1980Ap&SS..69..133M

CHEMICAL EVOLUTION COEFFICIENTS FOR THE STUDY OF GALACTIC EVOLUTION

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(Received 20 August, 1979)

Abstract. A new evaluation of chemical evolution coefficients has been made using recent stellar evolution and nucleosynthesis data. The role of the low and intermediate mass stars in galactic nucleosynthesis has been emphasized. A significant amount of ^{4}He, ^{12}C and neutron-rich species is found to be contributed by these stars. Comparison with observed abundances suggests a primary origin of ^{14}N. The simple model of galactic evolution with the new coefficients has been used to derive the ratio of helium to heavy element enrichment in the Galaxy. The new stellar evolution data do not explain the large value of this ratio that has been determined observationally.

1. Introduction

The chemical evolution of a galactic system depends on the evolution of the unit stars as well as on such collective properties as the rate of star formation and its variation with time, the distribution of stellar masses at birth and the dynamics of the gas–star system. Each generation of stars contributes to the chemical enrichment of the galaxy by processing new material in the stellar interiors through successive thermonuclear cycles and returning to the interstellar gas a fraction of the total mass containing both processed and unprocessed matter during the various phases of stellar mass ejection, including the terminal planetary nebula and supernova phenomena. The next generation of stars then forms from this gas and evolves according to the laws of stellar evolution in an ever-continuing process. To describe in detail this process of enrichment it is essential to know how and when stars eject matter and how much material – processed and unprocessed – is returned to the interstellar medium at these moments. What we need is a complete specification, as a function of stellar mass, of the fractional amount of each elemental species returned to the interstellar gas as well as the fraction of the mass contained in the stellar remnants permanently lost as far as further processing is concerned. With the progress in our knowledge of the theory of stellar evolution it is now possible to obtain these data for a large range of stellar masses from theoretical models. It has been amply demonstrated over the years that most of the nucleosynthesis in the Galaxy is effected through the evolution of massive stars with their intrinsic capacity for very efficient thermonuclear processing during their short lifetime. Most of this processed material is injected into the interstellar medium through supernova events. However, following the discovery of He-shell flashes in stars of low and intermediate masses (Schwarzschild and Härm, 1965; Weigert, 1966; Rose, 1966) it has been realized

Astrophysics and Space Science **69** (1980) 133–145. 0004–640X/80/0691–0133$01.95

that these stars are also capable of contributing to some elemental enrichment of the Galaxy.

Talbot and Arnett (1973, hereafter referred to as TA) have developed a formalism which is ideally suited for galactic chemical evolution studies. This formalism permits a complete description of the ejected nucleosynthesis products from stars in terms of a production matrix which involves all elemental species. Recent improvements in stellar nucleosynthesis data allow a more up-to-date estimate of the elements of this matrix. The matrix elements are essential inputs into chemical evolution models. Also, without having to consider detailed models, a number of important conclusions regarding abundances can be reached when this matrix is evaluated.

Iben and Truran (1978, hereafter referred to as IT) have considered in detail the evolution of low- and intermediate-mass stars which evolve through a thermal pulsing phase to planetary nebulae and white dwarfs. These stars are now known to be an important source of enrichment of ^{4}He, ^{12}C, ^{13}C, ^{22}Ne and *s*-process elements. The IT models derive the amounts of ejected material in the various species through all phases of mass ejection. Arnett (1978) has estimated the absolute yields of primary nucleosynthesis products – e.g., He, C, O, Ne, Mg, Si and Fe – for stars in the mass range 10–95 $M_\odot$. Along with the IT data, this gives complete coverage of the stellar mass spectrum in which we are interested for chemical evolution studies of galaxies.

In the following we have used the data from these two sources (IT, 1978; Arnett, 1978) to evaluate the elements of the production matrix. A comparison is made with the previous determination of the production matrix to see the changes due to refinements in stellar evolution theory. The implications of the new chemical evolution coefficients have then been discussed.

From studies of H II regions and planetary nebulae, Peimbert and coworkers (Peimbert, 1977, and references quoted therein) have shown that the overall helium enrichment in the Galaxy is proportional to the heavy element enrichment and that this ratio, $\Delta Y/\Delta Z$, is approximately 3. We have used the simple galactic evolution model of Talbot and Arnett (1973) to derive this ratio with the new production matrix. We have found that the use of the new stellar evolution data does not resolve the discrepancy existing between theory and observations as the theory still predicts a much smaller ratio than has been found observationally. This was also noted by Gingold (1977) and Hacyan *et al.* (1976) from earlier data of stellar evolution.

