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ABSORPTION OF INTENSE ELECTROMAGNETIC BEAMS IN A MAGNETOPLASMA

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Abstract. The multiphoton inverse bremsstrahlung absorption of two intense electromagnetic beams passing through a magnetized plasma is studied. The rate of absorption of electromagnetic energy by the electrons is calculated by deriving a kinetic equation for the electrons. It is found that the absorption enhances when the frequency of one electromagnetic beam is more, and that of the other electromagnetic beam is less, than the electron–cyclotron frequency. A possible application to extragalactic radio sources is discussed.

1. Introduction

The study of interaction of an intense electromagnetic beam, as a laser beam, with plasma is of great importance due to its possible role in the controlled thermonuclear fusion research (Krishan *et al.*, 1976). The high-intensity electromagnetic beam impinges on a plasma and spends a fraction of its energy in heating the particles. A good review of the various processes operative in a laser plasma system has been given by Brueckner and Jorna (1974). The two-laser absorption has been considered experimentally and theoretically by Capjack and James (1974). The multiphoton inverse bremsstrahlung of a single laser was studied by Seely and Harris (1973) and later the effect of a magnetic field was included by Seely (1974), who showed that the multiphoton absorption coefficient decreases as the laser frequency approaches the electron–cyclotron frequency. Presently we consider the absorption of two laser beams propagating through a plasma in a uniform magnetic field in the strong field limit. A quantum mechanical approach, following Harris (1969), is adopted here. The wave function and the energy of an electron is determined in the presence of two laser beams propagating parallel to the uniform magnetic field. These electrons then suffer scattering in the Coulomb field of the ions and thus get heated.

2. Theory

The vector potential due to the two laser beams and the magnetic field can be written as

$$\mathbf{A}(\mathbf{x}, t) = \mathbf{A}_1(t) + \mathbf{A}_2(t) - B_o y \hat{e}_x, \tag{1}$$

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where

$$\mathbf{A}_1(t) = a_1[\hat{e}_x \cos \omega_1 t + \hat{e}_y \sin \omega_1 t],$$

$$\mathbf{A}_2(t) = a_2[\hat{e}_x \cos \omega_2 t + \hat{e}_y \sin \omega_2 t], \tag{2}$$

in which B_0 is the strength of the uniform magnetic field in the z-direction, ω_1 and ω_2 are the laser frequencies, a_1 and a_2 are the laser amplitudes, and $\hat{e}_x$ and $\hat{e}_y$ are unit vectors along the x and y directions, respectively.

The lasers are treated in the dipole approximation. The solution of the time-dependent Schrödinger equation is found to be

$$\varphi_{pn}(\mathbf{x}, t) = \exp\left\{-\frac{i}{2m\hbar}\int^t \left[\left|\mathbf{p} - \frac{e\mathbf{A}_1(t')}{c} - \frac{e\mathbf{A}_2(t')}{c}\right|^2 - (p_x - G(t'))^2\right] \mathrm{d}t'\right\}$$
$$\times \exp\left\{\frac{i}{\hbar}(\mathbf{p}\cdot\mathbf{x} - E_n t)\right\} \bar{H}_n(\xi), \tag{3}$$

where

$$\mathbf{p} = (p_x, Q(t), p_z),$$

$$G(t) = -\frac{e\omega_c}{c}\left[\frac{a_1 \cos \omega_1 t}{\omega_1 - \omega_c} + \frac{a_2 \cos \omega_2 t}{\omega_2 - \omega_c}\right],$$

$$Q(t) = -i\frac{e\omega_c}{c}\left[\frac{a_1 \sin \omega_1 t}{\omega_1 - \omega_c} + \frac{a_2 \sin \omega_2 t}{\omega_2 - \omega_c}\right]$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c; \quad \omega_c = \frac{|e| B_0}{mc}; \qquad n = 0, 1, 2 \ldots$$

where c is the velocity of light in vacuum; and

$$\bar{H}_n(\xi) = \left(\frac{m\omega_c}{\pi\hbar}\right)^{1/4} (2^n n!)^{-1/2} \exp(-\xi^2/2)\, H_n(\xi)$$

where $H_n(\xi)$ is the Hermite polynomial;

$$\xi = \left(\frac{m\omega_c}{\hbar}\right)^{1/2} y - (m\omega_c\hbar)^{-1/2}(p_x - G(t)).$$

