RADIO FREQUENCY EMISSION IN ELECTRON BEAM–PLASMA AND BEAM–BEAM INTERACTION

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(Received 16 May, 1977)

Abstract. A linear excitation of electromagnetic modes at frequencies $\simeq(n + \frac{1}{2})\omega_{ce}$, in a plasma through which two electron beams are contra-streaming along the magnetic field is investigated. This may be a source of the observed $\frac{3}{2}\omega_{ce}$ emissions at auroral latitudes.

The stability of a beam–plasma (Young *et al.*, 1973) system has been studied quite extensively in various contexts and is well understood. But from the point of view of space physics (Zhulin *et al.*, 1972), the study of contra-streaming beams passing through a low-density plasma in the presence of a magnetic field (taken along the z-axis) becomes specially important. Recently, Bernstein *et al.* performed an experiment (Bernstein *et al.*, 1975) with counter-streaming beams by appropriately simulating conditions prevalent in the magnetosphere. They observed radio-frequency emission near $(n + \frac{1}{2})\omega_{ce}$, where n is an integer and ω_{ce} is the electron cyclotron frequency. Their plasma system consists of (i) a low pressure gas in a uniform magnetic field, and (ii) an energetic electron beam streaming through the gas along the magnetic field. (This primary electron beam hits a negatively biased collector plate to produce a less energetic and less dense contra-streaming secondary beam.)

In this note, we give a theory that explains some of the features of their experimental findings. To simplify the physics and avoid the numerical solution of the dispersion relation, we shall consider the two counter-streaming beams and the stationary plasma as three independent subsystems and then find the normal electromagnetic modes of each system. To see whether the total system as a whole is unstable or not, we search for negative and positive energy modes in each independent subsystem. The growth rate will then be given by interaction between one negative and one positive energy mode if the energy for the two-mode coupling thus involved is conserved – i.e.

$$|\omega_1| \operatorname{sgn}\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon_i)\right]_{\omega=\omega_1} + |\omega_2| \operatorname{sgn}\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon_j)\right]_{\omega=\omega_2} = 0, \tag{1}$$

where i and j refer to any one of the subsystems, ω_1 and ω_2 are their normal modes, such that ω_1 and ω_2 are the solutions of

$$\varepsilon_t(\vec{q}, \omega) = \frac{q^2c^2}{\omega^2}. \tag{2}$$

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Astrophysics and Space Science **53** (1978) 13–19. *All Rights Reserved*

The dielectric function $\varepsilon_i(\vec{q}, \omega)$ has been calculated and is given by

$$\begin{aligned}\varepsilon_i(\vec{q}, \omega) = 1 &- \frac{\omega_{pi}^2 q_z^2}{2q^2[\omega - sq_z u_i]^2} - \frac{\omega_{pi}^2 q_\perp^2}{4q^2[(\omega - sq_z u_i)^2 - \omega_{ce}^2]} - \\ &- \alpha_i \left[\frac{\omega_{pi}^2 q_z^2}{4q^2[\omega - sq_z u_i]^2} + \frac{\omega_{pi}^2 q_z^2}{q^2} \frac{[(\omega - sq_z u_i)^2 + \omega_{ce}^2]}{[(\omega - sq_z u_i)^2 - \omega_{ce}^2]} + \right. \\ &+ \left. \frac{\omega_{pi}^2 q_\perp^2}{q^2[(\omega - s_i q_z u_i)^2 - 4\omega_{ce}^2]}\right] - \frac{\alpha_i^2}{16} \frac{q_z^2}{q^2} \frac{\omega_{pi}^2}{q^2} \frac{[(\omega - sq_z u_i)^2 + 4\omega_{ce}^2]}{[(\omega - sq_z u_i)^2 - 4\omega_{ce}^2]^2} \\ &- \frac{6\alpha_i^2}{128} \frac{\omega_{pi}^2 q_\perp^2}{q^2[(\omega - sq_z u_i)^2 - 9\omega_{ce}^2]} - \\ &- \frac{\alpha_i^3}{48 \times 8} \frac{\omega_{pi}^2 q_z^2[(\omega - sq_z u_i)^2 + 9\omega_{ce}^2]}{[(\omega - sq_z u_i)^2 - 9\omega_{ce}^2]^2} - \\ &- \frac{\alpha_i^3}{96} \frac{\omega_{pi}^2}{q^2} \frac{q_\perp^2}{[(\omega - sq_z u_i)^2 - 16\omega_{ce}^2]} - \\ &- \frac{5\alpha_i^4}{2^{12} \times 3} \frac{q_\perp^2}{q^2} \frac{\omega_{pi}^2}{[(\omega - sq_z u_i)^2 - 25\omega_{ce}^2]} - \\ &- \frac{\alpha_i^5}{6!\, 2^{12}} \frac{q_\perp^2}{q^2} \frac{\omega_{pi}^2}{[(\omega - sq_z u_i)^2 - 36\omega_{ce}^2]} + \\ &+ O(\alpha_i^6), \end{aligned} \tag{3}$$

