PLASMA ACCELERATION BY ION-ACOUSTIC TURBULENCE

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Abstract. An energetic proton beam passing through a stationary ionized medium, excites ion-acoustic turbulence. The ion-acoustic instability saturates due to the non-linear indirect wave-particle scattering. The electric field associated with the ion-acoustic waves accelerates the plasma particles. Applicability of the results to cometary tails is discussed.

1. Introduction

An electron–ion plasma system, in which the electrons move with a drift velocity smaller than the electron thermal velocity and larger than the speed of the ion-acoustic wave is known to be unstable towards ion-acoustic oscillations. The amplitude of the ion-acoustic oscillations grows with time if the electron temperature T_e is more than the ion temperature T_e. There is another way of generating ion-acoustic turbulence. A proton beam with a speed greater than the ion-sound speed, passing through a plasma is also capable of exciting ion-acoustic oscillations through resonant instability Mikhailovskii (1974). In this piece of work excitation of ion-acoustic instability is studied in a system consisting of (i) a proton beam of density n_p and velocity V passing through (ii) an electron-ion plasma, the electronic component of which has a drift velocity $V_e < V$. All velocities are measured relative to the stationary ions. The linear analysis of the instability is carried out in Section 2. Section 3 deals with the saturation mechanism. Expressions for the rate of change of momentum and the acceleration of the plasma particles are derived in Section 4. Application of the results to the cometary tails is discussed in Section 5.

2. Linear Analysis

The dispersion relation for ion-acoustic waves in a proton beam-plasma system is given by Mikhailovskii (1974):

$$\varepsilon = 1 + \frac{1}{q^2 d_e^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_p^2}{(\omega - q_z v)^2} = 0\,, \tag{1}$$

where

(ω, q) are the frequency and the wave vector of the wave,

$d_e =$ Debye length for electrons,

$\omega_{pi}^2 = \dfrac{4\pi n_0 e^2}{m_i}$, n_0 is plasma density,

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$m_i =$ mass of the ion,

$\omega_p^2 = \dfrac{4\pi n_p e^2}{m_p}$, n_p is the density of proton beam and m_p is the mass of a proton,

V is the streaming velocity of the proton beam.

In deriving Equation (1), the following assumptions have been made:

$$V > V_{Tp}, \qquad V_e < V < V_{Te},$$

$$\omega > qV_{Ti}, \qquad |\omega - q_z V| > qV_{TP},$$

where V_{Tp}, V_{Ti}, and V_{Te} are the thermal velocities of the protons, ions and electrons respectively. V_e is the drift speed of electrons.

In order to study the excitation of the ion-acoustic instability, Equation (1) can be solved for complex roots Ω such that

$$\Omega = \omega + i\gamma$$

and

$$\omega \sim \frac{\omega_{pi}}{1 + \dfrac{1}{q^2 d_e^2}}. \tag{2}$$

The growth rate γ and the wave vector q is to be determined from the following equations:

$$\frac{3\gamma^2}{\omega^2} - \frac{\alpha\omega^2}{\gamma^2}\frac{\beta^2 - 1}{(\beta^2+1)^2} = 0 \tag{3}$$

and

$$1 - \frac{2\gamma^2}{\omega^2} - \frac{\alpha\omega^3\beta}{\gamma^3(\beta^2+1)^2} = 0, \tag{4}$$

where

$$\alpha = n_p m_i / n_0 m_p$$

and

$$\beta^2 = (\omega - q_z V)^2/\gamma^2.$$

While arriving at Equations (3) and (4), no assumptions about the magnitudes of α and β have been made.

3. Saturation Field

There have been many and varied attempts to explain the saturation of the ion-acoustic instability. Kadomtsev (1964) invoked the ion–nonlinear Landau damping as the saturation mechanism, which was shown to be inadequate by Sloan and Drummond (1970). Sagdeev and Galeev (1969) put their trust in mode–mode coupling whereas Tsytovich (1971) preferred the process of one ion sound wave scattering into two ion sound waves. Krishan (1978) suggested the indirect wave mediated wave-particle scattering as the saturation mechanism of the ion-acoustic instability. This mechanism furnished estimates of anomalous resistivity which agreed very well with the observed results in solar flares. In the present proton beam–plasma system, however, an additional contribution from the proton beam needs to be included. Since the details of the calculations for this process are given in Krishan (1978), here we will only give the final results as applicable to the proton beam–plasma system. The various processes are shown in Figures 1, 2a and 2b. The

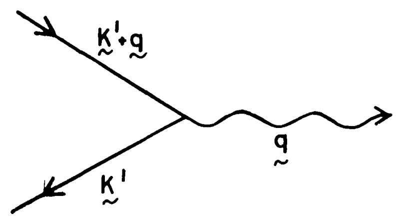

Fig. 1. Direct wave-particle scattering. The solid lines stand for the particles and the wavy lines for the ion-sound waves.

