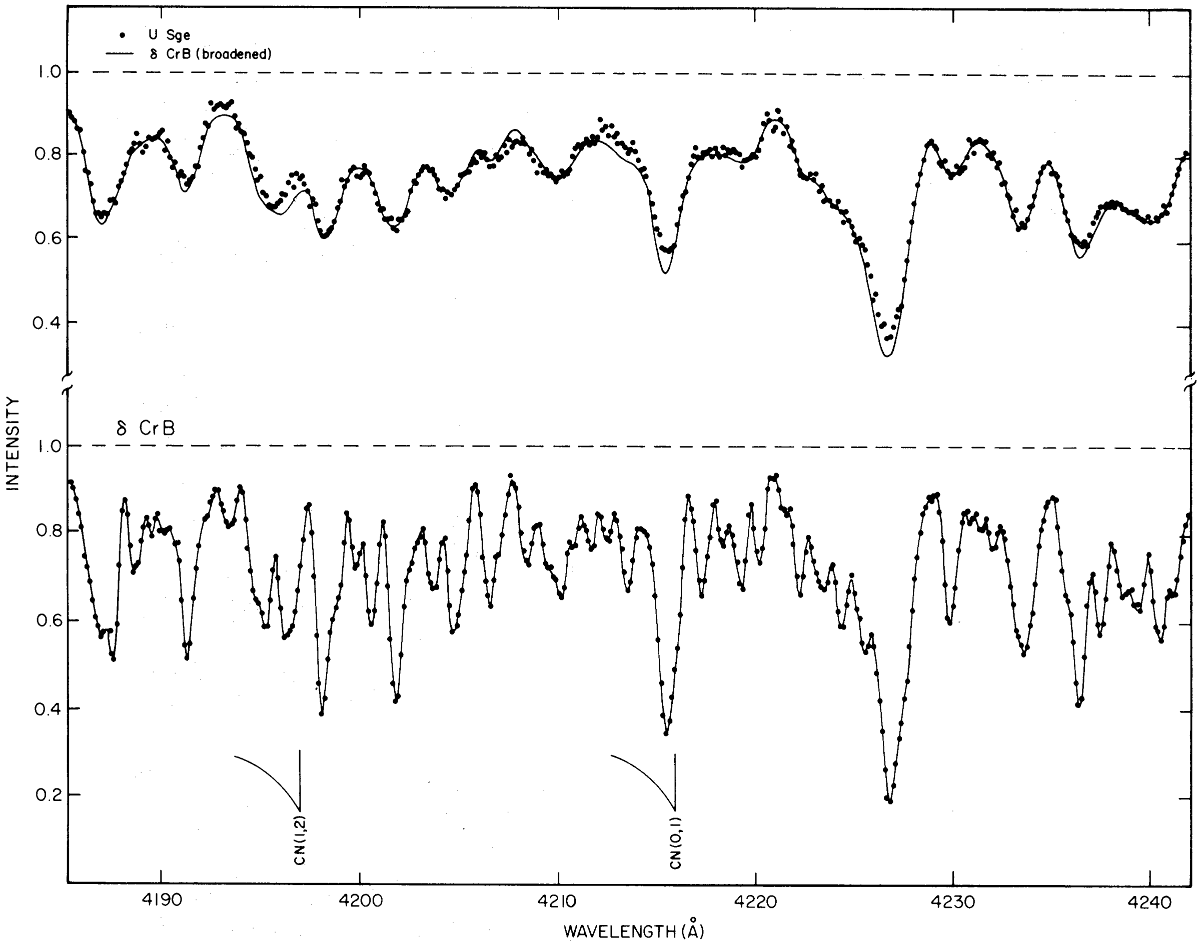

Figure 6. Spectra near 4210 Å of the secondary of U Sge and the standard star δ CrB. See caption to Fig. 5. Location of two bandheads belonging to the CN $B^2\Sigma^+–X^2\Sigma^+$ $\Delta v = -1$ sequence are shown. CN is not the major contributor in this interval.

convolved with the instrumental profile. The synthetic spectra of the secondaries of U Cep and U Sge were then rotationally broadened, to compare with the observed spectra.

