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# On the Balmer Progression in the Expanding Shell of Pleione

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**Abstract.** An attempt has been made to explain the Balmer progression in velocities seen in Pleione and other shell stars. Equations of conservation of mass and momentum are simultaneously solved with assumptions which simplify the calculations of the solution. We have considered the radiation pressure due to lines and continuum. It is found that the high-excitation lines are formed where the velocity gradients are high.

*Key words:* Balmer progression—velocity gradients—shell stars

## 1. Introduction

One of the very interesting results of the extensive work by Merrill on shell stars is the detection of a progression in the velocities of the absorption lines of Balmer series of hydrogen atom, which occurs at a certain phase of the shell episode. This is the so-called Balmer progression. Merrill (1952) studied this behaviour over an extended interval in the case of Pleione. The displacements of the centres of gravity of all hydrogen lines from their positions of rest were the same between 1938—when the shell episode of Pleione began—and 1945. These corresponded roughly to the radial velocity of Pleione and therefore the Pleiades cluster. Around 1941, the average radial velocity was found to be $+ 5{\cdot}5$ km s$^{-1}$ (Struve and Swings 1943), whereas the radial velocity of the entire cluster was found to be $+ 7$ or $+ 8$ km s$^{-1}$. From this difference in the radial velocities between Pleione and the Pleiades cluster, it can perhaps be interpreted that the shell of Pleione was expanding very slowly. Since 1946, the displacements of different Balmer lines changed in a systematic way. Higher the Balmer quantum number, more negative was the displacement of the line. That is, higher excitation Balmer lines showed higher radial velocities. Balmer progression had become quite large around 1950 and the lines were sharp. At the extreme of the progression in 1951, the $H_\beta$ core continued to be strong and cores of lines of higher excitation potential had become weak and asymmetrical.

There was another shell outburst which started in the early seventies. Higurashi and Hirata (1978) report that until 1978, no Balmer progression has been noticed.

Balmer progression has been noticed in several shell stars by Merrill. In HD 33232, the radial velocities changed very slowly with Balmer quantum number till 1948, and thereafter, the change was as dramatic as in Pleione. However, the shell star 48 Librae showed no such Balmer progression (Merrill 1952).

In this paper, we shall make an attempt to understand the Balmer progression without undertaking the complicated calculations of non-LTE radiative transfer and hydrodynamics of the shells. We would like to show qualitatively, that the high excitation Balmer lines of hydrogen are formed in a region where the velocity gradients are high and therefore these lines show high radial velocities.

## 2. Velocity gradients in the shell

In this section, we shall study how radiation field affects the gas motion through the pressure due to line radiation. Our aim is, in particular, to study the varying effect that it produces on lines of different quantum numbers. There are two main aspects that we should study: the hydrodynamics of the gas flow and how this motion affects the Balmer lines.

To understand the motion of the gases in the shell, one must solve simultaneously, the equation of continuity and equations of conservation of momentum and energy. We have to take into account the radiation pressure due to continuum and lines. It is difficult to obtain a simultaneous solution (analytical or numerical) without making a few simplifying assumptions.

For a spherically-symmetric flow in steady state, the equation of momentum together with the equation of continuity is written as (Mihalas 1978, p. 562)

$$\left(1-\frac{a^2}{v^2}\right) v \frac{dv}{dr}=\frac{2a^2}{r}-2a\frac{da}{dr}-\frac{GM(1-\Gamma_e)}{r^2}+g_{r,l} \tag{1}$$

where $a$ is the thermal velocity, $v$ is the gas velocity, $M$ is the mass of the central star, $\Gamma_e$ is the ratio of radiative acceleration (due to a continuum dominated by electron scattering) and acceleration due to gravity given by,

$$\Gamma_e \simeq 2{\cdot}5 \times 10^{-5}\left(\frac{L_*}{L_\odot}\right)\left(\frac{M_\odot}{M_*}\right) \tag{2}$$

(Mihalas 1978, p. 554). The quantity $g_{r,l}$ expressed as (Mihalas 1978, p. 558),

$$g_{r,l}=\frac{2\pi}{c\rho}\sum_l \int_0^1 d\mu \int_0^\infty d\nu\, \chi_l(\nu)\, I_\nu(0)\, \mu \exp(-\tau_\nu/\mu) \tag{3}$$

is the acceleration due to radiation force in the lines. The sum extends over all lines. The specific intensity decays exponentially and therefore we have written