where $\vec{q}$, ω are the wave vector and frequency of the mode; $q_\perp = \sqrt{q_x^2 + q_y^2}$; $\omega_{pi}^2 = (4\pi n_i e^2)/m$ is the plasma frequency (n_i being the density); $u_{i=1} = u_1$, $u_{i=2} = u_2$, $u_{i=0} = 0$; $s = 0$, 1 and -1 respectively for $i = 0$, 1 and 2 corresponding to stationary plasma, primary and secondary electron beams; $\sqrt{\alpha_i} = (q_\perp V_{\text{th},i})/\omega_{ce} \ll 1$ is the ratio of the wavelength to the Larmor radius, $V_{\text{th},i}$ being the thermal velocity of the electrons. The ion dynamics has been neglected as unimportant.

Since, in the experiment (Bernstein *et al.*, 1975), the modes $(n + \frac{1}{2})\omega_{ce}$ disappear in the absence of the secondary beam ($i = 2$, $s = -1$), the interaction of this beam with the plasma as well as the contra-streaming beam seems to be the possible mechanism of excitation of these modes. Therefore, let us first solve the equations

$$\varepsilon_1(\vec{q}, \omega) = \frac{q^2 c^2}{\omega^2}, \tag{4}$$

$$\varepsilon_2(\vec{q}, \omega) = \frac{q^2 c^2}{\omega^2}, \tag{5}$$

to show that modes given by (4) can interact with those given by (5) via a two-mode coupling of negative and positive energy modes. Equation (4) gives the following roots under the condition $q_\perp \gg q_z$:

$$\omega_{1\pm} = q_z u_1 \pm \omega_{ce} + \delta_1 \equiv \omega_{1\pm}^{(0)} + \delta_1, \tag{6}$$

$$\omega_{2\pm} = q_z u_1 \pm 2\omega_{ce} + \delta_2 \equiv \omega_{2\pm}^{(0)} + \delta_2, \tag{7}$$

$$\omega_{3\pm} = q_z u_1 \pm 3\omega_{ce} + \delta_3 \equiv \omega_{3\pm}^{(0)} + \delta_3, \tag{8}$$

$$\omega_{4\pm} = q_z u_1 \pm 4\omega_{ce} + \delta_4 \equiv \omega_{4\pm}^{(0)} + \delta_4, \tag{9}$$

$$\omega_{5\pm} = q_z u_1 \pm 5\omega_{ce} + \delta_5 \equiv \omega_{5\pm}^{(0)} + \delta_5, \tag{10}$$

$$\omega_{6\pm} = q_z u_1 \pm 6\omega_{ce} + \delta_6 \equiv \omega_{6\pm}^{(0)} + \delta_6, \tag{11}$$