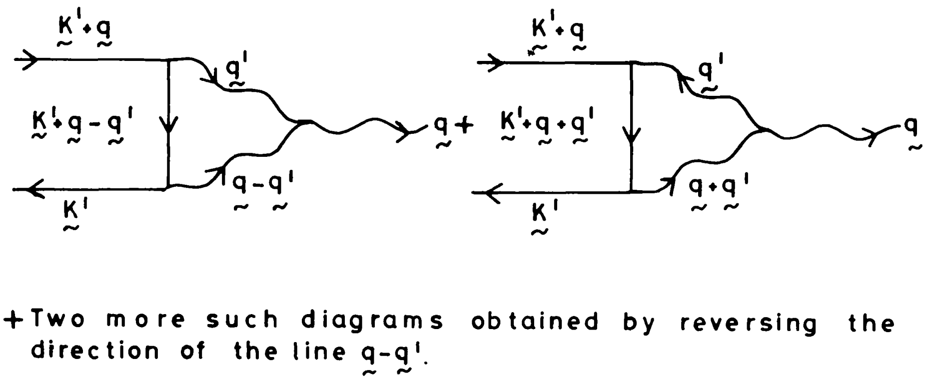

Fig. 2a. Indirect wave-particle scattering. The electron or the ion is scattered with the emission of two waves, which then interact to yield the third wave, which is to be stabilized.

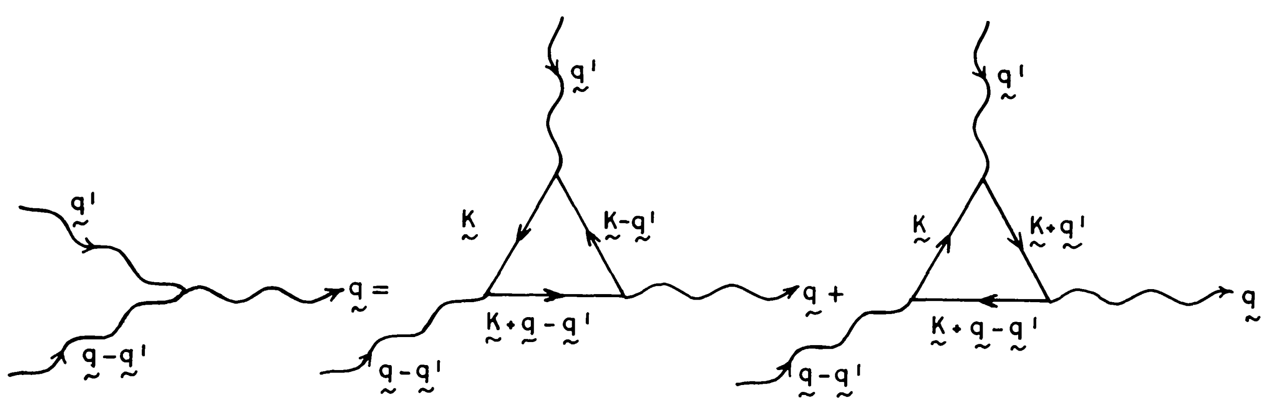

Fig. 2b. The details of the three-wave vertex appearing in Figure 2a.

effective matrix elements for the direct and indirect electron–ion sound wave scattering, ion–ion sound wave scattering and the proton–ion sound wave scattering are found to be:

$$W^e_{\text{eff}} = -|W_{\text{oe}}| \left[1 - \frac{4\pi I(t)}{3N_{\text{D}} m_e} \left(2 - \frac{n_p}{n_0} \frac{m_e^2}{m_p^2} \frac{V_{Te}^4}{V^4} \right) \times \times \int \frac{\mathrm{d}\psi}{\left[\frac{\theta^2 c_s}{2} - \theta V_{\perp e} \cos(\psi - \phi_e) \right]^2} \right], \tag{5}$$

$$W^i_{\text{eff}} = |W_{\text{oe}}| \left[1 + \frac{4\pi I(t)}{3N_{\text{D}} m_i} \left(2 - \frac{n_p}{n_0} \frac{m_e^2}{m_p^2} \frac{V_{Te}^4}{V^4} \right) \times \times \int \frac{\mathrm{d}\psi}{\left[\frac{\theta^2 c_s}{2} - \theta V_{\perp i} \cos(\psi - \phi_i) \right]^2} \right], \tag{6}$$