$I_\nu(\tau_\nu) = I_\nu(0) \exp(-\tau_a/\mu)$, $\tau_\nu$ being the optical depth at frequency $\nu$, $c$ the velocity of light, $\rho$ the material density and $\chi_l(\nu)$ the absorption coefficient at frequency $\nu$ in the line $l$. When summed up over all the lines, Equation (3) is given by,

$$g_{r,l} = \frac{S_e L}{4\pi c r^2} M(t), \tag{4}$$

where $S = n_e \sigma_e/\rho_e$, $n_e$ being the electron density and $\sigma_e$ being Thomson scattering coefficient. The quantity $M(t)$ is called the radiation force multiplier and if we manage to obtain a suitable form of this function, we can estimate $g_{r,l}$. Castor, Abbot and Klein (1975) evaluated this function assuming a C III spectrum and taking the abundance of $C^{++}$ equal to $10^{-3}$. They have also assumed line-forming ions on the basis of LTE approximation. They fitted $M(t)$ as

$$M(t) = kt^{-\alpha} \tag{5}$$

with $k \sim 1/30$ and $\alpha = 0{\cdot}7$. The quantity $t$ is calculated by the relation,

$$t = \sigma\, v_{\text{th}} \left(\frac{dv}{dr}\right)^{-1} \tag{6}$$

where $v_{\text{th}}$ is the thermal velocity of the gas.

Let us assume that the matter in the shell of Pleione is in radiative equilibrium and the temperature can be taken to vary as

$$T(r) = T_0 \left(\frac{r_0}{r}\right)^{1/2}. \tag{7}$$

We can estimate various quantities in Equation (1) and they are given by,

$$\frac{dT(r)}{dr} = -\tfrac{1}{2} T_0 \frac{1}{r} \left(\frac{r_0}{r}\right)^{1/2},$$

$$a(r) = 1{\cdot}2842 \times 10^4\, T_0^{1/2} \left(\frac{r_0}{r}\right)^{1/4},$$

$$\frac{da(r)}{dr} = -\, 3{\cdot}2105 \times 10^3\, T_0^{1/2} \frac{1}{r} \left(\frac{r_0}{r}\right)^{1/4},$$

$$\frac{2a^2(r)}{r} = -\,2a(r)\frac{da(r)}{dr} \simeq 8{\cdot}248 \times 10^7\, x\, r^{-3/2}. \tag{8}$$

with

$$x = r_0^{1/2}\, T_0 (1 + T_0^{1/2}) \tag{9}$$

and also

$$\frac{dv(r)}{dr} = \frac{\dfrac{2a^2(r)}{r} - 2a(r)\dfrac{da(r)}{dr} - \dfrac{GM(1-\Gamma e)}{r^2} + g_{r,l}}{v(r)\,[1 - a^2(r)/v^2(r)]}. \tag{10}$$

By substituting Equations (2)–(9) in Equation (10), we obtain

$$\frac{dv(r)}{dr} = v(r)\left[8{\cdot}2484 \times 10^{7}\, x\, r^{-3} - GMr^{-3}\left\{1 - 2{\cdot}5 \times 10^{-5}\left(\frac{L_*}{L_\odot}\right)\left(\frac{M_\odot}{M_*}\right)\right\}\right.$$

$$\left. + S_e L\,(4\pi c r^2)^{-1} M(t)\right] \Big/ \left\{v^2(r) - 1{\cdot}6492 \times 10^{8}\, T_0 \left(\frac{r_0}{r}\right)^{1/2}\right\}. \qquad (11)$$

We calculate the velocity gradient from Equation (11) by using appropriate values of parameters for Pleione. Mass, luminosity, radius and temperature of a star of spectral type are obtained from Allen (1973). The shell has been divided into several sectors of equal geometrical length. The velocity gradient $dv/dr$ in Equation (11) is calculated in each of these sectors. The calculation is started with initial velocity