The carbon abundance was found by matching the synthetic and observed spectra in the 4290–4328 Å region. The continuum was set from the relatively line-free region just longward of 4315 Å. The CH density in the stellar atmosphere is affected by both the C and the O abundances; however, at $T_{\rm eff} = \sim 5600$ K there is little loss of C to CO. The oxygen abundance of δ CrB was derived using the 6363 Å [O I] line. For κ Gem and for the secondaries of U Cep and U Sge, the ratio [O/Fe] = 0 is assumed. This assumption is supported by recent determinations of the O abundance in G and K giants and is consistent with theoretical predictions (Lambert & Ries 1981). Synthetic spectra (Fig. 7) computed for several different C abundances show that the CH sensitive regions are best fit with the C abundances listed in Table 3. Our analysis of the standard star spectra confirms that these stars are slightly C deficient, as expected for G and K giants with deep convective envelopes (Lambert & Ries 1981).

In the synthesis of the CN regions, we employ the C abundance derived from the CH lines with our assumption about O abundance (i.e. [O/Fe] = 0). Figs 8 and 9 show the observed and synthetic spectra of the secondary of U Sge. Continuum location is perhaps the major source of uncertainty for the 3860–3886 Å region. We developed an iterative solution based on the λλ3883.75 to 3885 Å interval. In the λλ4190 to 4223 Å interval, a window at

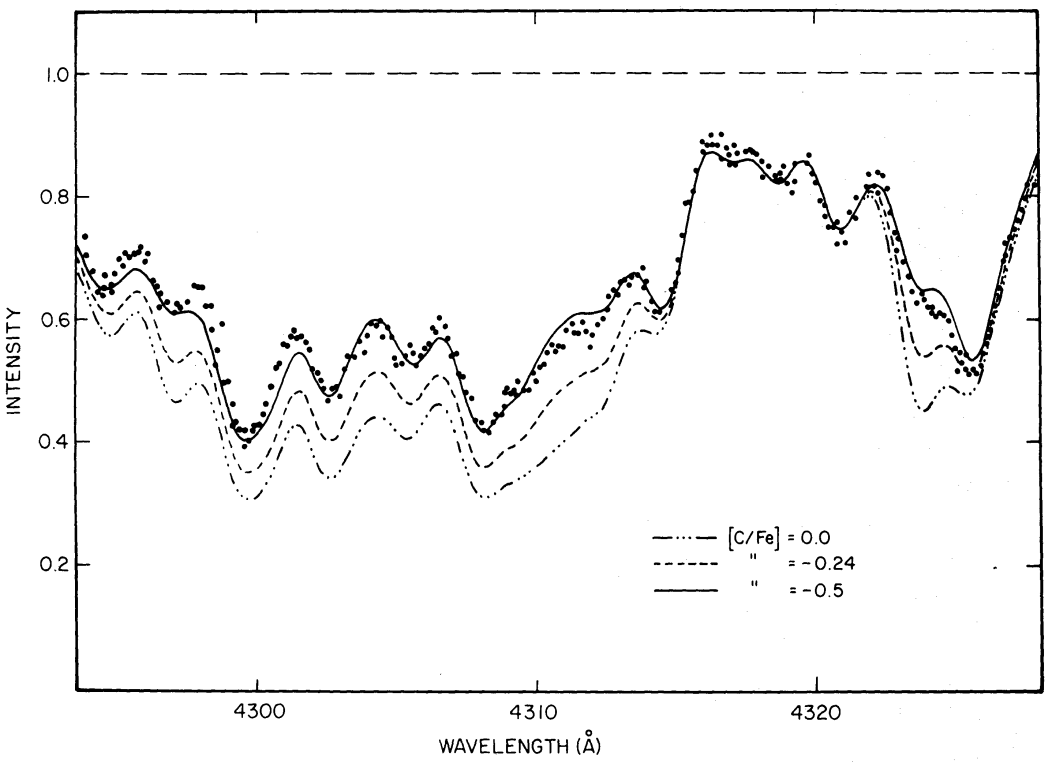

Figure 7. Observed (filled circles) and synthetic spectra (see text) of the secondary of U Sge near 4310 Å where the CH molecule provides many lines. Inspection shows that the synthetic spectrum corresponding to a modest C deficiency, [C/Fe] = − 0.5, fits the observed spectrum well.

