A TIME DEPENDENT MODEL FOR SPICULE FLOW

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Abstract. A time dependent model for the flow of gas in a spicule is studied. In this model, the flow occurs in a magnetic flux sheath. Starting from hydrostatic equilibrium, the flux sheath is allowed to collapse normal to itself. The collapse induces a flow of gas along the magnetic field and this flow is identified as a spicule. A variety of sheath geometries and velocity patterns for the normal flow have been studied. It is observed that a large curvature in the field geometry and a large initial value for the normal flow are necessary to achieve spicule-like velocities. The duration for which a large velocity of normal flow is required is much shorter than the average lifetime of a spicule. It is proposed that the initial rapid collapse occurs during an 'impulsive spicule' phase and it is the subsequent gradual relaxation of the flow which is observed as a spicule.

1. Introduction

It is well known from observations that the magnetic field on the Sun is concentrated into elements called flux tubes. These often take part in processes due to which their geometry can be altered. A change in geometry of a flux tube may in certain cases have a significant effect on the dynamics of gas within it. If the change in geometry is sufficiently rapid, it is possible for a flow with a fairly large velocity to be initiated in the flux tube. In this paper we examine the possibility that spicules are a manifestation of such a process.

Spicules are dense jets of gas moving upward with a velocity around 20 km s^{-1}. Observations have established firmly that spicules originate near the boundaries of supergranules, thereby identifying them with regions of magnetic field concentrations. Owing to the high conductivity of the solar plasma, the spicule flow will be channelled along the magnetic field. The driving mechanism for spicules is still debatable, although several theories have been proposed (see Athay, 1976, for a review). We investigate a model described qualitatively by Hollweg (1972), in which spicules are identified with the gas expelled upwards when a magnetic flux sheath undergoes lateral compression. Using the argument of mass conservation, Hollweg calculates that it is possible for gas to be expelled upwards with a velocity of 40 km s^{-1} in a sheet 1000 km high and 700 km thick undergoing compression with a lateral velocity of 0.4 km s^{-1}. However, before this model can be accepted, it is necessary to consider the effect of compressibility and to include the dynamics in the analysis. Recently, this has been done in Hollweg (1979) using a linear approach. According to the findings in this paper, a large vertical flow can be generated as a result of a 'resonance' between a fast MHD wave and gravito-accoustic wave. The existence of such a 'resonance' is doubtful (Venkatakrishnan and Hasan, 1981). Furthermore, the conditions for the generation of large flows based on a linear analysis cannot be applied to spicule flow, as such conditions become invalid once

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nonlinear effects set in. Thus, the seed flow generated due to 'resonance' may actually die out instead of being enhanced. The same criticism applies to the conclusions of Roberts (1979), who also advocates a resonance condition for the buildup of spicule flow.

In this paper, we examine quantitatively, using the nonlinear MHD equations, the hypothesis that the collapse of a magnetic flux sheath may actually drive a spicule. Since spicules have a finite lifetime, we consider the time dependent problem and attempt to demonstrate different possible phases which may occur during their lifetime.

For purposes of mathematical simplicity and also to focus our attention on the vertical flow, we solve the MHD equations in a specified stream geometry in the same spirit as Kopp and Pneuman (1976), who however, studied a totally different problem.

2. Basic Equations

Let us envisage the spicule as a two dimensional structure in the $y-z$ plane with gravity acting in the negative z-direction. It is convenient to transform to a system of co-ordinates (s, n) 'moving' with a field line where s denotes a distance measured along a field line and n denotes a distance measured along a normal curve (in the same plane).

Following Kopp and Pneuman (1976), we see that the unit vectors s and n satisfy the following geometric relations:

$$\frac{\partial \hat{s}}{\partial t}=\hat{n}\frac{\partial \theta}{\partial t}, \qquad \frac{\partial \hat{s}}{\partial s}=\frac{\hat{n}}{R}, \qquad \frac{\partial \hat{s}}{\partial n}=\hat{n}\frac{\partial \theta}{\partial n},$$

$$\frac{\partial \hat{n}}{\partial t}=-\hat{s}\frac{\partial \theta}{\partial t}, \qquad \frac{\partial \hat{n}}{\partial s}=-\frac{\hat{s}}{R}, \qquad \frac{\partial \hat{n}}{\partial n}=-\hat{s}\frac{\partial \theta}{\partial n},$$

where R denotes the radius of curvature of a field line and θ the angle the field makes with the z axis.

The equation of motion along the field can now be expressed as

$$\frac{\partial V_s}{\partial t}+V_s\frac{\partial V_s}{\partial s}+V_n\frac{\partial V_s}{\partial n}=$$

$$=-\frac{1}{s}\frac{\partial P}{\partial s}-g\cos\theta+V_n\frac{\partial \theta}{\partial t}+\frac{V_nV_s}{R}+V_n^2\frac{\partial \theta}{\partial n}, \qquad (2.1)$$

where V denotes velocity, P pressure ρ mass density, and g acceleration due to gravity.