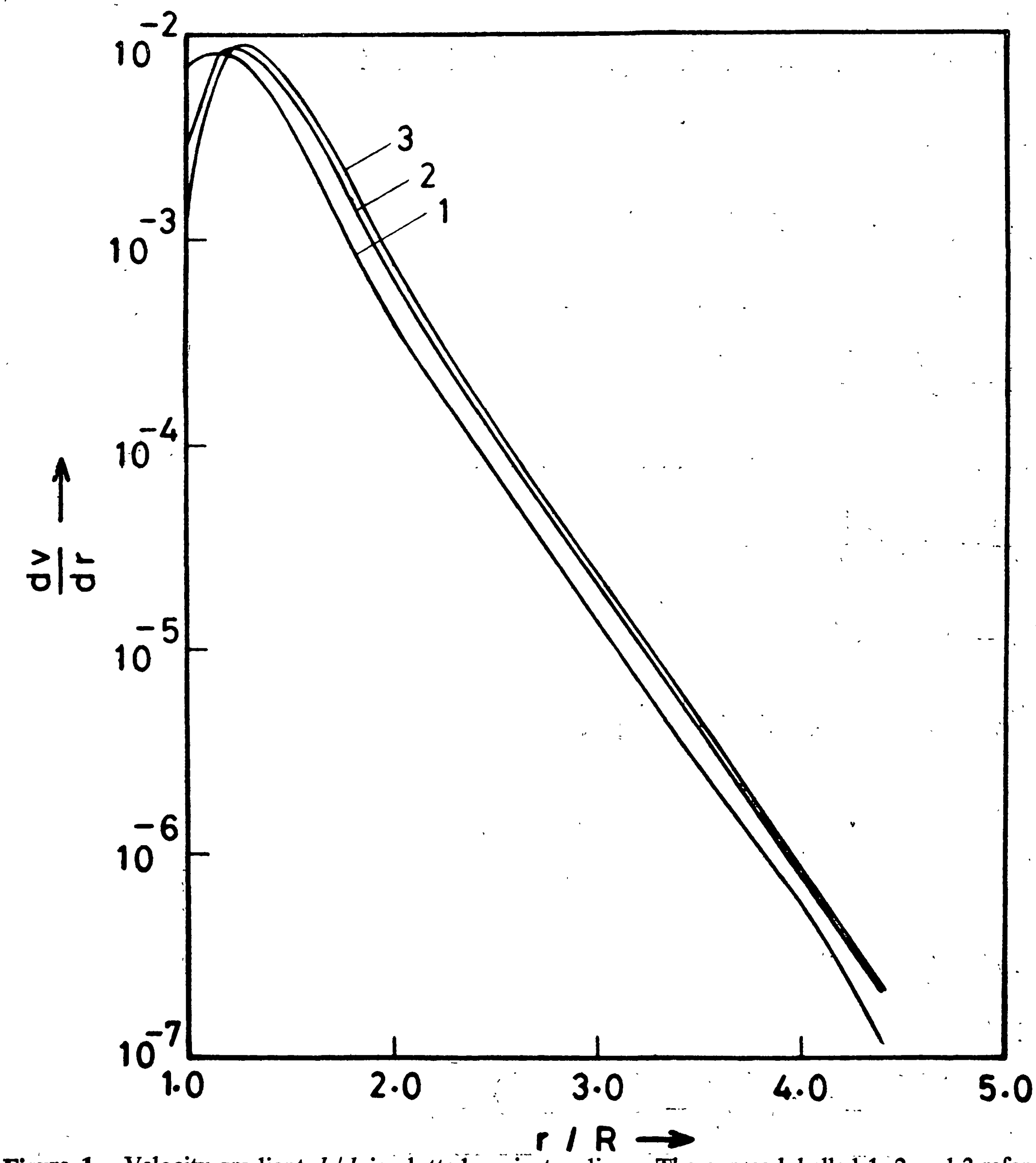


**Figure 1.** Velocity gradient $dv/dr$ is plotted against radius. The curves labelled 1, 2 and 3 refer to 2, 4 and 8 times the mean thermal values of $a$ at $r/R = 1$, respectively.

$V_0 = pa(r_0)$ $(p = 2,4,8)$ and $t$ is calculated with an assumed electron density. In the subsequent sector, we utilize velocity and the new velocity gradient calculated in the previous sector. New thermal velocity and velocity gradients have been used to calculate the factor $t$. We normally stopped the calculation of $dv/dt$ as soon as it becomes negative because negative $t$ will not allow us to estimate $M(t)$. One must remember that the values of $k$ and $\alpha$ in Equation (5) are taken to be 1/30 and 0·7. These values represent a star of an early spectral type and we have calculated the function $M(t)$ by using several different values for $k$ and $\alpha$. In all the cases, the nature of variation of the velocity gradient remains the same.

