CHARACTERISTICS OF DAMPING OF THE PULSES IN THE SUN

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Abstract. This paper is a sequel to our earlier paper on the mathematical modelling in determining the rotational frequency and the density of an ionized medium. The technique is based on the measurement of the group travel time for a wave propagating in a rotating ionized medium and finally a simple approximate formula determines the rotation and the density of the medium. The present paper calculates the damping of the pulse-waves in the rotating Sun and leads to a mathematical development to estimate more physical parameters of the solar system.

1. Introduction

The study of the rotating stars, since observed first by Schlesinger (1911), has taken a great deal of interest to focus on the advantages of the dynamics of the rotating stars. Several workers have studied the rotating stars in different aspects to give a close relation between the theory and the observations. In earlier stages, the studies were based on the observations while, in the later, several mathematical developments have been made. Chandrasekhar (1953a, b, c) was the first to show that the magnetohydrodynamic (MHD) wave in a rotating gaseous medium receives an influence of the Coriolis acceleration $2\Omega \times \vartheta$ and finally established a well known fact that the Coriolis force exhibits always a great influence in the Sun. Later, Lehnert (1954) shows that the Coriolis force might change many important points in the Alfvén theory on sunspot (Alfvén, 1950). Lehnert (1954) shows that, for the typical value of rotation $\Omega = 2 \times 10^{-6}\ \mathrm{s}^{-1}$ and Alfvén velocity $V = 2\ \mathrm{m\ s}^{-1}$ together with an assumption on polar strength of less than 25 G, the Coriolis force is equal to the 14 times of the magnetic force (where the wave length is taken larger than a hundredth of a solar radius). Taking all the evidences of Coriolis force effects on MHD waves in the Sun, Das (1979) very recently considered a model of ionized medium to study the waves in due account of the effects of Coriolis force in isolation. Das (1979) derived the dispersion relation for the wave traveling along the magnetic field and estimated the time taken by the wave pulses from the sources to the solar surface. The method is based on the measurement of group travel time of the wave along the path and is given by the line integral

$$t(\omega) = \int_h^0 \frac{\mathrm{d}h}{v_g},$$

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where v_g is the group velocity of wave along the path h. Many situations may arise near the solar surface, sunspots, ionisphere as well as in the laboratory where the travel time could be employed to estimate the physical parameters. Very recently, Das (1979) derived the travel time for a rotating medium and discussed the determination of the rotational frequency as well as the density of the medium. In sequel to this paper, we present here now the possible damping phenomena in magnetohydrodynamic waves generated in the Sun which could be employed to estimate more physical parameters of the Sun. The mathematical formulation of the dynamics of the waves are very much similar to those presented by Das (1979) and, in Section 2, we review the earlier work. Finally we estimate also the distribution function of the particle and the relative amplitude of the magnetic field of the wave as a function of the group travel time and the rotational frequency.

2. Dispersion Relation and Group Travel Time

In this section, we give a short review of the paper Das (1979) for the derivation of the group travel time for a wave along the axis of rotation. We consider an ionized medium consisting of electrons (subscript e) and ions (subscript i). We assume provisionally that the pulse-waves originated at the centre of a dipole field in the Sun travel along the magnetic field. In general, it may be that the axis of the magnetic field makes an angle with the axis of rotation. But, from the theoretical point of view, it is very likely that the magnetic field lines coincide with the rotational axis. The observational evidence is in favour of considering the medium rotating with an angular velocity (say $\mathbf{\Omega}$) around the magnetic field lines. The basic equations (with respect to a rotating frame of reference) are:

Equation of continuity:

$$\frac{\partial n_\alpha}{\partial t} + \operatorname{div}(n_\alpha \boldsymbol{\vartheta}_\alpha) = 0 . \tag{1}$$

Equation of motion:

$$\frac{\partial \boldsymbol{\vartheta}_\alpha}{\partial t} + \boldsymbol{\vartheta}_\alpha \cdot \nabla \boldsymbol{\vartheta}_\alpha = \frac{q_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{\boldsymbol{\vartheta}_\alpha \times \mathbf{H}}{c}\right] + 2\boldsymbol{\vartheta}_\alpha \times \mathbf{\Omega} , \tag{2}$$

together with Maxwell's equations:

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\sum_\alpha q_\alpha n_\alpha \boldsymbol{\vartheta}_\alpha + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} , \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} , \tag{4}$$

$$\nabla \cdot \mathbf{H} = 0 , \tag{5}$$

$$\nabla \cdot \mathbf{E} = 4\pi \sum_\alpha q_\alpha n_\alpha , \tag{6}$$

where $\alpha = i$, e, and v_α is the velocity of the α-type particle having the mass m_α and number density n_α and $q_\alpha = e$ when $\alpha = i$ and $q_\alpha = -e$ when $\alpha = e$.