2. Stellar Mass Ejection and Nucleosynthesis Products

Stars of all masses lose matter at various stages of their evolution. We must distinguish here between the quiescent mass-loss phenomena (e.g., stellar winds) and the more catastrophic terminal mass ejection events such as supernovae and planetary nebula (PN) formation. The evolutionary implications of quiescent mass-loss phenomena have not yet been fully realized. For the evolution of massive stars ($M \geqslant 10\ M_\odot$) there is no currently available study to estimate the magnitude and effects of this loss.

Calculations of the surface enrichment due to convective dredging of ^{14}N, ^{4}He, etc., are non-existent. It is conceivable that in these stars substantial mass-loss will have profound effects on the more advanced stages of evolution – especially on further processing – but no quantitative data are available to assess them. We therefore assume that all these stars ($M \geqslant 10\ M_\odot$) have only one mass ejection event, which is at the terminal phase of their evolution. They end their lives as supernovae, leaving behind in each case a small stellar remnant; the mass of this remnant has been tabulated by Arnett (1978).

It is generally believed that all low-mass stars end their lives as white dwarfs. Although no white dwarf may exist with a mass larger than about 1.4 $M_\odot$, the Main-Sequence precursor of a white dwarf may be of a considerably higher mass since mass loss during the subsequent phases of evolution can reduce the parent mass below the Chandrasekhar limit. The rate of mass loss then determines the upper mass limit of a white dwarf progenitor. There is strong observational evidence and theoretical support for the premise that all stars destined to become white dwarfs pass through the planetary nebula phase where they shed their outer hydrogen-rich envelope due to some pulsational instability (Osterbrock, 1973; Wood and Cahn, 1977). It is also known that all stars with Main-Sequence masses less than or equal to 8 $M_\odot$ develop degenerate C–O cores following central He-exhaustion (Iben, 1974). With favourable mass loss rates it is possible that all these stars ($M \leqslant 8\ M_\odot$) become white dwarfs after going through a planetary nebula phase. If carbon ignition takes place under degenerate conditions, which it must if the mass loss is not that efficient, then it is not possible at the moment to decide on the final fate of the stars in which this may happen (Tinsley, 1977). For our purpose we shall assume that all stars in the mass range 1–8 $M_\odot$ shed enough material during their evolution to end up as white dwarfs after passing through a PN phase, and for the masses of the remnant and planetary nebula we shall use the values given by IT.

The production matrix element Q_{mij} of a star of initial mass m is defined as the fraction of the mass of the star initially in the form of species j which is eventually ejected as species i. Thus, if X_j denotes the fraction by mass of species j, then

$$R_{mi} = \sum_j Q_{mij} X_j \tag{1}$$

is the fractional mass ejected into the ISM from the star of mass m in the form of species i. For a generation of stars characterized by an initial mass function (IMF) Ψ_m, the production matrix is given by

$$q_{ij} = \int_m \Psi_m Q_{mij}\, \mathrm{d}m . \tag{2}$$

The IMF is a normalized function such that

$$\int_m \Psi_m\, \mathrm{d}m = 1 . \tag{3}$$

In the most general case both Q_{mij} and q_{ij} are functions of X_j's since the structure and evolution of stars depend upon the composition. However, it is not possible at the present time to evaluate a composition-dependent Q_{mij} and treat Q_{mij} as a continuous function of time. Indications are that the bulk of the dependence of the ejected mass fraction on the composition is contained in the linear dependence of R_{mi} on X_j's. We treat Q_{mij} as independent of X_j's and hence of time. The production matrix for the generation q_{ij} may still vary with time if the IMF varies with time. In detail models of galactic evolution we may include such dependence explicitly. Most models, so far, have used time-independent IMFs.

The fraction of mass ejected from a generation of stars is denoted by f, and given by

$$f = \sum_i \int_m R_{mi} \Psi_m \, \mathrm{d}m. \tag{4}$$

The fraction of species i which is ejected unprocessed is given by

$$u_i = q_{ii} \tag{5a}$$

and the yield of species i by

$$p_i = \frac{\sum_{j \neq i} q_{ij} X_j}{1 - f}. \tag{5b}$$

PRESCRIPTION FOR Q_{mij}

We divide the stars into three mass categories:

(1) Low-mass stars: $1 \leqslant m \leqslant 3$,
(2) Intermediate-mass stars: $3 < m \leqslant 8$,
(3) Massive stars: $m \geqslant 10$.

The sources of our stellar evolution and nucleosynthesis data are:

(a) for low- and intermediate-mass stars – Iben and Truran (IT); and
(b) for massive stars – Arnett (1978).