4219 Å was used to set the continuum. Figs 8 and 9 show the effect on the synthetic spectra of a change of the N abundance for fixed C and O abundances.

Our final abundance estimates of the secondaries of U Cep and U Sge and their corresponding standard stars κ Gem and δ CrB are summarized in Table 3. The uncertainties are typically ± 0.2 dex. However, the principal results – the underabundance of C and a parallel overabundance of N – seem quite definite on the basis of the direct intercomparison of observed and synthetic spectra of standard stars and secondaries of U Cep and U Sge. Two important limitations must be noted: (i) the assumption about the O abundance affects both the C and N abundance determinations, and (ii) errors in the C abundance derived from the CH analysis necessarily affect the N abundance derived from the CN bands. Just possibly, spectra of very high signal-to-noise might through spectrum synthesis of the [O I] lines yield an O abundance. Unfortunately the CN molecule appears to be the sole source of an N abundance.

Table 3. Carbon and nitrogen abundances.

Star	Spectral type	[Fe/H]	[C/Fe]	[N/Fe]	[O/Fe]
U Cep	G8 III	0.0	−0.45	0.50	0.0★
U Sge	G2 III	0.0	−0.50	0.55	0.0★
δ CrB	G5 III	0.0	−0.30	0.35	0.02
κ Gem	G8 III	0.0	−0.35	0.40	0.0★
GK giant†		0.0	−0.24	+0.38	0.0

★ Assumed – see text.
† Mean values from a sample analysed by Lambert & Ries (1981).

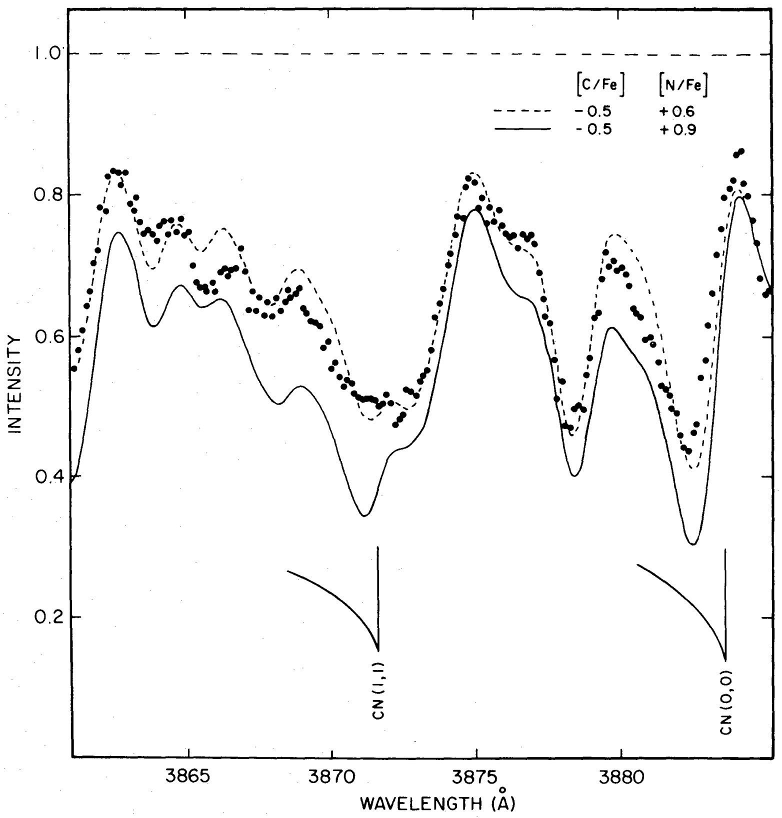

Figure 8. Observed (filled circles) and synthetic spectra (see text) of the secondary of U Sge near 3880 Å where the CN molecule provides many lines. The synthetic spectrum corresponding to [N/Fe] = +0.6 and [C/Fe] = − 0.5 (see Fig. 7) matches the observed spectrum.