and

$$W^p_{\text{eff}} = |W_{\text{oe}}| \left[1 - \frac{4\pi I(t)}{3N_{\text{D}} m_p} \left(2 - \frac{n_p}{n_0} \frac{m_e^2}{m_p^2} \frac{V_{Te}^4}{V^4} \right) \times \times \int \frac{\mathrm{d}\psi}{\left[\frac{\theta^2 c_s}{2} - \theta V_{\perp p} \cos(\psi - \phi_p) \right]^2} \right], \tag{7}$$

where

$|W_{\text{oe}}| = \left[\frac{4\pi e^2 \hbar \omega_q}{V q^2 F_q} \right]^{1/2}$ = matrix elements for direct wave-particle scattering,

N_{D} = number of electrons in the Debye sphere,

$I(t) = \hbar N_q \omega_q$ = energy in the mode q,

N_q = number of phonons in the ion-sound wave;

ϕ_e, ϕ_i, ϕ_p are the azimuthal angles of velocity vectors V_e, V_i, and V_p,

c_s = ion-sound speed = $\sqrt{\frac{T_e}{m_i}}$,

$\theta \sim \frac{c_s}{V}$;

$V_{e\perp}$, $V_{i\perp}$, and $V_{p\perp}$ are the components of electron, ion and proton velocities perpendicular to q.

The energy balance equation can be written as

$$\left.\frac{\mathrm{d}N_q}{\mathrm{d}t}\right|_j = \sum_{k'} \frac{2\pi}{\hbar} |W^i_{\text{eff}}|^2 [(N_q + 1) f_j(k' + q)(1 - f_j(k')) - - N_q f_j(k')(1 - f_j(k' + q))] \delta(\omega_q - q\, V_j), \tag{8}$$

where $j = e$, i, and p.

The growth rate or the damping rate of the ion sound wave on electrons, ions and protons is given by

$$\gamma_j = \frac{1}{2Nq}\frac{\mathrm{d}N_q}{\mathrm{d}t}\bigg|_j$$

and one finds:

$$\gamma_e = \sqrt{\frac{\pi}{8}}\frac{C_s}{V_{Te}}(qV_e - \omega_q)\exp\left[-\frac{m}{2\chi T_e}\left\{\left(\frac{\omega_q}{q} - V_e\right)^2\right\}\right]\times$$
$$\times\left[1 - \frac{8I(t)}{3N_\mathrm{D}\chi T_e\theta^2}\left(2 - \frac{n_p}{n_0}\frac{m_e^2}{m_p^2}\frac{V_{Te}^4}{V^4}\right)\right], \qquad (9)$$

$$\gamma_i = -\sqrt{\frac{\pi}{8}}\frac{C_s^3}{V_{Ti}^3}\omega_q e^{-T_e/2T_i}\left[1 + \frac{4\pi^2 I(t)}{3N_\mathrm{D}\theta^4 m_i C_s^2}\left(2 - \frac{n_p}{n_0}\frac{m_e^2}{m_p^2}\frac{V_{Te}^4}{V^4}\right)\right], \qquad (10)$$

and

$$\gamma_p = \gamma + \sqrt{\frac{\pi}{8}}\frac{C_s}{V_{T_p}}(qV - \omega_q)\frac{n_P}{n_0}\exp\left[-\frac{m_P}{2\chi T_p}\left(\frac{\omega_q}{q} - V\right)^2\right]\times$$
$$\times\left[1 - \frac{8I(t)}{3N_\mathrm{D}m_p V^2\theta^2}\left(2 - \frac{n_p}{n_0}\frac{m_e^2}{m_p^2}\frac{V_{Te}^4}{V^4}\right)\right]. \qquad (11)$$