Consider a frame of reference fixed to a given field line. In such a frame, moving normal to itself with velocity V_n, let us denote the space and time derivatives as D/Ds

and D/Dt, respectively. These satisfy the following operator relationships:

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + V_n \frac{\partial}{\partial n}, \tag{2.2a}$$

$$\frac{\mathrm{D}}{\mathrm{D}s} = \frac{\partial}{\partial s}. \tag{2.2b}$$

From (2.1) and (2.2), we finally have:

$$\frac{\mathrm{D}V_s}{\mathrm{D}t} + V_s \frac{\mathrm{D}V_s}{\mathrm{D}s} = -\frac{1}{\rho}\frac{\mathrm{D}P}{\mathrm{D}S} - g\cos\theta + V_n \frac{\mathrm{D}\theta}{\mathrm{D}t} + V_n V_s/R. \tag{2.3}$$

In a similar manner, the equation of continuity takes the form:

$$\frac{\mathrm{D}V_s}{\mathrm{D}s} + \frac{\mathrm{D}\ln\rho}{\mathrm{D}t} + V_s \frac{\mathrm{D}\ln\rho}{\mathrm{D}s} + V_s \frac{\partial\theta}{\partial n} - \frac{V_n}{R} + \frac{\partial V_n}{\partial n} = 0. \tag{2.4}$$

We close the set of equations by choosing an isothermal equation of state:

$$P = \rho a^2, \tag{2.5}$$

where $a = \sqrt{\mathscr{R}T}$ is the sound speed, assumed constant, $\mathscr{R}$ is the gas constant, and T the gas temperature. The unknown variables in Equations (2.3)–(2.5) are P, ρ, and V_s. We shall assume that the stream geometry is known at each instant of time by specifying V_n and the initial field configuration. Noting that Equations (2.3) and (2.4) are hyperbolic, we use the method of characteristics to arrive at a solution (Courant and Friedrichs, 1948). The characteristic equations are

$$\frac{\mathrm{d}\xi_+}{\mathrm{d}t} = A + B/a \quad \text{on} \quad C_+, \tag{2.6a}$$

$$\frac{\mathrm{d}\xi_-}{\mathrm{d}t} = A - B/a \quad \text{on} \quad C_-, \tag{2.6b}$$

where

$$\xi_\pm = \ln P \pm V_s/a, \tag{2.7a, b}$$

$$B = -g\cos\theta + V_n \frac{\mathrm{D}\theta}{\mathrm{D}t} + V_n V_s/R, \tag{2.8}$$

$$A = -V_s \frac{\partial\theta}{\partial n} + V_n/R - \partial V_n/\partial n. \tag{2.9}$$

Equations (2.6a) and (2.6b) hold on the characteristic curves C_+ and C_- respectively, whose equations are

$$C_+: \quad \frac{\mathrm{d}s}{\mathrm{d}t} = V_s + a, \tag{2.10a}$$

$$C_-: \quad \frac{ds}{dt} = V_s - a \,. \tag{2.10b}$$

Having transformed the equations into characteristic form, we are now in a position to obtain a solution to our problem. Before attempting this, let us first specify the stream geometry.

3. Stream Geometry

Consider first the magnetic field configuration at some initial time, say $t = 0$, before the collapse i.e. when $V_n = 0$. We choose the field to have the following potential form:

$$B_y = B_0 \exp(-kz) \sin ky \,,$$

$$B_z = B_0 \exp(-kz) \cos ky \,,$$

where k and B_0 are constants. The equation of a field line is easily determined as

$$\sin ky = \sin ky_B \exp(kz) \,, \tag{3.1}$$

where y_B is the lateral distance of the line base from the axis of the flux sheath. Furthermore, we see that

$$\theta = ky \,, \tag{3.2}$$

$$R = (k \sin ky)^{-1} \,. \tag{3.3}$$

Equations (3.1)–(3.3) specify the initial geometry of a field line. To determine the geometric properties of the field once the collapse starts, it is necessary to prescribe V_n explicitly. In order to keep the analysis sufficiently general, we consider the following two different forms for V_n:

(a) $$V_n = V_0 \sin ky / \sin ky_B \,, \tag{3.4}$$

where

$$V_0 = V_1 \mathrm{H}(t - t_1) + V_2 \mathrm{H}(t_1 - t) \,,$$

and

$$V_j = -V_{nj} \cos ky_{Bj} \sin(y_B/y_j) / [\sin(y_{Bj}/y_j) \cos(ky_B)] \qquad (j = 1, 2) \,.$$

In the above, H denotes the Heaviside function and t_1, V_{n1}, y_{B1}, y_1, V_{n2}, y_{B2}, and y_2 are constants chosen in such a way that at $t = t_1$ the velocity is continuous.

(b) $V_n = V_0 \, e^{-z/H}$, where V_0 and H are constants.

In (a) and (b), V_0 denotes the normal velocity at the base of a field line. The form given in (a) guarantees that the geometric relationships (3.1)–(3.3) hold at all instants of time, so that the initial form of the field is preserved. For the second from, the stream geometry for $t > 0$, can be determined by solving the following differential equations:

$$\frac{dy}{dt} = V_n \cos\theta\,, \tag{3.5}$$

$$\frac{dz}{dt} = -V_n \sin\theta\,, \tag{3.6}$$

$$\frac{d\theta}{dt} = \frac{\partial V_n}{\partial s}\,. \tag{3.7}$$

4. Method of Solution

For the purposes of our study, it is sufficient to consider a single magnetic line of force and examine the flow along it as it moves normal to itself in a specified manner. The geometric properties of a given field line are determined from Equations (3.5)–(3.7), whereas the dynamical variables are found from Equations (2.6) and (2.10).

Since we are studying an initial value problem, all variables must be specified at $t = 0$. We shall assume that initially the gas is in hydrostatic equilibrium. Thus at $t = 0$, we have:

$$P = P_0 \exp(-gz/a^2)\,,$$

$$V_s = 0\,,$$

$$V_n = 0\,,$$

where P_0 denotes the pressure at the base of the line.