We have plotted in Fig. 1, the velocity gradient as a function of stellar radius. We notice that intitially, in the first twenty per cent of stellar radius of the shell, the velocity gradient increases sharply and then falls rapidly. At about $r/R \approx 4$, the velocity gradient becomes almost negligible and the shell moves with constant velocity. At about $r/R \approx 4{\cdot}5$, the velocity gradient becomes negative. This is because the radiation force $g_{r,l}$ is decreasing faster than the gravity term. Therefore, the quantity $t$ becomes negative and hence no further calculations of $dv/dr$ are possible. It is interesting to note that the velocity gradients are extremely large in the sector between $r/R = 1$ and 2. This has an important consequence, in that the atoms of higher Balmer number are more abundant in this sector.

The number of hydrogen atoms in various stages of excitation, is calculated by using the Boltzmann equation (Aller 1963)

$$\log \frac{N_i}{N_2} = -\theta \chi_{2,i} + 2 \log i - 0{\cdot}6021$$

where $\theta = 5040/T$ and the quantum numbers are given by $i = 3, 4, \ldots 30$; $\chi_{2,i}$ is the excitation potential measured in electron volts. We have calculated the radial variation of the temperature (see Table 1) using Equation (7).

Utilizing the temperature distribution shown in Table 1, we have evaluated $N_i/N_2$ along the radius vector. These results are shown in Fig. 2. It is interesting to note that the atoms of higher Balmer quantum numbers are concentrated in the sector bounded by $r/R=1$ and 2. Therefore these lines are formed in this region.

If we examine Fig. 1, we notice that the gas in this sector is moving with a very high velocity gradient. This leads us to the conclusion that the lines of higher Balmer quantum numbers are formed where the velocity gradients are large, thus explaining the Balmer progression observed by Merill (1952). For a quantitative theoretical

**Table 1.** Radial distribution of temperature in the model envelope.

| $r/R$ | $T$(K) |
|---|---|
| 1 | 12000 |
| 1·5 | 9797 |
| 2 | 8485 |
| 2·5 | 7589 |
| 3 | 6928 |
| 3·5 | 6414 |
| 4 | 6000 |
| 4·5 | 5656 |
| 5 | 5366 |

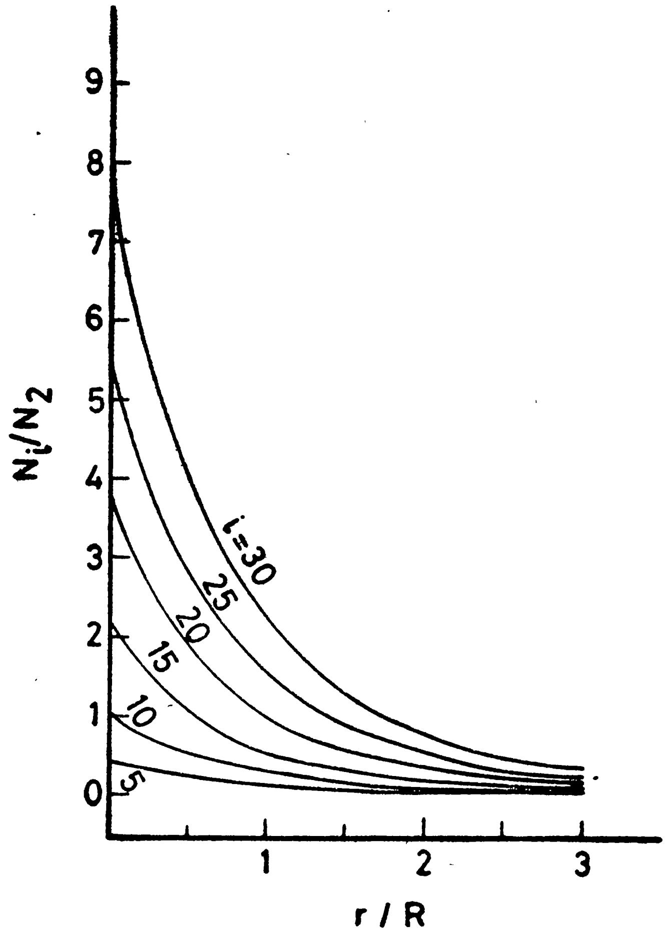


**Figure 2.** The radial distribution of high excitation Balmer atoms is described. *i* represents the Balmer quantum number.

estimate of the velocities, one has to obtain the simultaneous solution of the equations of gas dynamics and radiative transfer with non-LTE physics. Presently this is under investigation.

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