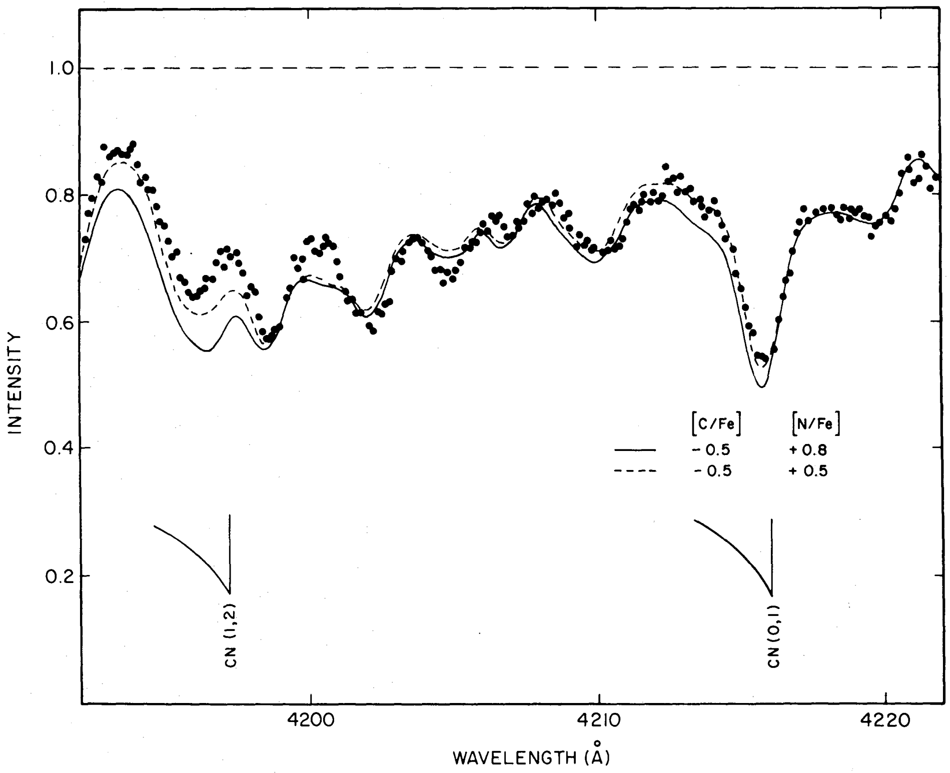

Figure 9. Observed (filled circles) and synthetic spectra (see text) of the secondary of U Sge near 4210 Å where CN lines are present. Although the synthetic spectra show that the CN contribution is modest an abundance [N/Fe] = + 0.5 with [C/Fe] = − 0.5 (see Fig. 7) gives the best fit to the observed spectrum. This N abundance is consistent with that obtained from the CN $\Delta v = 0$ bands (Fig. 8).

4 Discussion

The C deficiency and N overabundance discovered here for the mass-losing secondaries of U Cep and U Sge are more marked than the C deficiencies and N overabundances that are characteristic of field G and K giants (Lambert & Ries 1981). In both types of object the observed C and N abundances are the result of conversion, in the stellar interior during the star's main-sequence lifetime, of C to N by the CN portion of the CNO tricycle, followed by convective mixing to the surface when the star evolved into a red giant. These abundance results are therefore observational evidence in favour of the generally accepted idea that the secondaries of semi-detached systems are in a post-main-sequence stage of evolution.

The C deficiencies and N overabundances in the secondaries of U Cep and U Sge are very similar and are also more marked than in the typical field G or K giant. From Table 3 we find that their average C and N abundances with respect to a field G or K giant are −0.25 and +0.15 dex, respectively. We think that these differences are real and that they are a consequence of the mass loss suffered by the secondaries before the development of deep convection. Note that further mass loss after deep convection does not affect the observed C and N abundances, so the C and N abundances do not say anything about the mass loss that has occurred since the onset of deep convection. Convection does not develop until a star reaches the red giant side of the Hertzsprung gap so we identify the mass loss that occurred prior to its onset with the phase of rapid mass transfer that took place when the secondary left the neighbourhood of the main sequence and crossed the Hertzsprung gap.