For solving Equations (2.6a) and (2.6b), a two dimensional grid is prepared by drawing the C_+ and C_- curves. It is important to point out that only one family of characteristics reach points on the boundary when the flow is subsonic. Consequently, it is necessary to specify an independent relation between P and V_s so that the flow properties are known throughout the region of interest. For simplicity, we assume that at the base of the line, the pressure remains constant and at the top the velocity is specified by extrapolating the interior values.

We solved Equations (2.6a), (2.6b), (2.10a), and (2.11) numerically using a predictor-corrector method. An inverse marching scheme was used to advance the equations in time (for details see Hasan and Venkatakrishnan, 1980). To ensure numerical stability, the time step was chosen small enough so that the Courant–Friedrichs–Lewy criterion was satisfied.

5. Results

Let us now examine the results of the computations. Figures 1 to 5 depict the spatial and temporal distribution of the flow variables for different choice of parameters.

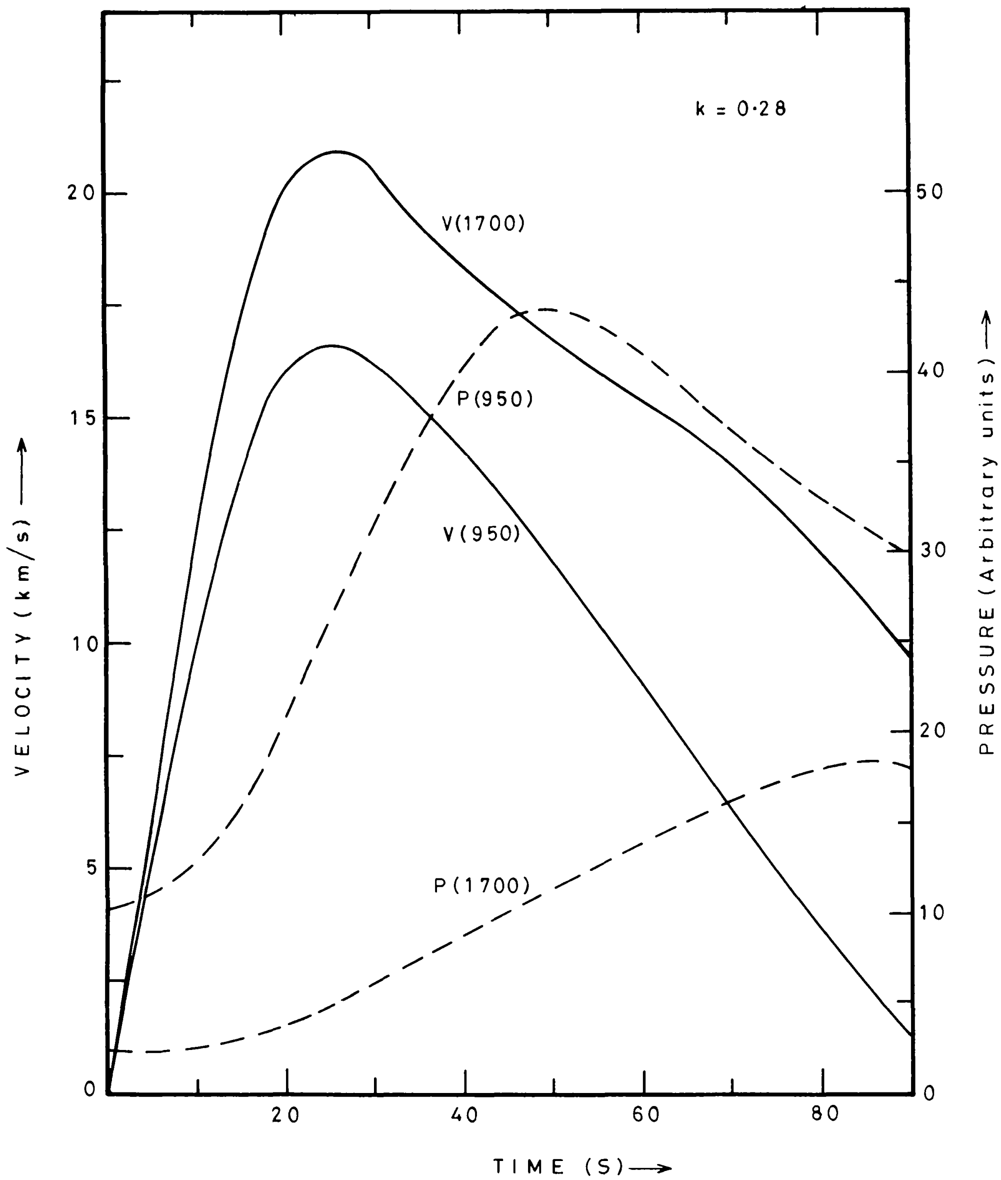

Fig. 1. The run of parallel velocity (solid lines) and pressure (dashes) with time is shown at different space points (indicated in brackets). A starting value, $V_{nb} = 60$ km s^{-1}, was chosen for the normal velocity at the base. The units of k are 10^{-3} km^{-1}.

In order to reduce computation time we considered a field line having a length of 2000 km for most of the numerical study. A representative temperature of 10^4 K was chosen, though it was seen that the results were insensitive to its precise choice.

Case I: $V_n = V_0 \sin ky/\sin ky_B$.