where the δ's are small corrections to the frequency ω and are given by

$$\begin{aligned}
\frac{\delta_1}{\omega_{ce}} &= -\frac{\omega_{p1}^2 q_\perp^2}{8q^2\omega_{ce}^2}\frac{\omega_{1\pm}^{(0)2}}{q^2c^2},\\
\frac{\delta_2}{\omega_{ce}} &= -\frac{\omega_{p1}^2 q_\perp^2}{4q^2\omega_{ce}^2}\frac{\omega_{2\pm}^{(0)2}}{q^2c^2},\\
\frac{\delta_3}{\omega_{ce}} &= -\frac{\alpha^2\omega_{p1}^2 q_\perp^2}{128\omega_{ce}^2 q^2}\frac{\omega_{3\pm}^{(0)2}}{q^2c^2},\\
\frac{\delta_4}{\omega_{ce}} &= -\frac{\alpha^3\omega_{p1}^2 q_\perp^2 \omega_{4\pm}^{(0)2}}{2^5 \times 4!\,\omega_{ce}^2 q^2 q^2 c^2},\\
\frac{\delta_5}{\omega_{ce}} &= -\frac{\alpha^4\omega_{p1}^2 q_\perp^2 \omega_{5\pm}^{(0)2}}{2^{10} \times 4!\,\omega_{ce}^2 q^2 q^2 c^2},\\
\frac{\delta_6}{\omega_{ce}} &= -\frac{\alpha^5\omega_{p1}^2 q_\perp^2 \omega_{6\pm}^{(0)2}}{2^{12} \times 5!\,\omega_{ce}^2 q^2 q^2 c^2}.
\end{aligned} \tag{12}$$

Similarly, the solution of Equation (4) gives the real roots

$$\Omega_{1\pm} = -q_z u_2 \pm \omega_{ce} + \delta_1', \tag{13}$$

$$\Omega_{2\pm} = -q_z u_2 \pm 2\omega_{ce} + \delta_2', \tag{14}$$

$$\Omega_{3\pm} = -q_z u_2 \pm 3\omega_{ce} + \delta_3', \tag{15}$$

$$\Omega_{4\pm} = -q_z u_2 \pm 4\omega_{ce} + \delta_4', \tag{16}$$

$$\Omega_{5\pm} = -q_z u_2 \pm 5\omega_{ce} + \delta_5', \tag{17}$$

$$\Omega_{6\pm} = -q_z u_2 \pm 6\omega_{ce} + \delta_6', \tag{18}$$

with δ''s again being the small corrections and can be obtained from Equation (11) by replacing ω_{p1}^2 by ω_{p2}^2. Now the modes ω_{1+}, ω_{2+}, ω_{3+}, ω_{4+}, ω_{5+} and ω_{6+} are negative energy modes because, under the condition $q_z u_1/\omega_{ce} > 1$ and $q_z < 0$, it makes

$$F_1 \equiv \frac{1}{\omega}\frac{\partial}{\partial\omega}\left[\frac{\omega^2}{2}\,\varepsilon_1(\vec{q},\omega)\right] < 0.$$

Similarly, under the condition $q_z u_2/\omega_{ce} < 1$, the modes $\Omega_{1\pm}$, $\Omega_{2\pm}$, $\Omega_{3\pm}$, $\Omega_{4\pm}$, $\Omega_{5\pm}$ and $\Omega_{6\pm}$ are positive energy modes for $q_z \gtrless 0$. However, we shall have to take $q_z > 0$, as the momentum has also to be conserved in the two-mode coupling. As a result, any frequency in the positive energy set can interact with any frequency in the negative energy set, giving rise to various instabilities provided (1) is satisfied. The interaction of the modes due to stream $i = 1$ as well as $i = 2$ with the stationary stream gives usual integral harmonics. However, we shall not concentrate on this at present. We shall return to it when we discuss the case $u_2 = 0$ as in the experiment. The growth rate for each of the instabilities can be calculated by calculating the matrix elements for two-wave coupling (Krishan and Krishan, 1975; Krishan and Ravindra, 1975).