Note that p_x and p_z are constants of motion and the state of the electron is specified by p_x, p_z and n. The function $\varphi_{pn}(\mathbf{x}, t)$ can be shown to be orthonormal. The average electron energy is found to be

$$\epsilon = E_n + \frac{1}{2m}\left[\left(G - \frac{e}{c}A_{1x}(t) - \frac{e}{c}A_{2x}(t)\right)^2 + \left(Q - \frac{e}{c}A_{1y}(t) - \frac{e}{c}A_{2y}(t)\right)^2\right]. \tag{5}$$

To determine the transition probability for electron–ion Coulomb scattering, the scattering potential $V(\mathbf{x})$ will be treated as a perturbation. $V(\mathbf{x})$ can be written as

$$V(\mathbf{x}) = -4\pi Z e^2 \hbar^2 \sum_{\mathbf{q}} \frac{1}{q^2} \exp[i\mathbf{q}\cdot(\mathbf{x} + \mathbf{x}_\alpha)/\hbar], \tag{6}$$

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where $\mathbf{x}$ and $\mathbf{x}_\alpha$ refer to electron and ion positions respectively, and Z_e is the ionic charge. The transition probability amplitude $M(1\to 2)$ for an electron to make a jump from the state (p_{1x}, p_{1z}, n_1) to the state (p_{2x}, p_{2z}, n_2) is given by

$$M(1\to 2) = -\frac{i}{\hbar}\int_{-\tau/2}^{\tau/2} \mathrm{d}t \int \mathrm{d}^3x\varphi^*_{p_2n_2}(\mathbf{x}, t)\, V(\mathbf{x})\varphi_{p_1n_1}(\mathbf{x}, t). \tag{7}$$

Substituting for φ's from Equation (3) and performing the integrations, we find the transition probability per unit time to be given by

$$\gamma(1\to 2) = |M(1\to 2)|^2/\tau = 4Z^2e^4N_i(2\pi\hbar)^3$$
$$\times \sum_{s_1, s_2, q_y}\left\{q_0^{-4}\,|J(n_1, n_2, \rho_0)|^2 \times J^2_{s_1}\left(\frac{\lambda_1}{\hbar\omega_1}\right) J^2_{s_2}\left(\frac{\lambda_2}{\hbar\omega_2}\right)\right.$$
$$\left.\times N_e(1)[1-N_e(2)]\delta\left[E_{n_2} - E_{n_1} + \frac{(p^2_{2z}-p^2_{1z})}{2m} - s_1\hbar\omega_1 - s_2\hbar\omega_2\right]\right\}, \tag{8}$$

where N_i is the ion density,

$$\mathbf{q}_0 = (p_{2x} - p_{1x}, a_y, p_{2z} - p_{1z}),$$

$$J(n_1, n_2, \rho_0) = (n_1!n_2!)^{-1/2}\exp(-\rho_0/2)\rho_0^{(n_1+n_2)/2}\cdot {}_2F_0\left(-n_1, -n_2, -\frac{1}{\rho_0}\right),$$

$$\rho_0 = \frac{(q_x^2 + q_y^2)}{2m\hbar\omega_c},$$

${}_2F_0(-n_1, -n_2, -1/\rho_0)$ is the hypergeometric function, s_1 and s_2 are integers ranging from $-\infty$ to $+\infty$, J_{s_1} and J_{s_2} are Bessel functions of the first kind, $\lambda_1 = eE_1q_{0\perp}/m(\omega_1-\omega_c)$, $\lambda_2 = eE_2q_{0\perp}/m(\omega_2-\omega_c)$, $N_e(2)$ is the square of the matrix elements of the fermion destruction operator and $[1-N_e(2)]$ is the square of the matrix elements for the fermion creation operator. The physical content of Equation (8) can be brought out clearly if we write

$$\gamma(1\to 2) = \sum_{s_1, s_2=-\infty}^{\infty} \mathcal{T}[s_1, s_2; 1\to 2]N_e(1)[1-N_e(2)], \tag{9}$$

where $\mathcal{T}(s_1, s_2; 1\to 2)$ represents the probability of a transition from state $|1\rangle$ to state $|2\rangle$ through the absorption or emission of s_1 photons of frequency ω_1 and s_2 photons of frequency ω_2. The summation over s_1 and s_2 therefore implies that the transition occurs via the absorption or emission of any number of photons.