The low- and intermediate-mass stars evolve through a core H-burning, a shell H-burning, a core He-burning and a double-shell-source (DSS) phase. They contribute to element enrichment in three stages. In the first red giant phase deep envelope convection in these stars brings to the surface products synthesized exterior to the H-burning zone: namely, ^{3}He, ^{13}C and ^{14}N. At the same time ^{12}C is depleted by the amount ^{14}N is enhanced. There may be a small contribution to ^{14}N from the conversion of ^{16}O in the ON cycle. We have neglected this and have assumed the ^{16}O abundance in the envelope to have remained unchanged in this phase. According to the mass-loss rates computed by Reimers (1975) and Fusi-Pecci and Renzini (1976), the low-mass stars ($m < 3.08$) lose matter through stellar winds in this phase and contribute to some enrichment of the species mentioned above. Stars with initial masses larger than 3.08 $M_\odot$ do not suffer appreciable mass loss in this phase. However, the changes in surface abundances need to be evaluated to assess further changes in the same species during subsequent evolutionary phases. All these stars make a second excursion to the red giant branch following core He exhaustion. The intermediate-mass stars ($m > 3$) undergo a further surface composition change prior to the DSS phase as the convection digs deep this time into zones processed through H-burning and carries, most importantly, synthesized ^{4}He and ^{14}N upwards. Corresponding changes occur in the surface abundance of ^{12}C, ^{16}O and ^{1}H. We have used the IT equations (19)–(22) to obtain a direct estimate of the abundance change in these species. Finally, thermal pulsing in all these stars ($m \leqslant 8$) produce further changes in the surface abundances of ^{12}C, ^{22}Ne, the elements between ^{22}Ne and ^{56}Fe (denoted by l in IT) and the heavy elements beyond ^{56}Fe (denoted by $\mathscr{S}$ in IT) for which ^{56}Fe act as the seed nuclei. ^{12}C is produced from ^{1}H and ^{4}He during the pulses. Similarly, ^{22}Ne comes totally from ^{14}N which in turn came from the initial ^{12}C, ^{16}O and ^{14}N. The l elements are further synthesis products from the initial CNO nuclei. It is possible that the ^{12}C produced in the pulses is partially converted to ^{14}N at the base of the convective envelope during the interpulse phase and contribute to the primary synthesis of nitrogen (TA). It is not possible at this moment to settle this question theoretically and estimate the amount of ^{14}N of primary origin. For the present we have neglected this possibility. Since we have been interested mainly in the abundant species, no explicit calculation has been made for the heavy elements ($\mathscr{S}$) either. During the entire second red giant phase, stellar winds carry material out into the interstellar medium and eventually the remainder of the outer envelope is blown off as a planetary nebula with a markedly different composition from the initial one. The final abundances in both the wind component and the PN component, as well as the masses ejected in the two components, have been tabulated by IT. We have used their Tables 1–9 to derive the values for each Q_{mij} element for all the stellar masses considered. These have been listed in Table II.

All massive stars ($m \geqslant 10$) burn carbon under non-degenerate conditions in their core and proceed towards more advanced burning stages where the ashes from one cycle provide the fuel for the next, and more and more advanced species are synthesized. Eventually the massive star develops an onion-skin structure with an Fe–Ni core, followed by successive shells containing products of O-, C-, He- and H-burning (Clayton and Woosley, 1974; Schramm, 1977). In the supernova event the entire mass outside of the core is assumed to be ejected. During the event further explosive nucleosynthesis may occur, but the extent and manner in which the bulk yields change as a result are yet to be assessed (Arnett, 1978). If the event were not very violent, the major nucleosynthesis species may undergo only minor readjustments. Following Arnett we have neglected further processing of the bulk elements, He, C, O, Ne, Mg, Si and Fe, in the explosion.

Arnett's models are concerned with the evolution of He cores of different masses M_α. Each M_α is related to an M, the initial Main-Sequence mass of the star, that develops a He core of mass M_α. We adopt the M–M_α transformation given by Arnett and note that the unprocessed fraction of the various species comes primarily from the outermost zone in each star whose mass is M–M_α. The diagonal elements of the production matrix are thus close to $1 - M_\alpha/M$ with minor modifications as indicated below. The tabulated bulk yields directly give, as a function of M_α, the mass of species j ejected, denoted by M_j^{ej}. To transform these to Q_{mij}'s we remember the onion-skin structure and note the following:

(a) ^{1}H comes only from the outermost envelope which has not evolved through any major thermonuclear phase. However, in the inner part of this envelope both ^{12}C and ^{16}O are converted to ^{14}N. According to Iben (1977), the fraction of the mass of a star over which ^{12}C is converted completely to ^{14}N is larger than 0.5. Lacking exact estimates we have set this fraction equal to 0.5. This leads to an overestimate of unprocessed ^{12}C from the massive stars. It is yet more difficult to define a fraction of the mass over which ^{16}O is completely converted to ^{14}N (Iben, 1977). We have here neglected any ^{16}O–^{14}N conversion in the outermost envelope of the massive stars and set the transition edge mass $M_{\rm ON}$ equal to M_α.