If the mass transfer in U Cep and U Sge has been conservative then the minimum masses of the main-sequence progenitors of the present secondaries were half the total mass of the systems, i.e. 3.5 and 3.8 $m_\odot$, respectively. Actual progenitor masses were probably in the range 4–6 $m_\odot$. The variation of composition with mass fraction for a 5 $m_\odot$ star at the onset of shell H burning, i.e. just before crossing the Hertzsprung gap, shown in fig. 2 of Iben (1967) is therefore relevant. The precise mass of the main-sequence progenitor is not crucial because the variation of composition with mass fraction is nearly independent of mass, over a wide mass range. There is an envelope of mass fraction ~ 0.5 which has not been affected by CN cycling and interior to this a shell of mass fraction ~ 0.25 in which C has been almost completely converted to N by CN cycling. (There has not been significant burning of H to He or conversion of O to N by the CNO cycle in either of these regions.) Mass lost during the rapid mass-transfer phase was entirely unprocessed envelope material. When the secondary arrived on the red giant side of the Hertzsprung gap convection developed, mixing the interior CN-processed material with the remainder of the unprocessed envelope material. The C deficiency and N overabundance characteristic of the CN cycle is more marked in the secondary than in a single, but otherwise similar G or K giant because, at the time of convective mixing, there was less unprocessed material and therefore less dilution of the processed material than in the single G or K giant.

In the Appendix this scenario is used to estimate what fraction of their mass the secondaries lost during the rapid mass-transfer phase. The 0.25 dex C deficiency of the secondaries compared to single G and K giants indicates that the fraction of mass lost during this phase was 0.35. The 0.15 dex overabundance of N requires a fractional mass loss of 0.30. Luck probably plays a part in the surprisingly good agreement of these two mass loss estimates, which should, in principle, be the same. The N abundance is less reliable than the C abundance because it is derived from CN and therefore incorporates any error in the C abundance used. Also the N based estimate of the mass loss assumes an initial C/N ratio; the solar value (C/N = 4) was used for the result quoted above. Neither of these uncertainties affects the C based estimate of mass loss, so it is more reliable and is preferred.

The error of the C abundance of the secondaries relative to the G and K giants is about