Figure 1 shows the run of velocity and pressure with time at different space points for which V_n has the form (a). To achieve spicule-like velocities an initial normal velocity $\gtrsim 60$ km at the base was found necessary. All curves show a sharp rise in the first few seconds followed by a subsequent gradual decline. The decline is due to the rapid diminution of the lateral flow after around 15 s. A sudden decrease in the normal flow does not lead to a similar decrease in the parallel flow, which relaxes on a

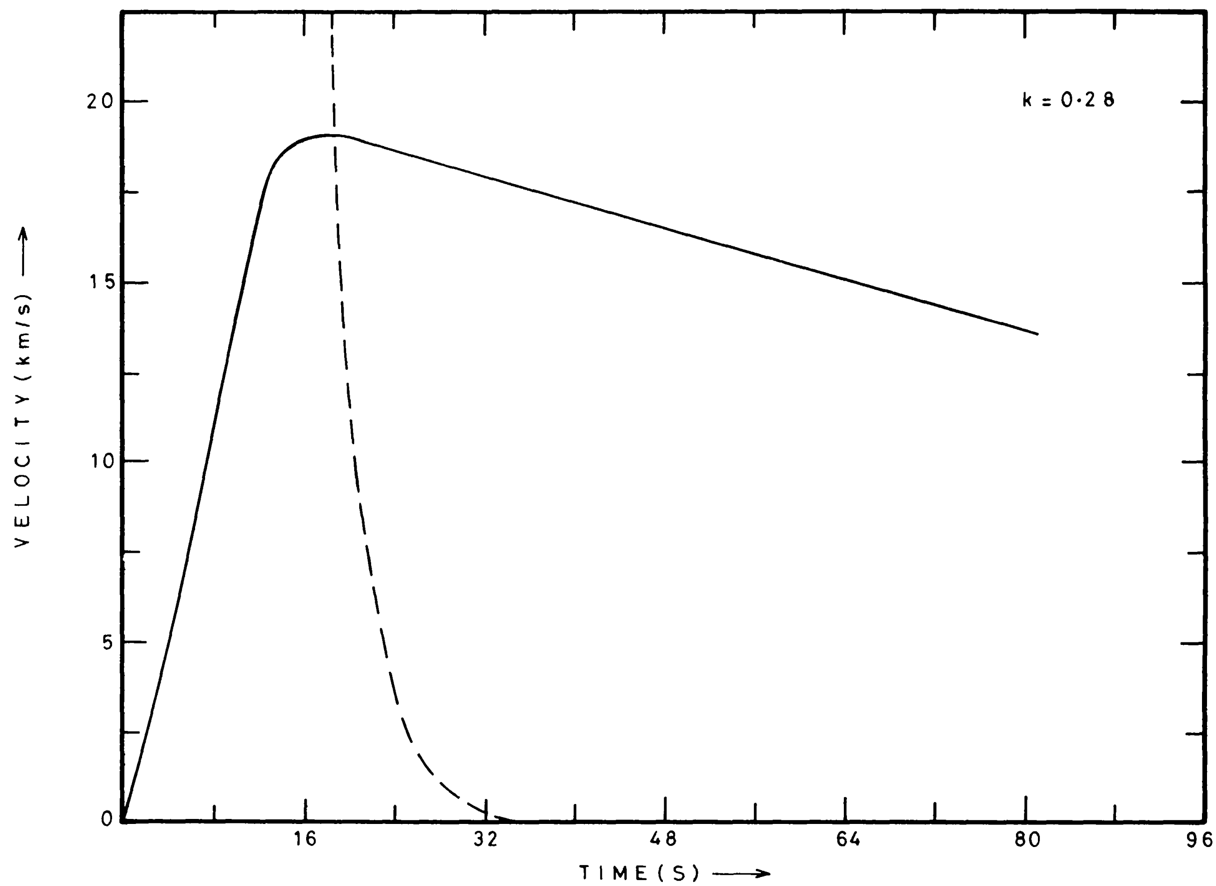

Fig. 2. The variation of parallel velocity (solid line) and normal velocity (dashes) with time is depicted at $s = 2900$ km for a total field length of 4000 km and $V_{nb} = 60$ km s^{-1} initially.