(b) Unprocessed ^{4}He comes from the zone M–M_α as well as from the shell processed through just H-burning, while the processed ^{4}He comes only from the latter. From the mass of He ejected M_4^{ej}, it is thus possible to estimate both $Q_{4,1}$ and $Q_{4,4}$.

(c) No unprocessed ^{12}C and ^{16}O come from within M_α since all have been converted through the CNO bi-cycle to ^{14}N. ^{12}C is processed in a shell that has evolved through He-burning and has yet to undergo C-burning, while ^{16}O comes from shells processed through both He- and C-burning. Then M_{12}^{ej} and M_{16}^{ej} are easily expressed in terms of $Q_{12,i}$ and $Q_{16,i}$, where i stands for the progenitors ^{1}H and ^{4}He.

(d) It is difficult to make exact estimates of the matrix elements for Ne and Mg. These elements are produced in C- and O-burning and come from the innermost zones of the onion-skin star. For simplicity we have neglected the contribution to unprocessed Ne from more inward zones than the He zone, and unprocessed Mg from more inward zones than the C zone. We have also assumed that the C zone consists entirely of ^{12}C and ^{16}O, each contributing an equal mass. While ^{20}Ne and ^{24}Mg are primary synthesis products, the isotopic species ^{22}Ne and ^{25,26}Mg are produced in various nuclear reactions involving mostly the initial CNO nuclei. In the ejected mass of Ne and Mg the respective isotopic species are also included, but the amounts for the individual species are not given. To obtain a complete description of the production matrix, we have assumed that the isotopic species are produced in these zones in the same ratios as they are found in the solar system. Explosive nucleosynthesis calculations by Arnett and coworkers vindicate this assumption (Arnett and Clayton, 1970; Trimble, 1975). The solar system ratios for these species have been taken from Cameron (1973).

(e) Elements heavier than Mg are all denoted by h. The prominent constituents are Si and Fe. Following Arnett we have written $M_h^{ej} = M_\alpha \langle X_{\rm Si} \rangle$ and thus obtained the element $Q_{h,i}$. The diagonal element $Q_{h,h} = 1 - m_r/m$.

To obtain the matrix elements, we require the initial abundances by mass of the species considered. These have been adopted from Cameron (1973) and are given in Table I.

TABLE I

Atomic species considered

Element	Symbol	Adopted abundance
Hydrogen	H	0.770
Helium	He	0.214
Carbon	C	3.43×10^{-3}
Oxygen	O	8.33×10^{-3}
Nitrogen	N	1.27×10^{-3}
Neon	Ne	1.67×10^{-3}
Magnesium	Mg	6.16×10^{-4}