We consider a plane which propagates in such a way that all the perturbed quantities are assumed to vary as $\exp[i(kr - \omega t)]$. Following Uberoi and Das (1970), the dispersion relation for the waves along the magnetic field can be written as

$$D_{\mp}(\omega, k) \equiv c^2k^2 - \omega^2 + \frac{\omega\omega_{pi}^2}{\omega \mp \pi_i} + \frac{\omega\omega_{pe}^2}{\omega \pm \pi_e} = 0\,, \tag{7}$$

where $\pi_i = \omega_{ci} + 2\Omega$ and $\pi_e = \omega_{ce} - 2\Omega$, and all the conventional symbols have their usual meanings. The $\pm$ signs represent the right and left hand circularly polarized waves. We restrict our investigation on wave frequencies near to twice the rotational frequency and following Das (1979), the group velocity for the left circularly polarized (LCP) wave in a medium free from the applied magnetic field is given by

$$u_g = \frac{c(2\Omega)^{1/2}[2\Omega - \omega]^{3/2}}{\omega_p[2\Omega - \omega/2]}\,, \tag{8}$$

where $\omega_p^2 = \omega_{pi}^2 + \omega_{pe}^2$.

The simplification in making the medium free from an applied magnetic field is not for mathematical simplicity. The inherent reason will be clear later. Using (8), the group travel time reads as

$$t(\omega) = \frac{\omega_p(0)[2\Omega(0)]^{1/2}}{2c\Omega'(0)[2\Omega(0) - \omega]^{1/2}}\,, \tag{9}$$

where the variation of rotation along the path z is assumed linear and is approximated by the linear relation $\Omega(z) = \Omega(0) + z\Omega'(0)$, where $\Omega(0)$ is the rotation at the source of the pulses and $\Omega'(0)$ is the gradient along the axis of rotation. The Equation (9) is the required equation in determining the rotational frequency by observing the different pulses and consequently the density is obtained as

$$n_e(0) = \frac{c\theta m_e}{4\pi e^2} \frac{2\Omega'(0)}{[2\Omega(0)]^{1/2}}\,, \tag{10}$$

where θ is the slope of the straight line obtained from a plot of $t(\omega)$ against $[2\Omega(0) - \omega]^{1/2}$.

3. Further Use of Group Travel Time

It is a well known feature that the Coriolis force in an ionized medium introduces an equivalent magnetic field as well as having a similar behaviour as extra ions in introducing the critical frequency at which a polarization reversal occurs. Now consider the case, when the charged particles moving along the equivalent magnetic field may observe an oscillation of the perpendicular electric field at the frequency near to twice the rotational frequency. In this case the particles absorb the energy from the field and consequently the absorption of the energy will cause the wave to damp out. In order to calculate the damping rate, we assume a maxwellian velocity distribution for the particles and following Stix (1962), the dispersion relation of the LCP wave along the equivalent magnetic field can be written in the following form:

$$n^2 = 1 - \frac{\omega_p^2}{2\Omega(\omega - 2\Omega)} + \frac{i\sqrt{\pi}\omega_p^2}{k\omega\vartheta_{\text{th}}} \exp(-z^2), \tag{11}$$

where

$$z = \frac{\Delta\omega}{k\vartheta_{\text{th}}} \quad \text{with} \quad \Delta\omega = 2\Omega - \omega$$

and ϑ_{th} is the thermal velocity of the particle. Assuming now either ω or k to be complex, one can estimate the term for the temporal and spatial damping rates of the pulses. First, we consider k is real and ω is expressed as $\omega = \omega_r + i\omega_i$ with $|\omega_i| \ll |\omega_r|$. The separation of real and imaginary parts gives approximately the damping rate as

$$\omega_i = \frac{c\sqrt{\pi}[\Delta\omega(0)]^{5/2}}{\omega_p(0)[2\Omega(0)]^{1/2}\vartheta_{\text{th}}} \exp(-\eta(0)), \tag{12}$$

where

$$\eta(s) = \frac{c^2[\Delta\omega(s)]^3}{2\Omega(s)\omega_p^2(s)\vartheta_{\text{th}}^2}.$$

The above expression is derived for the pulses originated in the Sun and propagates along the equivalent magnetic field with the frequency near to twice its rotational frequency. Now substituting $\Delta\omega(0)$ [obtained from the group travel time equation (9)] into Equation (12), the damping rate (12) takes the following form:

$$\omega_i = At^{-5} \exp(-Bt^{-6}), \tag{13}$$

where

$$A = \frac{\sqrt{\pi}}{\vartheta_{\text{th}}} \left[\frac{\omega_p(0)}{c}\right]^4 \frac{[2\Omega(0)]^2}{[2\Omega'(0)]^5} \exp[-\eta(0)]$$

and

$$B = \left[\frac{\omega_p(0)}{c}\right]^4 \frac{[2\Omega(0)]^2}{[2\Omega'(0)]^6} \frac{1}{\vartheta_{\text{th}}^2}.$$