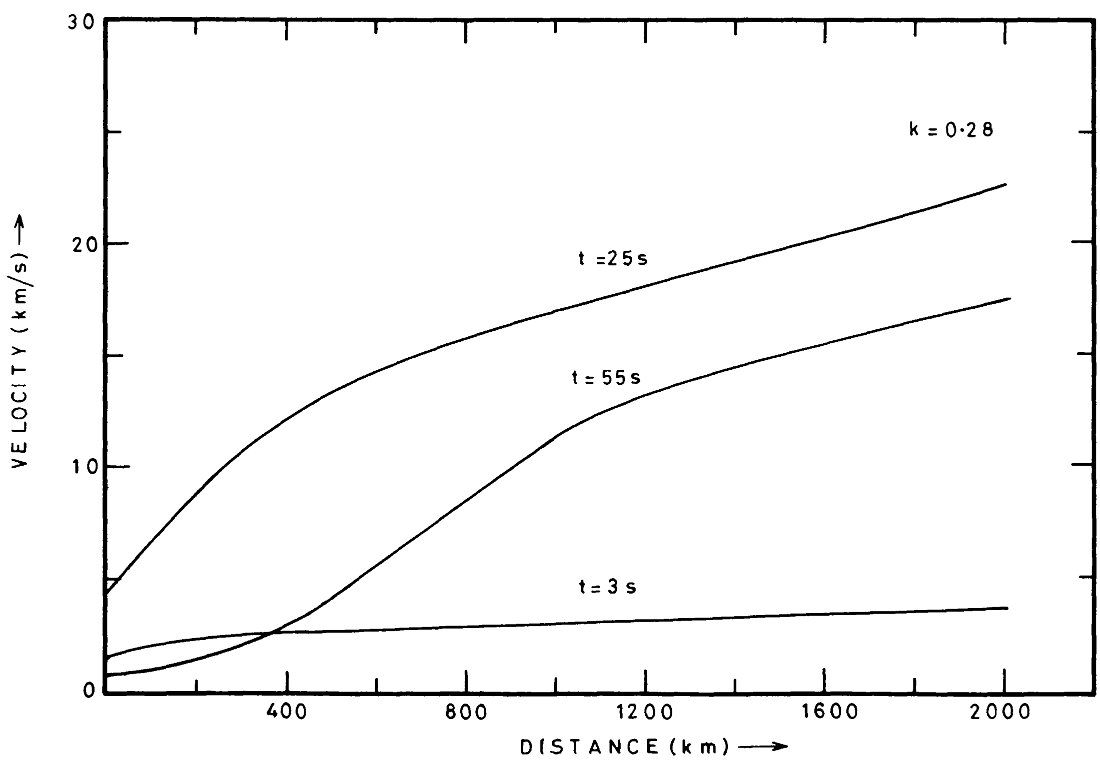

Fig. 3. The spatial distribution of parallel velocity at different times is shown for an initial choice $V_{nb} = 60$ km s^{-1}.

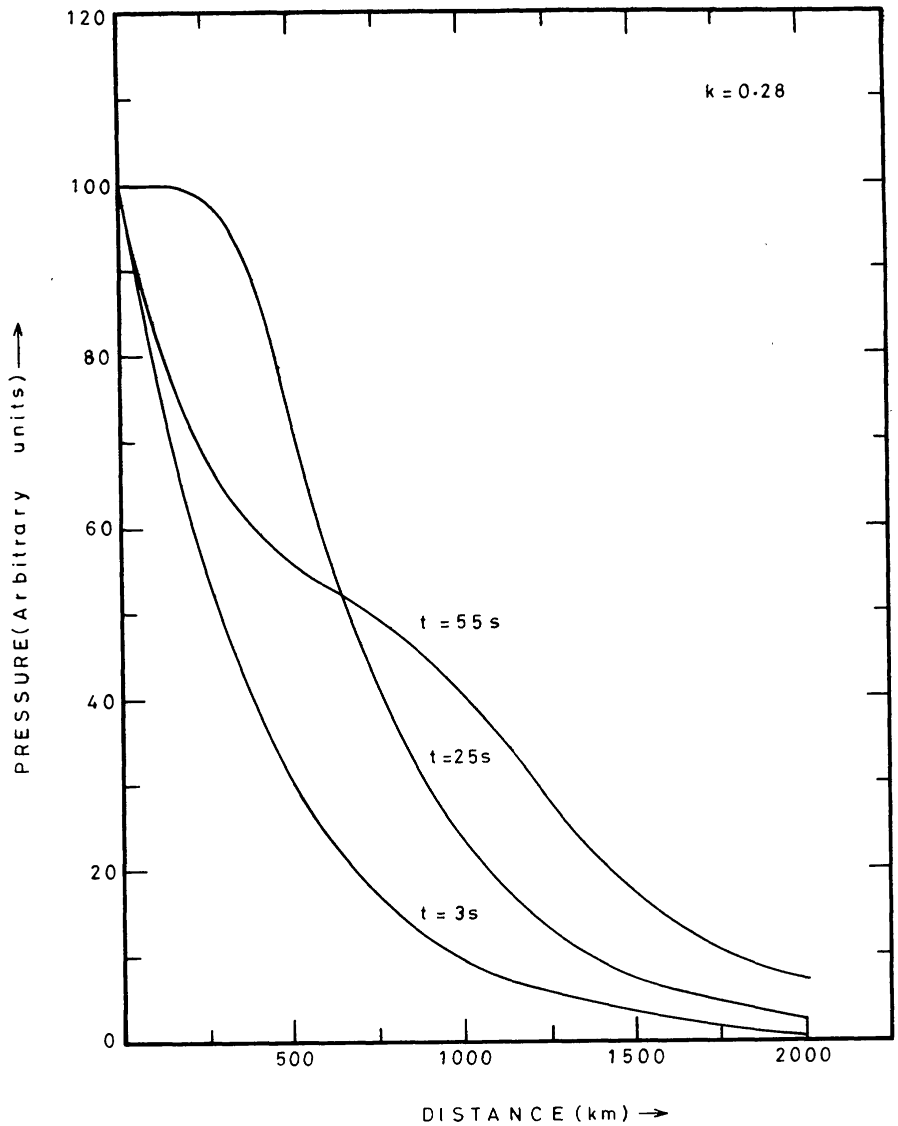

Fig. 4. The spatial distribution of pressure at different times is shown for an initial choice $V_{nb} = 60 \text{ km s}^{-1}$.

much larger time scale. This time scale varies from around 70 s for a spicule length of 2000 km to about 300 s when the length is 4000 km. Figure 2 shows the temporal behaviour of velocity in the latter case. The rapid decrease of V_n near 15 s is also shown.