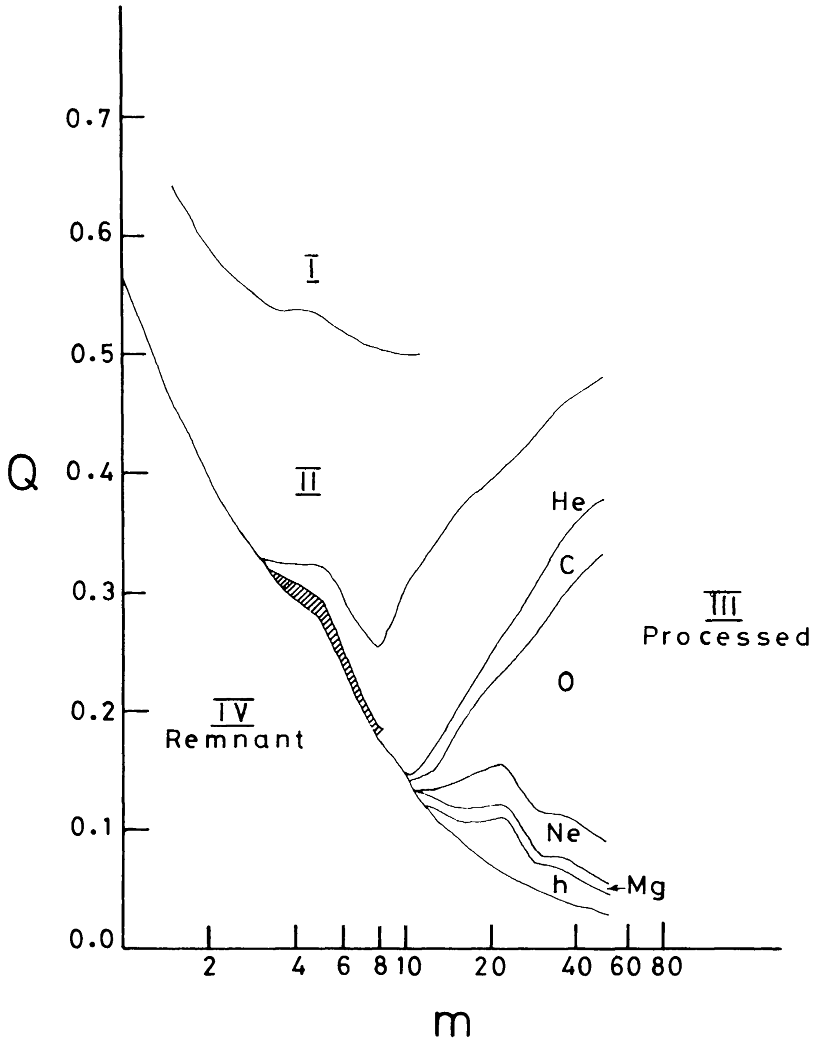

Fig. 1. The stellar mass fractions containing different elements at the time the mass is returned are shown as a function of the mass of the star. Zone I retains the initial composition of the star. Zone I retains the initial composition of the star. In Zone II, C and O have been converted to ^{14}N. Zone III contains the major nucleosynthesis products. The shaded region denotes the fraction of ^{12}C produced and ejected as a consequence of He-shell flashes. Zone IV gives the material locked in remnants.

In Table III the Q_{mij} elements for the massive stars are listed. Figure 1 displays graphically the various contributions of each star to galactic nucleosynthesis.

3. Chemical Evolution Coefficients

In order to evaluate the net galactic enrichment of the various species the fundamental data given in Tables II and III have to be weighted by the IMF and integrated over all stellar masses. We have used two different IMFs.

The first IMF, due to TA, is given by

$$\Psi_m = \zeta(\mu - 1)m^{-\mu}, \tag{6a}$$

with $\zeta = 0.25$ and $\mu = 1.55$. The production matrix for a generation of stars, q_{ij}, has been computed by numerical integration of Equation (2). The lower limit of the integral is dictated by the age of the Galaxy. We have assumed that stars less massive than 1 $M_\odot$ have not had time to evolve in the lifetime of the Galaxy. The upper mass limit is chosen as 75 $M_\odot$. The mass points for integration are not evenly spaced between 1 $M_\odot$ and 75 $M_\odot$. For convenience in integration, interpolated values have been used where necessary. In Table IVa the q_{ij} matrix is displayed. This is to be compared with Table 5 of TA to see the changes, if any, in the chemical evolution coefficients due to the new data.

The second IMF, due to Tinsley (1979), is given by

$$\begin{aligned} \phi(m) &= 0.156m^{-2.3}, \quad && 2 < m \leqslant 50; \\ &= 0.127m^{-2.0}, && 1 < m \leqslant 2; \\ &= 0.127m^{-1.25}, && 0.8 < m \leqslant 1; \end{aligned} \tag{6b}$$

where $\phi(m)$ is the number of stars formed in the interval $(m, m + \mathrm{d}m)$ per unit total mass of stars formed. This is a flatter IMF and obviously predicts different yields for the various species. The q_{ij} matrix with this version of the IMF is shown in Table IVb.

A comparison of Table 5 of TA and our Table IVa reveals very little change in the matrix elements for the primary synthesis species despite the vastly improved stellar evolution and nucleosynthesis data used here. Of course, it has now been possible to include explicitly more species (e.g., Ne and Mg) as also to break up CO into C and O. Compared to the TA specification, a smaller fraction of the stellar mass per generation is returned to the interstellar pool because of the different values used by us for the remnant masses. We note that Talbot and Arnett had chosen the mass fraction ejected in the form of heavy species as one-third the fraction ejected in the form of C and O to match the solar system abundance distribution. Our explicit calculation of these fractions shows this assumption to be correct. If we calculate the yields per generation according to Equation (5b), we find that

$$\begin{aligned} \frac{\sum_{i>16} p_i}{p_{\mathrm{CO}}} &= 0.395 \quad \text{from Table IVa,} \\ &= 0.389 \quad \text{from Table IVb.} \end{aligned}$$