The kinetic equation for the electrons can be derived by summing the diagrams shown in Figure 1. Using Equation (9) we can write the rate of change of $N_e(2)$ as

$$\frac{\partial N_e(2)}{\partial t} = \sum_{s_1, s_2}\sum_{\mathbf{p}_1, n_1} \mathcal{T}(s_1, s_2; 1\to 2)[N_e(1) - N_e(2)]. \tag{10}$$

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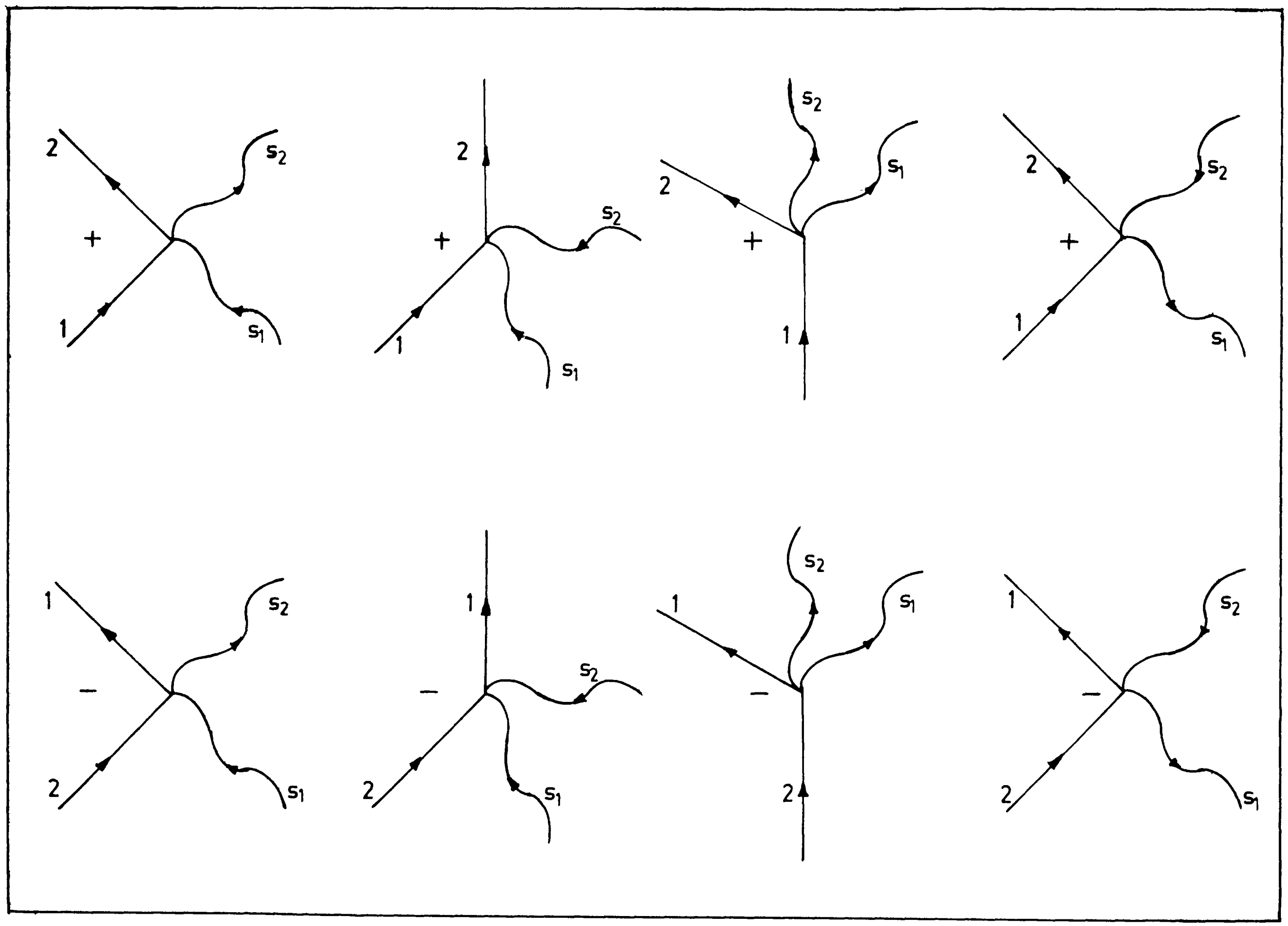