Figures 3 and 4 depict the spatial distribution at different times for velocity and pressure respectively. We can notice the presence of a slight kink propagating downstream with an approximate speed of 25 km s^{-1}. This kink is probably due to the sudden onset of the lateral flow. The effect of curvature on the parallel flow is demonstrated in Figure 5. We see that as k increases, which corresponds to a decrease in the radius of curvature of the field line, the velocity also increases. However, the position of the peak remains roughly the same. Figure 6 shows the

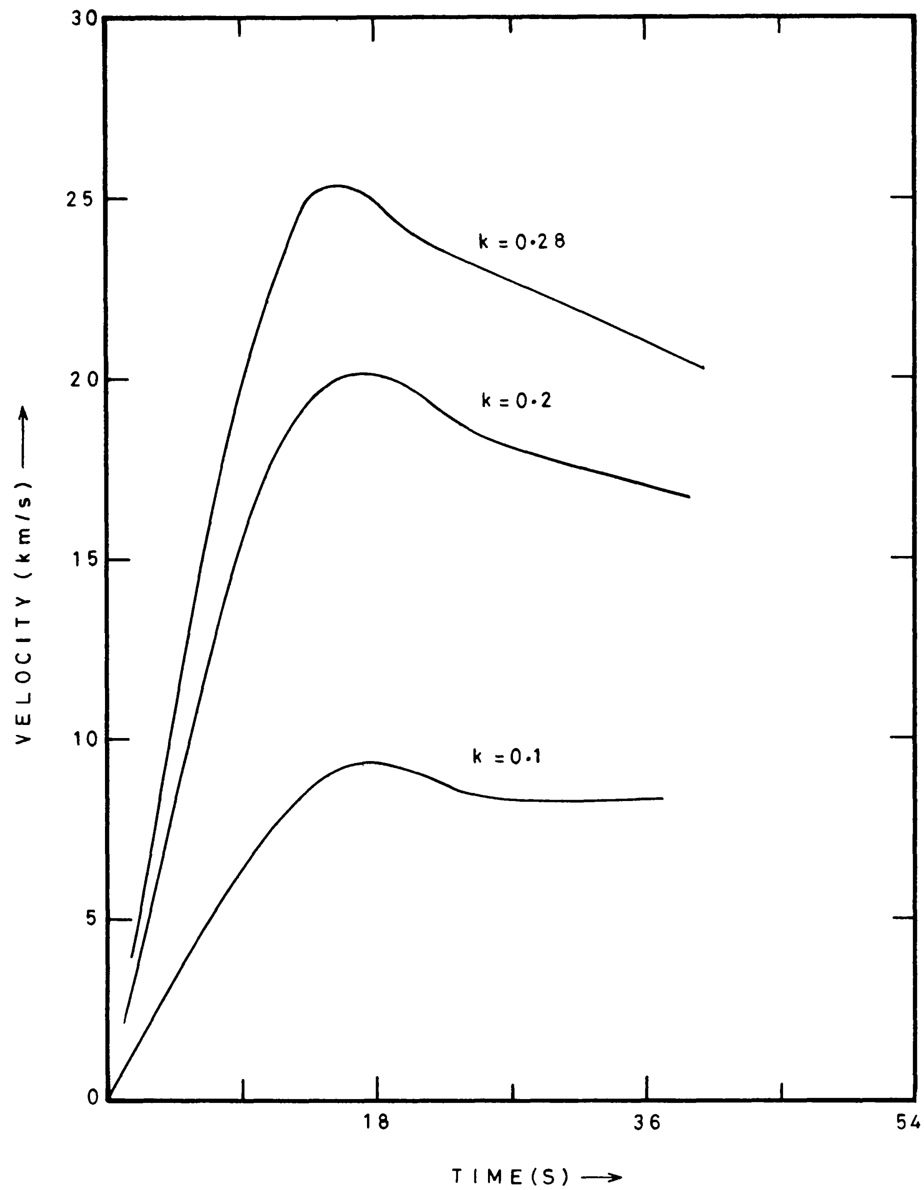

Fig. 5. The effect of curvature on the temporal distribution of velocity at $s = 1700$ km is depicted. A value of 20 000 K for the temperature and an initial normal velocity $V_{nb} = 80$ km s^{-1} were used.

stream geometry at different instants of time. The field lines have an initial rapid motion followed by a sudden decrease in velocity.

Case II: $V_n = V_0 \, e^{-z/H}$.

In order to demonstrate that the results obtained by us are sufficiently general, we considered a second from for V_n. Figure 7 shows the spatial velocity distribution for different times for cases when the velocity increases with height (solid curves) and when it decreases with height (dashed curves). We can thus conclude that there is an updraft or downdraft depending on whether V_n increases or decreases with height respectively.