TABLE II

m	m_C	$Q_{1,1}$	$Q_{4,1}$	$Q_{4,4}$	$Q_{12,i}$ $(i = 1,4)$	$Q_{12,12}$	$Q_{14,12}$	$Q_{14,16}$	$Q_{22,i}$ $(i = 12, 14, 16)$	$Q_{l,i}$ $(i=12,14,16)$
1.0	0.56	0.440	—	0.440	—	0.356	9.75(−2)	—	—	—
1.5	0.70	0.533	—	0.533	—	0.362	1.99(−1)	—	—	—
2.0	0.79	0.605	—	0.605	2.40(−4)	0.411	2.26(−1)	—	—	—
2.5	0.89	0.644	—	0.644	7.40(−4)	0.438	2.40(−1)	—	7.00(−3)	1.23(−4)
3.0	0.99	0.670	—	0.670	2.19(−3)	0.456	2.50(−1)	—	1.69(−2)	3.27(−3)
3.5	1.08	0.677	1.41(−2)	0.691	5.47(−3)	0.461	2.69(−1)	1.23(−2)	—	—
4.0	1.18	0.678	2.69(−2)	0.705	9.28(−3)	0.461	2.85(−1)	2.36(−2)	4.14(−2)	7.34(−2)
5.0	1.38	0.679	4.48(−2)	0.724	1.71(−2)	0.462	3.06(−1)	3.92(−2)	2.30(−2)	2.32(−1)
6.0	1.40	0.708	5.91(−2)	0.767	1.59(−2)	0.481	3.33(−1)	5.17(−2)	1.70(−2)	1.59(−1)
7.0	1.40	0.730	7.01(−2)	0.800	1.37(−2)	0.496	3.54(−1)	6.13(−2)	1.24(−2)	1.33(−1)
8.0	1.40	0.746	7.87(−2)	0.825	1.13(−2)	0.508	3.70(−1)	6.88(−2)	6.61(−3)	9.88(−2)

Notes: (i) m is the initial Main-Sequence mass and m_C the mass of the stellar remnant, both in solar units.

(ii) $Q_{i,j}$ implies Q_{i_A, j_B} i_A being the product nucleus and j_B the progenitor.
i = 1, 4, 12, 14, 16, 22, respectively, denote ^{1}H, ^{4}He, ^{12}C, ^{14}N, ^{16}O, ^{22}Ne; l denotes nuclei in the interval $22 < A < 56$.

(iii) $X.X(Y)$ is equivalent to $X.X \times 10^Y$.

TABLE III

m	$\frac{m_\alpha}{m}$	$\frac{m_r}{m}$	$Q_{1,1}$	$Q_{4,1}$	$Q_{4,4}$	$Q_{12,i}$ $(i = 1, 4)$	$Q_{16,i}$ $(i = 1, 4)$	$Q_{14,12}$	$Q_{\mathrm{Ne},i}$ $(i = 1, 4)$	$Q_{\mathrm{Ne},i}$ $(i = 12, 14, 16)$	$Q_{\mathrm{Ne,Ne}}$	$Q_{\mathrm{Mg},i}$ $(i = 1, 4)$	$Q_{\mathrm{Mg},i}$ $(i = 12, 14, 16)$	$Q_{\mathrm{Mg,Mg}}$	$Q_{h,i}$ $(i = 1\ 4)$	$Q_{h,h}$
10	0.300	0.142	0.700	0.155	0.855	4.80(−3)	3.23(−2)	0.355			0.855	1.10(−5)	2.20(−4)	0.865		0.858
12	0.333	0.115	0.667	0.173	0.840	1.62(−2)	1.41(−2)	0.340	2.89(−3)	0.027	0.840	4.29(−3)	9.65(−2)	0.872	8.56(−3)	0.885
16	0.375	0.088	0.625	0.164	0.789	1.83(−2)	4.92(−2)	0.289	2.42(−2)	0.223	0.789	1.26(−2)	2.84(−1)	0.825	1.68(−2)	0.912
22	0.364	0.059	0.636	0.143	0.779	2.63(−2)	7.66(−2)	0.279	3.18(−2)	0.294	0.779	9.75(−3)	2.19(−1)	0.831	1.57(−2)	0.941
28	0.429	0.056	0.571	0.130	0.701	4.45(−2)	1.40(−1)	0.201	3.39(−2)	0.313	0.701	8.79(−3)	1.98(−1)	0.788	1.54(−2)	0.944
35	0.457	0.039	0.543	0.118	0.661	4.69(−2)	1.79(−1)	0.161	3.71(−2)	0.342	0.661	9.16(−3)	2.06(−1)	0.753	4.55(−2)	0.961
50	0.480	0.029	0.520	0.102	0.622	4.67(−2)	2.38(−1)	0.122	3.60(−2)	0.333	0.622	9.90(−3)	2.23(−1)	0.714	3.64(−2)	0.971
75	0.427	0.030	0.573	0.090	0.663	2.78(−2)	2.25(−1)	0.163	2.85(−2)	0.263	0.663	1.03(−2)	2.31(−1)	0.718	1.62(−2)	0.970