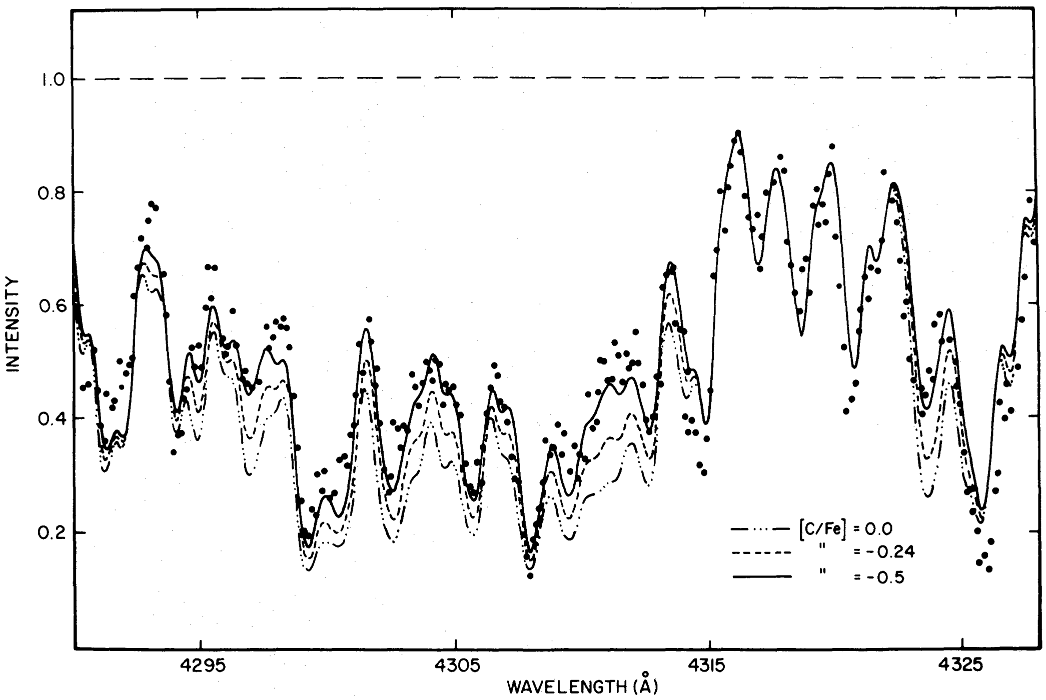

Figure 10. Observed (filled circles) and synthetic spectra of the secondary of S Cnc near 4310 Å. Although the spectrum is rather noisy because clouds stopped the exposure, it shows that CH is present and that C (and H) must therefore be present in the atmosphere of the 0.2 $m_\odot$ secondary. The synthetic spectra are computed for a model atmosphere with $T_{\text{eff}} = 4750$ K; $\log g = 2.25$; [Fe/H] = 0.0; with $\xi = 2.0\ \text{km s}^{-1}$ and different C abundances. The C deficiency is similar to that found for the secondary of U Cep and U Sge.

± 0.1 dex. The C abundance difference of -0.25 ± 0.1 dex corresponds to an estimated fractional mass loss of $0.35^{-0.07}_{+0.04}$, during the rapid mass-transfer phase. We note that there is a definite upper limit of 0.5 on the rapid mass-transfer related mass loss. If it has been greater than this all the unprocessed envelope would have been lost and there would have been no dilution of the CN cycle processed material, so the [C/H] of the secondaries would be $\lesssim -1.5$, a possibility that is excluded by the [C/H] $= -0.5$ that is actually observed.

Does our estimate of 0.35 fractional mass loss during the rapid mass-transfer phase account for all the mass loss that has occurred in U Cep and U Sge? The conservative mass transfer minimum masses of the main-sequence progenitors of the present secondaries are 3.5 and 3.8 $m_\odot$ and put lower limits of 0.20 and 0.50 on the fractional mass loss of the secondaries of U Cep and U Sge, respectively. Thus in U Cep there is no requirement for additional mass loss, while in U Sge there must have been further mass loss after the phase of rapid mass transfer, i.e. after mixing. The additional fractional mass loss required in U Sge is 0.15, or 0.6 $m_\odot$. Both U Cep and U Sge have current mass-transfer rates of 10^{-6}–10^{-7} $m_\odot\ \text{yr}^{-1}$ (Hall & Neff 1976). The additional mass loss required in U Sge must be due to the integrated effect of this mass transfer. (Note that we only have *lower limits* on the total mass loss of the secondaries, so it is quite possible that the secondary of U Sge has lost more than 0.6 $m_\odot$ since it became mixed. The secondary of U Cep may also have had significant mass loss since mixing.)

The idea that once the secondaries are mixed then further mass loss does not alter their observed composition can explain why secondaries that have suffered much more extensive mass loss than those of U Cep or U Sge do *not* show the extremes of composition that one would expect in the absence of mixing. The secondary of the semidetached system S Cnc, which has a mass of only $0.2\,m_\odot$ (Popper 1980), is an example of very extensive mass loss.* The low mass, which is too small to allow core H burning, means that the star must have an H burning shell surrounding a very condensed He core with no nuclear burning. The condensed He core is $0.19\,m_\odot$, thus the envelope is only $0.01\,m_\odot$ and must be a tiny remnant of the original envelope.† The mass of the He core of the secondary of S Cnc indicates that its main-sequence progenitor must have been $\sim 2\,m_\odot$. The secondary of S Cnc therefore differs from the secondaries of U Cep and U Sge, in that it had a less massive main-sequence progenitor. We recently made a Digicon observation of CH in the secondary of S Cnc during the total eclipse of the primary. A preliminary analysis shows that [C/H] = − 0.5 (see Fig. 10). It excludes the [C/H] of ~ − 1.5 that would be expected if there had been no mixing during the post-main-sequence evolution of the secondary.

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* S Cnc has a A0 V primary and a K2 secondary. Primary eclipse is total so that – as for U Cep and U Sge – the pure secondary spectrum can be observed during the total phase of primary eclipse.

† This description of the structure of the secondary of S Cnc follows Refsdal, Roth & Weigert's (1974) description of the secondary of AS Eri, which is also a semi-detached system with a very low mass ($0.2\,m_\odot$) secondary (Popper 1973). The present primaries of S Cnc and AS Eri have masses of 2.4 and $1.9\,m_\odot$ respectively (Popper 1980).