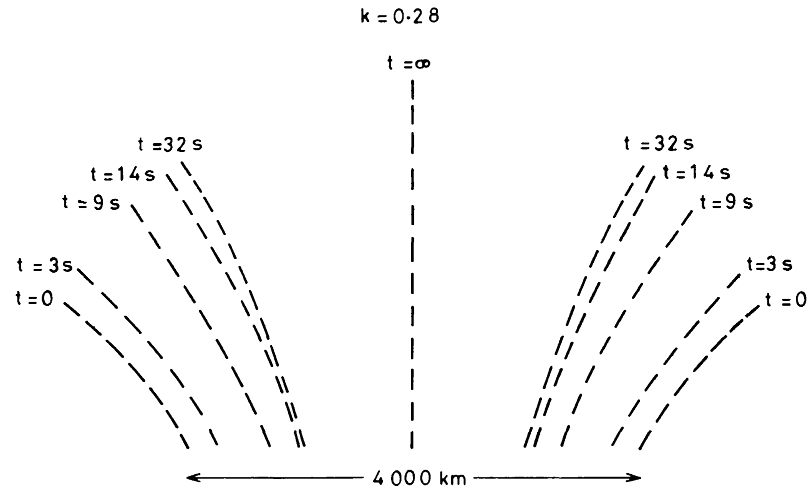

Fig. 6. The boundary streamlines at different instants during the collapse are shown for flow (a), assuming $V_{nb} = 60$ km s^{-1} initially.

6. Discussion

The main finding of this paper is that to set up an upward flow with a spicule-like velocity in a flux sheath, starting from hydrostatic equilibrium, the sheath must collapse normal to itself with a fairly high velocity. This result is in marked contrast to the qualitative findings of Hollweg (1972), described in Section 1, where a lateral velocity ~ 0.4 km s^{-1} was found sufficient to drive a spicule flow. The present study which includes the effects of compressibility and gravity suggests a lateral base velocity $\gtrsim 60$ km s^{-1} is necessary to drive a spicule. However, the duration for which such a large velocity is necessary to build up a spicule-like flow need at the most be around 15 s. Using an analogy with flares, we term this buildup phase as the 'impulsive spicule phase' (ISP for short). After the ISP, the collapse effectively stops and the parallel flow gradually begins to relax. We identify this relaxation phase as the 'main' spicule phase. The duration of this phase depends upon the time taken for a sound wave to traverse the total length of the flux sheath. Using a representative length of 4000 km, which corresponds to a height of 3500 km, we find a relaxation timescale of about 300 s, which is in agreement with the observational lifetime of spicules (Beckers, 1968). Regarding the temporal behaviour of spicule velocity at fixed points in space, there exists at present a great dearth of observations. The only observations in this respect that we are aware of are those of Title (1966). He finds (as quoted by Defouw, 1970) that the velocity of a typical upflow event was a 'rise to peak velocity in less than thirty seconds and then a decay for the next ninety seconds'. The time behaviour obtained from our model appears to be in qualitative agreement with this observed behaviour.

Let us now briefly discuss a possible mechanism which can provide the necessary trigger for the rapid collapse of a flux sheath during the ISP. It is useful to note that the large velocity of collapse ($\gtrsim 60$ km s^{-1}) needed during the ISP is much larger than the observed velocity of network flows ($\lesssim 0.5$ km s^{-1}). Thus, such flows cannot