In the following tables various instabilities obtained have been listed, together with their magnitudes, under different value of u_1 and u_2 taken from the experimental set-up, and also the expression for the growth rate of each instability is given to facilitate the comparison of their relative magnitudes. From Tables I–IV we observe:

(i) In the absence of a second stream ($u_2 = 0$), we get the usual integral cyclotron harmonics (see Table I).

(ii) Landauer and Muller (1966) did an experiment under complete reflection of the primary beam and observed $\omega \simeq (\frac{1}{2}n)\omega_{ce}$ modes. Now if we put $u_2 = u_1$, for complete reflection, we get $\omega = (\frac{1}{2}n)\omega_{ce}$ as listed in Table II. To calculate the growth rates of various modes in this situation we have put $\omega_{p2}^2 \simeq \omega_{p1}^2$ since the densities of the two beams, to a good approximation, are nearly equal. According to the experiment of Bernstein *et al.*, only modes $\omega = n\omega_{ce}$ are observed, starting with $n = 2$.

TABLE I

Frequency spectrum and growth rates when the secondary beam is absent, i.e. $u_2 = 0$, $\omega_{p2} = 0$

ω/ω_{ce}	$\gamma\left[\frac{\omega_{ce}c^4q_\perp^4}{\omega_{p0}^3\omega_{p1}}\right]$
1	$\frac{\omega^2}{8}$
2	$\sqrt{\frac{\alpha_0}{32}}\,\omega^2$
3	$\frac{\alpha_0}{32}\,\omega^2$
4	$\left[\frac{\alpha_0}{8}\right]^{3/2}\frac{\omega^2}{2\sqrt{3}}$
5	$\frac{\alpha_0^2}{2^8\sqrt{3}}\,\omega^2$
6	$\frac{\alpha_0^{5/2}}{2^9\sqrt{15}}\,\omega^2$

TABLE II

Frequency spectrum and growth rates under complete reflection of secondary beam when $u_1 = u_2$

ω/ω_{ce}	$\gamma\left[\dfrac{\omega_{ce}c^4q_\perp^4}{\omega_{p0}^2\omega_{p1}^2}\right]$
$\dfrac{2u_1 - u_2}{u_1 + u_2} = \dfrac{1}{2}$	$\sqrt{\dfrac{\alpha_2}{32}}\,\omega^2$
$\dfrac{3u_1 - u_2}{u_1 + u_2} = 1$	$\dfrac{\alpha_2}{32}\,\omega^2$
$\dfrac{4u_1 - u_2}{u_1 + u_2} = \dfrac{3}{2}$	$\dfrac{\alpha_2^{3/2}}{32\sqrt{6}}\,\omega^2$
$\dfrac{6u_1 - u_2}{u_1 + u_2} = \dfrac{5}{2}$	$\dfrac{\alpha_2^{5/2}}{2^9}\dfrac{\alpha^{1/2}}{\sqrt{15}}\,\omega^2$

(iii) When the primary beam is not reflected, Bernstein *et al.* observe modes with $\omega \simeq (n + \frac{1}{2})\omega_{ce}$. At the time of complete reflection of the primary beam, its return velocity in the spatial domain, where plasma interactions are observed, is a crucial parameter. To enable our theoretical calculations to match their experiment, we took

TABLE III

Frequency spectrum and growth rates for the value $u_1 = \frac{1}{5}u_2$. For $\alpha_2 > 2 \times 10^{-2}$, $\omega/\omega_{ce} = 1.5$ has the highest growth rate as in the experiment of Bernstein *et al.* (1975)