Fig. 1. The processes contributing to the change in the number of electrons $N_e(2)$ in state $|2\rangle$. The straight lines represent particles and the wavy line represents photons. The process with a plus sign gives a positive contribution to the change in $N_e(2)$ and that with minus sign gives a negative contribution.

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Taking the classical limit (Harris, 1969) – i.e.,

$$\hbar \to 0, \qquad n \to \infty,$$

$$\sum_{\mathbf{p}_1, n_1} \to \left(\frac{m}{2\pi\hbar}\right)^3 \int \mathrm{d}^3 v_1$$

and

$$\hbar\omega_c\left(n+\frac{1}{2}\right) \to \frac{1}{2} m v_\perp^2;$$

and assuming Maxwellian velocity distribution for the electrons in the initial state, we find that

$$\frac{\partial f_e(\mathbf{v}_2)}{\partial t} = 4Z^2 e^4 N_i N_e m^3 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int \mathrm{d}^3 v_1 \times \left\{\left[\exp\left(-\frac{mv_1^2}{2k_B T}\right) - \exp\left(-\frac{mv_2^2}{2k_B T}\right)\right] \sum_{q_y, s_1, s_2} \left[|J(n_1, n_2, \rho_0)|^2 \times J_{s_1}^2\left(\frac{\lambda_1}{\hbar\omega_1}\right) J_{s_2}^2\left(\frac{\lambda_2}{\hbar\omega_2}\right) \delta(\Omega - s_1\hbar\omega_1 - s_2\hbar\omega_2)\right]\right\}, \tag{11}$$

where

$$\Omega = E_{n_2} - E_{n_1} + (p_{2z}^2 - p_{1z}^2)/2m.$$

For strong lasers, the arguments of the Bessel functions are very large, and in this case the significant contribution is achieved only when the argument is equal to the order of the Bessel functions. The sum over s_1 and s_2 can be performed as

$$\sum_{s_1, s_2} J_{s_1}^2\left(\frac{\lambda_1}{\hbar\omega_1}\right) J_{s_2}^2\left(\frac{\lambda_2}{\hbar\omega_2}\right) \delta(\Omega - s_1\hbar\omega_1 - s_2\hbar\omega_2)$$
$$= \sum_{s_1} J_{s_1}^2\left(\frac{\lambda_1}{\hbar\omega_1}\right) \sum_{s_2} J_{s_2}^2\left(\frac{\lambda_2}{\hbar\omega_2}\right) \delta(\Omega - s_1\hbar\omega_1 - s_2\hbar\omega_2)$$
$$= \sum_{s_1} J_{s_1}^2\left(\frac{\lambda_1}{\hbar\omega_1}\right) J_{(\Omega - s_1\hbar\omega_1)/\hbar\omega_2}^2\left(\frac{\lambda_2}{\hbar\omega_2}\right) \sum_{s_2} \delta(\Omega - s_1\hbar\omega_1 - s_2\hbar\omega_2); \tag{12}$$

for $\lambda_2 \gg \hbar\omega_2$,

$$J_{(\Omega - s_1\hbar\omega_1)/\hbar\omega_2}^2\left(\frac{\lambda_2}{\hbar\omega_2}\right)$$

is a function which has maximum values near $(\Omega - s_1\hbar\omega_1) = \pm\lambda_2$. Therefore, Equation (12) becomes