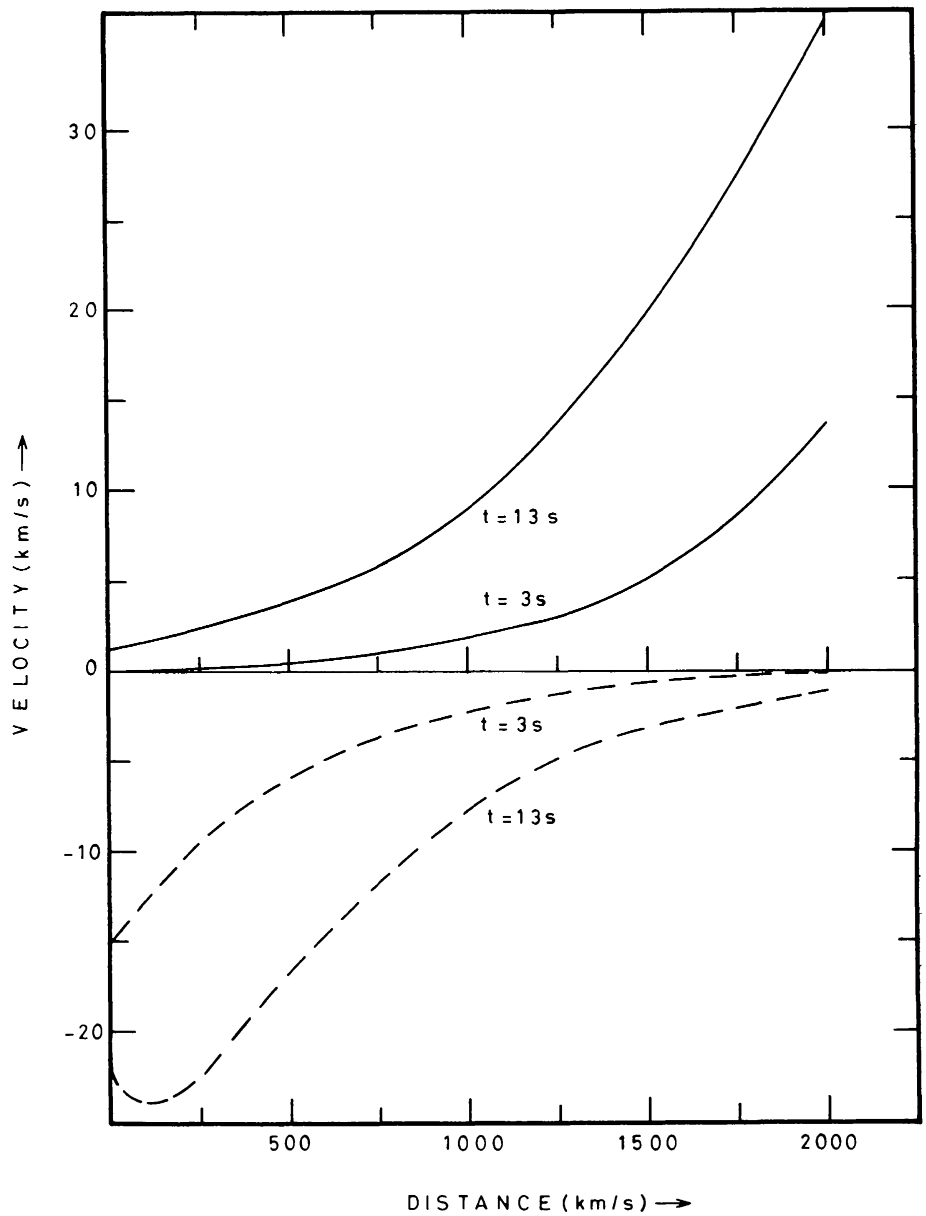

Fig. 7. The run of parallel velocity at different times is shown for $H = 1$ and $V_{nb} = 80\ \mathrm{km\ s^{-1}}$ (at $t = 0$) by solid lines, and $H = -1$ and $V_{nb} = 10\ \mathrm{km\ s^{-1}}$ (at $t = 0$), represented by dashes. The units for H are 10^3 km.

provide the required squeeze for spicules. A likely process which can lead to a collapse is a thermal instability, a suggestion first made in the context of spicules by Thomas and Athay (1961). It has been shown by Defouw (1970) that a thermal instability can occur in the upper chromosphere in the presence of a strong magnetic field. Even, if the collapse occurs locally, it will be rapidly transmitted to other portions of the flux sheath with the Alfvén velocity. For a temperature in the range 20 000 to 30 000 K the linear growth timescale at an electron density level of $3 \times 10^{11}\ \mathrm{cm^{-3}}$ is roughly 20 s (Defouw, 1970), which is compatible with our tentative picture for the ISP. We can also obtain an approximate lower limit for the velocity of collapse. Let ρ_1, r_1, and ρ_2, r_2 denote the density and radius respectively at some level before and after the collapse. Assuming axisymmetric collapse, mass conservation implies

$$\rho_1 r_1^2 = \rho_2 r_2^2$$

or

$$r_1 = (\rho_2/\rho_1)^{1/2}\, r_2\,. \tag{6.1}$$

Choosing $\rho_2/\rho_1 \sim 10$ and $r_2 \sim 700$ km, (6.1) gives $r_1 \sim 2100$ km. Thus, we can infer a mean collapse velocity $\sim(2100-700)/20 = 70$ km s^{-1}. We can also obtain a rough estimate for the magnetic field in a spicule. Let B_1 and B_2 represent the field strengths respectively before and after the collapse. Flux conservation leads to

$$B_1 r_1^2 = B_2 r_2^2$$

or

$$\frac{B_2}{B_1} \sim 10\,.$$

Thus an initial field of 10 G will be amplified to 100 G as a result of the collapse.

Another conclusion to emerge from our study is that the gas is accelerated in the direction of increasing V_n. To understand this behaviour, let us recall a well known result in fluid dynamics that for subsonic steady flow of gas through a nozzle with varying area of cross-section, the gas is accelerated in the direction in which the cross-section decreases (Zucrow and Hoffman, 1976). In the present study, the spatial variation of lateral flow provides such a nozzle, albeit a time dependent one. Thus, the acceleration of gas in the direction of increasing V_n can be understood simply in terms of nozzle flow, without the undesirable effect of choking, present in steady flow.

Let us now discuss the effect of the initial field geometry on the flow. We found that larger the curvature factor k, larger is the velocity attained by the fluid. A large value of k corresponds to a large curvature of the normal to the field line, resulting in a large centrifugal acceleration along the field.

Lastly, we find that the magnitude and form of the flow generated in the flux sheath is fairly insensitive to temperature. Our assumption of isothermal flow, although not strictly realistic, is unlikely to be a serious limitation since the essential nature of the flow mainly depends on V_n (Hasan and Venkatakrishnan, 1980). Such a model is, therefore, free from the necessity to assume large temperatures as in the case of Unno *et al.* (1974). In this steady state model for spicule flow (with $V_n = 0$), they had to invoke an 'effective temperature' of 25 000 K to explain the observed extension of spicules. They suggested that wave support could partly contribute to the temperature. Apart from their choosing a rather high value for the energy flux in the waves as pointed out by Athay (1976), the assumption of a steady state by Unno *et al.* (1974) to model spicule flow is not appropriate.

Before concluding, it is of some intererest to discuss how the ISP or more generally the 'birth' of a spicule would manifest itself observationally on the basis of our model. Unfortunately, the phenomenon presents several observational problems. Firstly, the 'birth' will occur in regions where there is heavy spicule overlap. Secondly, the phenomenon is a rapid one (~ 10 s) taking place on small length scales ($\sim 1''$). Finally, the initial density of the gas (when the lateral velocity is maximum) will be of the order of the ambient density, thus giving poor contrast. When the densities become spicule-like, the collapse would have ceased.

In summary, we have shown, on the basis of our study, that it is possible for a spicule to result when a flux sheath collapses normal to itself. We found that the spicule phenomenon can be represented by two distinct phases; an initial rapid phase followed by a gradual decay phase. The temporal behaviour of velocity obtained from our model appears to be compatible with existing observations.

Acknowledgements

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