The saturation energy of the ion sound wave is given by the condition

$$\gamma_e + \gamma_p + \gamma_i = 0\,. \qquad (12)$$

This furnishes

$$\left[I(t)\left(2 - \frac{n_p}{n_0}\frac{m_e^2}{m_p^2}\frac{V_{Te}^4}{V^4}\right)\Big/ 3N_\mathrm{D}\frac{C_s}{V}\chi T_e\right] =$$
$$= \left[\sqrt{\frac{8}{\pi}}\frac{\gamma}{\omega_q} - \sqrt{\frac{m_e}{m_i}}\left(1 - \frac{qV_e}{\omega_q}\right)e^{-V_e^2/V_{Te}^2} + \right.$$
$$\left. + \frac{V_{Te}}{V_{Tp}}\left(\frac{qV}{\omega_q} - 1\right)\frac{n_P}{n_0}e^{-V^2/V_{Tp}^2} - \left(\frac{T_e}{T_i}\right)^{3/2} e^{-T_e/2T_i}\right]\times \qquad (13)$$
$$\times\left[8\pi^2\left(\frac{T_e}{T_i}\right)^{3/2}\frac{V^3}{C_s^3}e^{-T_e/2T_i} - \frac{12\pi C_s^2}{VV_{Te}}e^{-V_e^2/V_{Te}^2}\left(1 - \frac{qV_e}{\omega_q}\right) + \right.$$
$$\left. + \frac{12\pi C_s^2}{VV_{TP}}\sqrt{\frac{m_e}{m_P}}\frac{V_{Te}^2}{V^2}\frac{n_P}{n_0}\left(\frac{qV}{\omega_q} - 1\right)e^{-V^2/V_{TP}^2}\right]^{-1}.$$

4. Acceleration of the Plasma Particles

The rate of change of energy and momentum of the plasma particles in the presence of ion-acoustic turbulence can be calculated by using the diffusion equation

(Lacombe and Mangeney, 1969 and Kaplan and Tsytovich, 1973). One writes

$$\left.\frac{\partial \mathbf{V}}{\partial t}\right|_j = \frac{1}{N_{0j}} \int \frac{\partial f_i(\mathbf{V})}{\partial t} \mathbf{V}\, \mathrm{d}^3 V, \tag{14}$$

where N_{0j} is the total number of particles, $f_j(\mathbf{V})$ is the velocity distribution function and

$$\begin{aligned}\frac{\partial f_j(V)}{\partial t} = \frac{2\pi}{\hbar^2} \sum_q \frac{4\pi e_j^2}{Vq^2} I_q[\{(1-f_j(k))f_i(k+q) - f_j(k)(1-f_j(k+q))\}\times \\ \times \delta(E_{k+q} - E_k - \hbar\omega_q) + \\ + \{(1-f_j(k))f_j(k-q) - f_j(k)(1-f_j(k-q))\}\times \\ \times \delta(E_{k-q} - E_k + \hbar\omega_q)]\,. \end{aligned} \tag{15}$$

Here e_j is the charge of the species and δ is the energy conserving Dirac delta function. Substituting for I_q from Equation (13), one can write down the rate of change of momentum for the electrons, the protons and the ions due to wave particle scattering:

$$\begin{aligned}\left.\frac{\partial V}{\partial t}\right|_e = \frac{\omega_{pe}}{72} \frac{\gamma}{\omega} V \left(\frac{q_z}{q}\right)^4 \left(\frac{q_z}{q} - \frac{V_e}{V}\right)\left(\frac{T_e}{T_i}\right)^{-3/2} \times \\ \times e^{T_e/2T_i} e^{-[(\omega_q/q)-V_e]^2/V_{Te}^2}, \end{aligned} \tag{16}$$

$$\begin{aligned}\left.\frac{\partial V}{\partial t}\right|_p = \frac{\omega_{pe}}{72} \frac{\gamma}{\omega} V \frac{V_{Te}}{V_{Tp}} \frac{n_0 m_e}{n_p m_p} \left(\frac{q_z}{q}\right)^4 \left(\frac{q_z}{q} - 1\right)\left(\frac{T_e}{T_i}\right)^{-3/2} \times \\ \times e^{T_e/2T_i} e^{-[(\omega_q/q)-V]^2/V_{TP}^2}, \end{aligned} \tag{17}$$

and

$$\left.\frac{\partial V}{\partial t}\right|_i = \frac{\omega_{Pe}}{72} \frac{\gamma}{\omega} V \frac{V_{Te}}{V_{Ti}} \frac{m_e}{m_i} \left(\frac{q_z}{q}\right)^5 \left(\frac{T_e}{T_i}\right)^{-3/2} e^{T_e/2T_i} e^{-T_e/2T_i}. \tag{18}$$

One can easily see that the ions gain negligibly small amount of momentum by direct wave-particle scattering. Therefore the major process by which the ions get accelerated is due to the relaxation of the electrons and ions. One can write the momentum balance equation as

$$n_0 m_e \left.\frac{\partial \mathbf{V}}{\partial t}\right|_e + n_P m_P \left.\frac{\partial \mathbf{V}}{\partial t}\right|_P + n_0 m_i \left.\frac{\partial V}{\partial t}\right|_i = 0 \tag{19}$$

and the acceleration imparted to the ions a_i is given by

$$n_0 m_i a_i = -n_0 m_e \left.\frac{\partial V}{\partial t}\right|_e - n_P m_P \left.\frac{\partial V}{\partial t}\right|_P. \tag{20}$$