Note: m_α is the mass of the He star and m_r the mass of the stellar remnant, both in solar units. The rest of the notations have the same meaning as in Table II.

TABLE IVa

q_{ij}

Product	Progenitor *j*							
i	H	He	C	O	N	Ne	Mg	*h*
H	1.39(−1)	—	—	—	—	—	—	—
He	9.78(−3)	1.50(−1)	—	—	—	—	—	—
C	2.11(−3)	2.11(−3)	1.00(−1)	—	—	—	—	—
O	5.15(−3)	5.15(−3)	—	1.39(−1)	—	—	—	—
N	—	—	5.41(−2)	9.56(−3)	1.50(−1)	—	—	—
Ne	1.14(−3)	1.14(−3)	1.23(−2)	1.23(−2)	1.23(−2)	1.50(−1)	—	—
Mg	4.42(−4)	4.42(−4)	1.37(−2)	1.37(−2)	1.37(−2)	—	1.52(−1)	—
h	9.37(−4)	9.37(−4)	—	—	—	—	—	1.58(−1)

TABLE IVb

q_{ij}

Product	Progenitor *j*							
i	H	He	C	O	N	Ne	Mg	*h*
H	2.24(−1)	—	—	—	—	—	—	—
He	2.20(−2)	2.51(−1)	—	—	—	—	—	—
C	4.90(−3)	4.90(−3)	1.69(−1)	—	—	—	—	—
O	1.42(−2)	1.42(−2)	—	2.24(−1)	—	—	—	—
N	—	—	9.02(−2)	2.16(−2)	2.51(−1)	—	—	—
Ne	3.00(−3)	3.00(−3)	3.08(−2)	3.08(−2)	3.08(−2)	2.51(−1)	—	—
Mg	1.05(−3)	1.05(−3)	3.28(−2)	3.28(−2)	3.28(−2)	—	2.56(−1)	—
h	2.51(−3)	2.51(−3)	—	—	—	—	—	2.71(−1)

TABLE V

IMF	C	O	Ne	Mg	h
Equation (6a)	2.46×10^{-3}	6.01×10^{-3}	1.53×10^{-3}	7.29×10^{-4}	1.09×10^{-3}
Equation (6b)	6.62×10^{-3}	1.91×10^{-2}	4.60×10^{-3}	2.00×10^{-3}	3.39×10^{-3}

The individual yields are given in Table V. According to the present stellar data, the low- and intermediate-mass stars ($1 \leqslant m \leqslant 8$) contribute significantly to the total yields of ^{4}He and ^{12}C. In fact, we find that the contribution of these stars to $q_{4,1}$ and $q_{12,1}$ is about 30% with the TA IMF and close to 20% with the Tinsley IMF. The lower relative yield in the second case is due to the use of a flatter IMF that increases the relative number of massive stars (which are responsible for the bulk of the nucleosynthesis) with respect to the less massive stars.

Strictly speaking, the observed abundances in a galaxy at the present time are determined by its chemical history and, as such, are dependent on the history of the stellar birth rate. However, most of the nucleosynthesis at any time is due to the massive stars which have an extremely short life compared to the lifetime of the galaxy. It is, therefore, possible to use the assumption of *instantaneous recycling*, which effectively equates the stellar lifetimes to zero. This vastly simplifies the equations of chemical evolution and make them independent of the stellar birth rate. Even then there is an implicit dependence of the abundance of any species on the total stellar birth rate through its dependence on the present gas to star mass ratio; but if we compare the relative abundance of two species, then this dependence also cancels out and it is possible to obtain directly from the chemical evolution coefficients some meaningful abundance ratios for comparison with the observed data. For all species other than He the initial abundance may be taken as zero; and since the present abundances are also quite small it can be shown that for any two primary synthesis species with abundances X_i and X_z,

$$\frac{X_i}{X_z} = \frac{p_i}{p_z} = \frac{q_{i,1}}{q_{z,1}}. \tag{7}$$

Similarly, for a secondary species with abundance X_k if its progenitor species has an abundance X_z, the ratio is of the form