$$\sum_{s_1, s_2} J_{s_1}^2\left(\frac{\lambda_1}{\hbar\omega_1}\right) J_{s_2}^2\left(\frac{\lambda_2}{\hbar\omega_2}\right) \delta(\Omega - s_1\hbar\omega_1 - s_2\hbar\omega_2)$$
$$= \sum_{s_1} J_{s_1}^2\left(\frac{\lambda_1}{\hbar\omega_1}\right) \frac{1}{2}[\delta(\Omega - s_1\hbar\omega_1 - \lambda_2) + \delta(\Omega - s_1\hbar\omega_1 + \lambda_2)]$$

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$$=\frac{1}{2}J^2_{(\Omega-\lambda_2)/\hbar\omega_2}\left(\frac{\lambda_1}{\hbar\omega_1}\right)\sum_{s_1}\delta(\Omega-s_1\hbar\omega_1-\lambda_2)$$
$$+\frac{1}{2}J^2_{(\Omega+\lambda_2)/\hbar\omega_2}\left(\frac{\lambda_1}{\hbar\omega_1}\right)\sum_{s_1}\delta(\Omega-s_1\hbar\omega_1+\lambda_2)$$
$$=\frac{1}{4}[\delta(\Omega-\lambda_2-\lambda_1)+\delta(\Omega-\lambda_2+\lambda_1)-\delta(\Omega+\lambda_2-\lambda_1)+\delta(\Omega+\lambda_2+\lambda_1)].$$

Of these four Dirac delta functions, $\delta(\Omega-\lambda_2-\lambda_1)$ gives the maximum contribution to the absorption probability when $\lambda_1 \gg k_BT$ and $\lambda_2 \gg k_BT$. The kinetic equation then is of the form

$$\frac{\partial f_e(\mathbf{v}_2)}{\partial t}=Z^2e^4N_iN_em^3\left(\frac{m}{2\pi k_BT}\right)^{3/2}\exp\left(-\frac{mv_2^2}{2k_BT}\right)$$
$$\times\int d^3v_1\left\{q_0^{-4}\,|J(n_1,n_2,\rho_0)|^2\right.$$
$$\left.\times\left[\exp\left(\frac{\lambda_1+\lambda_2}{k_BT}\right)-1\right]\delta(\Omega-\lambda_2-\lambda_1)\right\}.$$

We neglect the 1 in the square bracket compared to the first term. In the limit of cold-plasma approximation,

$${}_2F_0\left(-n_1,-n_2,-\frac{1}{\rho_0}\right)=1,$$

and as $n_2\to\infty$ and $\rho_2\to\infty$ it can be shown (cf. Seely, 1974) that

$$\sum_{q_y} q_2^{-4}|J(0,-n_2,\rho_2)|^2=2(2\pi)^{-1/2}\sum_{\rho_2} q_2^{-4}\delta_{n_2,n_1}$$
$$\simeq\sum_{q_y'} q_2^{-4}\delta_{q_y^2,mv_{2y}^2},$$

where $\mathbf{q}_0\to\mathbf{q}_2$ as we assume $f_e(\mathbf{v}_1)\to\delta(\mathbf{v}_1)$ and

$$\mathbf{q}_2=(p_{2x},q_y,p_{2z}).$$

After incorporating these simplifications, we find the kinetic equation to reduce to

$$\frac{\partial f_e(\mathbf{v}_2)}{\partial t}=\frac{Z^2e^4N_iN_e}{m}v_2^{-4}\delta\left(\frac{mv_2^2}{2}-Sv_{2\perp}\right),$$

where

$$S=\left[\frac{eE_1}{\omega_1-\omega_c}+\frac{eE_2}{\omega_2-\omega_c}\right]$$

and where E_1 and E_2 are the electric fields associated with the two lasers, $q_{2\perp}=mv_{2\perp}$. The rate of change of average kinetic energy of the electrons can be calculated from