5. Application to Cometary Tails

The theoretical system studied above can be easily identified with the solar wind comet system, where the solar wind of density n_P is propagating with a velocity V through the cometary plasma of density n_0. The condition of zero current requires the electrons to move with a velocity V_e, which is determined such that $n_P V = (n_0 + n_P) V_e$, $n_P < n_0$. Since $V_e \ll V$, one can consider the proton beam passing through a relatively stationary plasma. Even though the electrons have a net drift velocity V_e, the ion-acoustic oscillations are still described by the dispersion relation given in Equation (1), since $V_e \ll V_{Te}$. Chernikov (1975) has also considered the acceleration of cometary ions by ion-acoustic turbulence. The present treatment differs in two ways. First, here the proton beam is entirely responsible for the linear growth rate of the ion-acoustic instability, the electrons give a small contribution in case $V_e > C_s$. The growth rate as given by Equations (3) and (4) is much larger than that given by Chernikov (1975). Therefore, it seems that the resonant beam plasma instability is a more efficient mechanism of generating ion-acoustic turbulence. Secondly as will be shown below, the electrons are more efficient in accelerating the cometary ions than the protons. The rate of loss of momentum is, apart from other factors, proportional to $\exp\{-[(\omega_q/q) - V]^2/V_{Tp}^2\}$ for protons and to $\exp\{-[(\omega_q/q) - V_e]^2/V_{Te}^2\}$ for the electrons. Obviously the first factor is much smaller than the second factor, because $V_{Te} \gg V_{Tp}$. One gets from Equation (20):

$$a_i = -\frac{m_e}{m_i}\frac{\omega_{Pe}}{72}\frac{\gamma}{\omega} V\left(\frac{q_z}{q}\right)^4\left(\frac{q_z}{q} - \frac{V_e}{V}\right)\left(\frac{T_e}{T_i}\right)^{-3/2} e^{T_e/2T_i}\, e^{[(\omega_q/q) - V_e]^2/V_{Te}^2}.$$

We can make an estimate of the acceleration of the cometary ions, by using typical values of the solar wind–comet parameters:

For

$$n_0 = 100\ \text{cm}^{-3}, \qquad n_P = 5\ \text{cm}^{-3}, \qquad V \sim 3\times 10^7\ \text{cm s}^{-1},$$

$$m_i \sim 28 m_P, \qquad \frac{T_e}{T_i} \sim 20, \qquad V_{Te} \sim 3\times 10^8\ \text{cm s}^{-1},$$

$$C_s = \sqrt{\frac{T_e}{m_i}} \sim 0.03\, V, \qquad \alpha = \tfrac{28}{20}.$$

For $\alpha = \frac{28}{20}$, one finds that Equations (3) and (4) can be satisfied for $|\beta| \sim 3$ and therefore,

$$\omega - q_z V = -3\gamma,$$

$$\frac{q_z}{q}\frac{V}{C_s} \sim 1 + \frac{3\gamma}{\omega}, \quad \text{and} \quad \frac{\gamma}{\omega} \sim 0.4.$$

Substituting for all the quantities one gets $a_i \sim 500\ \text{cm s}^{-2}$. For $n_0 = 10\ \text{cm}^{-3}$, $\alpha = \frac{28}{2}$.

For $\alpha = \frac{28}{2}$, Equations (3) and (4) can be solved for $|\beta| \sim 6$ and therefore,

$$\frac{q_z}{q} \frac{V}{C_s} \sim 1 + \frac{6\gamma}{\omega} \quad \text{and} \quad \frac{\gamma}{\omega} \sim 0.5 \, .$$

Again one determines a_i to be

$$a_i \sim 2.5 \times 10^4 \text{ cm s}^{-2} \, .$$

6. Conclusion

A resonant ion-acoustic instability is excited by an energetic proton beam with velocity V passing through a plasma, the electronic component of which has a net drift velocity $V_e < V$, but $V_e > C_s$. The streaming electrons and protons lose their directional momentum due to wave-particle scattering. For the relaxation of the electrons and protons a force arises which then acts on the ions and accelerates them. It is found that the force produced for the relaxation of the electrons is much more than that for protons. The estimates of acceleration of the ions in the solar wind–cometary plasma agree very well with the observed results.

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