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Appendix

The quantities in the unprocessed envelope, the interior shell where C has been processed to N and the subsequent mixed region are denoted as follows:

	Number of atoms of element/g	Fraction of mass of star
Unprocessed envelope	N_u	m_u
Processed shell	N_p	m_p
Mixed	N_m	$m_u + m_p$.

We have:

$$\frac{N_m}{N_u} = \frac{(m_u N_u + m_p N_p)/(m_u + m_p)}{N_u}$$

$$= \frac{m_u}{m_u + m_p} + \frac{m_p}{m_u + m_p}\frac{N_p}{N_u}. \tag{1}$$

If the element is C then N_p is essentially zero so this simplifies to:

$$\frac{N(\mathrm{C})_m}{N(\mathrm{C})_u} = \frac{m_u}{m_u + m_p}.$$

Let superscripts 'b' and 's' identify the secondary of the binary and the single G or K giant respectively. Divide this equation for the secondary by the same equation for the single G or K giant:

$$\frac{N(\mathrm{C})_m^b}{N(\mathrm{C})_u^b} \Big/ \frac{N(\mathrm{C})_m^s}{N(\mathrm{C})_u^s} = \frac{m_u^b (m_u^s + m_p^s)}{m_u^s (m_u^b + m_p^b)}.$$

The binary and single G or K giant have the same initial composition so $N(\mathrm{C})_u^b = N(\mathrm{C})_u^s$ and this equation becomes

$$\frac{N(\mathrm{C})_m^b}{N(\mathrm{C})_m^s} = \frac{m_u^b (m_u^s + m_p^s)}{m_u^s (m_u^b + m_p^b)}.$$

The left side of this equation is the -0.25 dex abundance ratio of C in the secondaries and single G or K giants, $m_u^s = 0.5$ and $m_p^s = 0.25$ (fig. 2 of Iben 1977) and $m_p^b = m_p^s$. Solving for m_u^b with these numbers we obtain $m_u^b = 0.15$. In other words the fractional mass of the unprocessed envelope at the time of mixing was 0.15. It was 0.50 initially, so the fractional mass loss was 0.35.

If the element is N then in the processed material every C nucleus has been converted to an N nucleus. If the $N(\mathrm{C})/N(\mathrm{N})$ ratio of the unprocessed material – i.e. the initial composition – is R then $N(\mathrm{N})_\mathrm{p}/N(\mathrm{N})_\mathrm{u} = R+1$ and equation (1) becomes:

$$\frac{N(\mathrm{N})_\mathrm{m}}{N(\mathrm{N})_\mathrm{u}} = \frac{m_\mathrm{u} + (R+1)m_\mathrm{p}}{m_\mathrm{u} + m_\mathrm{p}}.$$

Again divide this equation for the secondary of the binary by the same equation for the single G or K giant:

$$\frac{N(\mathrm{N})^\mathrm{b}_\mathrm{m}}{N(\mathrm{N})^\mathrm{s}_\mathrm{m}} = \frac{[m^\mathrm{b}_\mathrm{u} + (R+1)m^\mathrm{b}_\mathrm{p}]\,(m^\mathrm{s}_\mathrm{u} + m^\mathrm{s}_\mathrm{p})}{[m^\mathrm{s}_\mathrm{u} + (R+1)m^\mathrm{s}_\mathrm{p}]\,(m^\mathrm{b}_\mathrm{u} + m^\mathrm{b}_\mathrm{p})}.$$

The left side of this equation is the +0.15 dex abundance ratio of N in the secondaries and single G or K giants and $m^\mathrm{s}_\mathrm{u} = 0.5$, $m^\mathrm{b}_\mathrm{p} = m^\mathrm{s}_\mathrm{p} = 0.25$ as before. Solving for m^b_u for different values of R we find:

R	m^b_u	Fractional mass loss ($= 0.50 - m^\mathrm{b}_\mathrm{u}$)
4	0.20	0.30
3	0.16	0.34
2	0.12	0.38.

The solar value of R $[= N(\mathrm{C})/N(\mathrm{N})]$ is 4 and the corresponding mass loss of 0.30 is adopted.