From (13), it is clear that the damping rate depends on t. For small t, ω_i is negligible small. Now when the damping is included in the pulses, the velocity distribution of the particles has a functional relation with the damping rate and is given by the following relation:

$$\omega_i = -\frac{2c\pi^2[\Delta\omega(0)]^{5/2}}{\omega_p(0)[2\Omega(0)]^{1/2}} F(v), \tag{14}$$

where $F(v)$ is the moment of a one particle velocity distribution. Combining Equations (13) and (14), $F(v)$ is expressed as

$$F(v) = \frac{1}{2\pi^{3/2}} \frac{1}{\vartheta_{\rm th}} \exp(-Bt^{-6}) \,. \tag{15}$$

Similarly, we now assume ω is real and $k = k_r + ik_i$ with $|k_i| \ll |k_r|$. As before, a separation of real and imaginary parts of k will give the damping length of the pulses and for small $\Delta\omega$, we have

$$k_r = \frac{\omega\omega_p}{c[2\Omega]^{1/2}[\Delta\omega]^{1/2}} \tag{16}$$

and

$$k_i = \frac{\sqrt{\pi}\,\Delta\omega}{2\omega_p^2\vartheta_{\rm th}} \exp(-\eta(s)) \,,$$

where η is given in (12). The total attenuation due to the damping of the pulses is obtained by integrating k_i over the path, and using the similar small $\Delta\omega$ we have the total attenuation, β as

$$\beta = \int_{\Delta\omega(h)}^{\Delta\omega(0)} \frac{\sqrt{\pi}}{2\Omega'(0)} \frac{\Delta\omega}{\vartheta_{\rm th}} \exp(-\eta)\, \mathrm{d}(\Delta\omega)$$

$$= \frac{\sqrt{\pi}\omega_p(0)}{3c} \left[\frac{\Delta\omega(0)}{2\Omega(0)}\right]^{1/2} \left[\frac{2\Omega(0)}{2\Omega'(0)}\right] \frac{\exp(-\eta(0))}{[\eta(0)]^{1/2}} \,. \tag{17}$$

Substituting the value of $\Delta\omega(0)$ from (9), we have β as

$$\beta = A't^2 \exp(-B't^6) \,, \tag{18}$$

where

$$A' = \frac{\sqrt{\pi}\vartheta_{\rm th}}{3} [2\Omega'(0)]$$

and $B' = B$, given in (13).

Thus the variation of the equivalent magnetic field, B_1 can be expressed as

$$B_1 \sim \exp(-\beta) \,, \tag{19}$$

i.e.

$$\sim \exp[-A't^2 \exp(-B't^{-6})] \,.$$

Now, in the case where the energy absorbed per unit bandwidth is taken constant, the Poynting flux observes as

$$|E_1||B_1| \sim \frac{\mathrm{d}\omega}{\mathrm{d}t}$$

and using Equation (12), we have

$$|E_1||B_1| \sim \left|\frac{d(\Delta\omega)}{dt}\right| \sim (\Delta\omega)^{3/2} \sim t^{-3}, \tag{20}$$

where t is defined in Equation (12). Again, for the constant Poynting flux, the ratio of the magnetic field to the electric field varies directly to the refractive index of the pulses and thus, we have

$$\frac{|B_1|}{|E_1|} \sim n \sim (\Delta\omega)^{-1/2} \sim t. \tag{21}$$

Combining Equation (19), (20), and (21), we have the relative wave amplitude of the equivalent magnetic field as a function of the group travel time and is given by

$$|B_1| \sim \frac{1}{t} \exp\left[-A' t^2 \exp\left(-B' t^{-6}\right)\right],$$

where A' and B' are the functions of plasma density, rotational frequency and its differential variation along the field lines.

Finally, emerging from the present investigation and the results presented by Das (1979) earlier, we have that the magnetohydrodynamic pulses in the Sun propagate along the magnetic field lines towards the solar surface and the observations of several pulses and the corresponding travel times from the sources of pulses to the solar surface could be employed to find out the rotational frequency and the density of the medium. The consideration of the damping of the pulses in the present paper yields information on the velocity distribution of the particules and the relative amplitude of the equivalent magnetic field of the pulses.

The effect of the magnetic field is not considered in the dynamics of the pulses not because of the mathematical simplicity but to show the effect of Coriolis force in isolation and which could be a useful technique to estimate the physical parameters. The presence of the magnetic field in the dynamics of pulses will have an equivalent rotational frequency and the similar mathematical development can be continued by replacing the rotational frequency 2Ω with an equivalent rotational frequency $2\bar{\Omega} = \omega_{c\alpha} + 2\Omega$ where $\omega_{c\alpha}$ is the cyclotron frequency of α-type particles. The present study is an ideal model but gives an insight of a technique in diagnosing the physical parameters of a solar system. The technique needs further investigation if we are to determine the physical parameters for a real model arising in the solar system.

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