ω/ω_{ce}	$\gamma\left[\dfrac{\omega_{ce}c^4q_\perp^4}{\omega_{p0}^2\omega_{p1}\omega_{p2}}\right]$
$\dfrac{u_1 - u_2}{u_1 + u_2} = 0.66$	$\dfrac{\omega^2}{8}$
$\dfrac{2(u_1 - u_2)}{u_1 + u_2} = 1.33$	$\dfrac{\sqrt{\alpha_1\alpha_2}}{4}\,\omega^2$
$\dfrac{2u_1 - u_2}{u_1 + u_2} = 1.5$	$\sqrt{\dfrac{\alpha_2}{32}}\,\omega^2$
$\dfrac{3u_1 - 2u_2}{u_1 + u_2} = 2.16$	$\dfrac{\alpha_2}{16}\sqrt{\dfrac{\alpha_1}{2}}\,\omega^2$
$\dfrac{3(u_1 - u_2)}{u_1 + u_2} = 1.98$	$\dfrac{\alpha_1\alpha_2}{2^8}\,\omega^2$
$\dfrac{3u_1 - u_2}{u_1 + u_2} = 2.33$	$\dfrac{\alpha_2}{32}\,\omega^2$

$u_2 = \frac{1}{5}u_1$. The various modes thus excited are listed in Table III along with their growth rates. The first few modes we obtain are $\omega/\omega_{ce} = 0.66, 1.5, 1.33, 1.98, 2.16, 2.33$. The modes with frequencies higher than these also exist, but they do not follow any particular pattern and have much lower growth rates. From these modes, if $(1 \gg)\alpha_2 > 2 \times 10^{-2}$, then the mode $\omega/\omega_{ce} = 1.5$ has the maximum growth rate whereas $\omega/\omega_{ce} = 1.98$ has the least growth. Further, from the two modes $\omega/\omega_{ce} = 2.16$ and 2.33, located close to each other, the latter has a growth rate higher than the former by a factor of $1/\alpha_2$. Similarly, the mode $\omega/\omega_{ce} = 1.33$ will not be observable as it is very close to the mode $\omega/\omega_{ce} = 1.5$, which has the highest growth rate. Thus the observable modes should be $\omega/\omega_{ce} = 0.66, 1.5, 2.33$, which are roughly given by $\omega/\omega_{ce} = (n + \frac{1}{2})\omega_{ce}$.

In the above experiment another factor was observed which is in agreement with our theory. Namely, if the collector voltage is made more negative, which increases u_2, the frequencies tend to decrease. One can see the same thing happening if we increase u_2 in our calculations. This is true of all the modes whether the growth rate is small or large. As an illustration, we have taken $u_2 = \frac{1}{4}u_1$ in Table IV to stress this point.

TABLE IV

Frequency spectrum and growth rates for the value $u_1 = \frac{1}{4}u_2$. For $\alpha_2 > 2 \times 10^{-2}$ the mode $\omega/\omega_{ce} = 1.4$ has the highest growth rate. Comparing it with Table III, one can notice a drop in the mode frequencies. This phenomenon persists as u_2 is increased, as observed in the experiment of Bernstein *et al.* (1975)

ω/ω_{ce}	$\gamma\left[\frac{\omega_{ce}c^4q_\perp^4}{\omega_{p0}^2\omega_{p1}\omega_{p2}}\right]$
$\frac{u_1 - u_2}{u_1 + u_2} = 0.6$	$\frac{\omega^2}{8}$
$\frac{2(u_1 - u_2)}{u_1 + u_2} = 1.2$	$\frac{\sqrt{\alpha_1\alpha_2}}{4}\omega^2$
$\frac{2u_1 - u_2}{u_1 + u_2} = 1.4$	$\sqrt{\frac{\alpha_2}{32}}\omega^2$
$\frac{3u_1 - 2u_2}{u_1 + u_2} = 2$	$\alpha_2\frac{\sqrt{\alpha_1}}{\sqrt{2}}\omega^2$
$\frac{3(u_1 - u_2)}{u_1 + u_2} = 1.8$	$\frac{\alpha_1\alpha_2}{2^8}\omega^2$
$\frac{3u_1 - u_2}{u_1 + u_2} = 2.2$	$\frac{\alpha_2}{32}\omega^2$

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