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$$\frac{\mathrm{d}\langle\epsilon\rangle}{\mathrm{d}t} = \int \mathrm{d}^3 v_2 \frac{m v_2^2}{2} \frac{\partial f_e(\mathbf{v}_2)}{\partial t},$$

and is given by

$$\frac{\mathrm{d}\langle\epsilon\rangle}{\mathrm{d}t} = \frac{\pi^2 Z^2 e^4 N_i N_e}{2\left[\frac{eE_1}{\omega_1 - \omega_c} - \frac{eE_2}{\omega_2 - \omega_c}\right]}.$$

From the equation it is obvious that for $\omega_1 > \omega_c$ and $\omega_c > \omega_2$, the absorption rate can shoot up if the electric fields E_1 and E_2 are of comparable strengths.

Let $\omega_1 - \omega_c = \delta = \omega_c - \omega_2$, then

$$\frac{\mathrm{d}\langle\epsilon\rangle}{\mathrm{d}t} = \frac{\pi^2 Z^2 e^3 N_i N_e}{2(E_1 - E_2)/\delta} = \epsilon_0 \nu_{\text{eff}},$$

where ϵ_0 is the oscillatory energy of the electron, which from Equation (5) is found to be

$$\epsilon_0 = \frac{e^2 E_1^2}{2m\delta^2}\left(1 - \frac{E_2}{E_1}\right)^2;$$

and ν_{eff}, the effective collision frequency, becomes

$$\nu_{\text{eff}} = \frac{\pi^2 Z^2 N_i N_e e m \delta^3}{E_1^3\left(1 - \frac{E_2}{E_1}\right)^3}.$$

This result reduces to that of Seely (1974) for $E_2 = 0$. We note that the collision frequency in the presence of two lasers could be much more than that for the single laser case. The precise condition for increasing ν_{eff} is given by

$$\frac{\left(1 - \frac{\omega_c}{\omega_1}\right)^3}{\left(1 - \frac{E_2}{E_1}\right)^3} > 1$$

or

$$\frac{E_2}{E_1} > \frac{\omega_c}{\omega_1}.$$

This condition effectively reduces the oscillatory energy of the electron and hence, physically, we can expect the collision frequency to increase, which obviously leads to increased absorption of laser energy.

3. Application to Strong Radio Sources

The extragalactic double radio sources are known to derive their energy from the centrally placed nuclear regions which may contain a large number of galaxies, pulsars or quasars. The energy is extracted from the gravitational field

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of the nucleus and may emanate in the form of relativistic particles or electromagnetic waves. Rees (1971) proposed this flow of energy mainly in the form of low-frequency, high-intensity electromagnetic waves which could get beamed due to self-focusing effects. These electromagnetic waves would then act on a working surface and spend some of its energy in heating the plasma particles. The details of the formation of two oppositely directed beams and the conversion of the bulk energy contained in the fluid into the random particle energy has been considered by Blandford and Rees (1974). Here we propose the multiphoton inverse bremsstrahlung as the heating mechanism of the plasma particles. As Rees (1971) has pointed out, the central nucleus emits waves over a whole range of frequencies. Since the electron–cyclotron frequency in the extragalactic medium is comparable to the frequency of these intense electromagnetic waves, it is natural to divide the electromagnetic waves into two classes, one with their frequency slightly higher than the electron–cyclotron frequency and the other with their frequency slightly lower than the electron–cyclotron frequency. Once this scenario is established, the results of the previous section can be easily applied. The advantage of invoking two kinds of electromagnetic beams are obvious as far as the effectivity of the inverse bremsstrahlung is concerned. In the case of single electromagnetic beam, the collision frequency decreases as the electromagnetic beam frequency approaches the electron–cyclotron frequency; but in the presence of two electromagnetic beams with frequencies and amplitudes close to each other such that $E_2/E_1 > \omega_c/\omega_1$, the collision frequency again shoots up. The multiphoton inverse bremsstrahlung which seems to lose ground in the single electromagnetic beam–magnetoplasma system resurrects itself back in the presence of the second beam. We point out that we have considered only the limiting case of non-relativistic plasma particles. To get a clearer picture we should consider the relativistic case and also the situation when the electromagnetic waves become incoherent. These cases are under investigation.

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