$$\frac{2X_k}{X_z^2} = \frac{q_{k,z}}{q_{z,1}}. \tag{8}$$

Thus it is possible to compare the observed abundance ratios with the predictions of Equations (7) and (8) where the chemical coefficients are already theoretically known. We make this comparison with the solar system abundances (Cameron, 1973). We find that for the primary synthesis species the agreement is quite good when coefficients from either Table IVa or IVb are used. There is a slight difference (of about a factor of 2) when the heavy element yield, p_h, is compared with a primary reference species, say ^{16}O; but given the uncertainties involved in the evaluation of the $q_{h,i}$, even here the agreement is deemed sufficiently good.

The situation changes when we consider ^{14}N, which is thought to be of secondary origin. If we use Equation (8) and compare $2X(^{14}\mathrm{N})/X(^{16}\mathrm{O})^2$ with the observed value in the solar system, we find that the theoretical prediction falls far below. There is much more ^{14}N than its purely secondary origin implies. It is difficult to see how any further improvement in the stellar evolution data can increase the ratio significantly to bring it closer to the observed solar system value. TA had noted this disagreement earlier, and had made a rather drastic assumption in an attempt to resolve this problem. They postulated a complete conversion of C and O to ^{14}N in the outer H-rich envelope of the stars. The new q_{ij} then yielded almost a solar system value for the abundance ratio of ^{14}N to C and O. Current stellar evolution models certainly do not support such a large conversion of C, O to ^{14}N. On the other hand, there is a growing belief that ^{14}N could be a primary synthesis product and that it originated in stars in the mass interval 1–3 $M_\odot$ (Edmunds and Pagel, 1978). It is possible that the ^{12}C produced in these stars during their thermal pulsing phase is processed further through the CN cycle to make ^{14}N. Although such a phenomenon is far from proven theoretically, we can evaluate its effect in the present case on the predicted abundances since we know the fraction of new ^{12}C produced in thermal pulses in stars in the interval 1–8 $M_\odot$. If we assume that the entire amount of ^{12}C produced in shell flashes is converted to ^{14}N before ejection in the stellar wind and PN shell, and recalculate the elements of the production matrix, we find from Equation (7)

$$\begin{aligned} \frac{q_{14,1}}{q_{16,1}} &= 0.13, \qquad \text{from Table IVa,} \\ &= 0.08, \qquad \text{from Table IVb;} \end{aligned}$$

while the solar system value is 0.15. Certainly the agreement is much better with an assumed partly primary origin of ^{14}N. However, two points must be noted:

(i) The bulk of the ^{12}C produced in thermal pulsing comes from stars in the mass range 4–8 $M_\odot$ and not from the low-mass stars. To establish the mass range of stars really responsible for the primary synthesis of nitrogen we need to calculate a more detailed evolution model, relaxing the assumption of instantaneous recycling. If the primary ^{14}N comes from low-mass stars, time-delayed effects have to be taken into account. We shall discuss this problem further in a future paper.

(ii) If indeed ^{12}C is converted to ^{14}N the predicted yield of ^{12}C (Table V) will be less by 20–30% and the agreement of the ^{12}C/^{16}O with the observed value will not be as good.

We emphasize that the presently available stellar evolution data fail to explain the abundance of nitrogen unless a primary origin for this element is invoked.

Finally, the yields obtained in Table V allow a determination of $\Delta Y/\Delta Z$, the helium to heavy element enrichment ratio, based on the simple model of TA. Since there is no drastic change in the q_{ij} with our new data, this ratio has not been substantially revised. With the coefficients from Table IVa we derive a value of 0.76 for this ratio, while from Table IVb $\Delta Y/\Delta Z \approx 0.65$. The lower value in the second case is due to the fact that the bulk of the production of ^{4}He takes place in stars of mass less than 12 $M_\odot$ and a flatter IMF tends to suppress the total ^{4}He generation by reducing the number of stars with mass less than 12 $M_\odot$ relative to the number with mass larger than 12 $M_\odot$. In any case, the value of $\Delta Y/\Delta Z$ falls far below the observed ratio from various classes of objects (Peimbert, 1977). A steeper IMF should certainly help to increase the theoretical value but its use may substantially worsen the otherwise good agreement between the abundance ratios. It is to be noted that since the relative production of various species depends upon the stellar mass, the yields place stringent constraints on the slope of the IMF. Therefore, we conclude that the present stellar evolution data do not explain the observed $\Delta Y/\Delta Z$. The problem must be discussed in the context of more detailed galactic evolution models for further elucidation.

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