NONLINEAR ANALYSIS OF TWO CONTRA-STREAMING ELECTRON BEAMS – PLASMA SYSTEM

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Abstract. The nonlinear theory of electromagnetic modes in the radio frequency range, excited from a system consisting of two contra-streaming electron beams, supports the experimental observation that out of $\omega \sim (n + \frac{1}{2})\omega_{ce}$, $\omega = \frac{3}{2}\omega_{ce}$ is the most prominent mode.

In an earlier paper (Krishan *et al.*, 1978) the stability of a magnetoplasma system carrying two contra-streaming electron beams was investigated in the search of radio frequency emissions near $(n + \frac{1}{2})\omega_{ce}$, where n is an integer and ω_{ce} is the electron cyclotron frequency. The incentive to study this system theoretically has been provided by a large-scale experiment recently performed by Bernstein *et al.* (1975). Their plasma system is composed of the following components: (1) a low-pressure gas in a uniform magnetic field, the strength of the magnetic field being maintained at that of the Earth's magnetic field; (2) a primary energetic electron beam whose energy can be varied from 200 eV to 600 eV, streaming through the gas along the magnetic field; and (3) a negatively biased collector plate which, when hit by the primary electron beam, produces a less energetic, less dense contra-streaming secondary electron beam. This has been done in an attempt to simulate the configuration and the conditions prevalent in the Earth's magnetosphere. A linear analysis of this plasma system has shown the existence of radio frequency excitations near $(n + \frac{1}{2})\omega_{ce}$ for some range of values of the ratio u_1/u_2, where u_1, and u_2 are the streaming velocities of the primary and secondary electron beams, respectively. Further, it was shown that out of all the $(n + \frac{1}{2})\omega_{ce}$ modes, the one corresponding to $n = 1$ has the maximum linear growth rate, thus supporting the experimental finding that $\omega = \frac{3}{2}\omega_{ce}$ is the most prominent of all the modes. This could be better confirmed by comparing the amplitudes of the various modes, rather than their growth rates.

The present work is based on the nonlinear analysis of this plasma system. The nonlinear theory enables one to determine the amplitudes of the modes $(n + \frac{1}{2})\omega_{ce}$, and thereby provides a stronger theoretical base to the experimental observations. To avoid the numerical solution of the dispersion relation, the two contra-streaming electron beams and the stationary plasma have been considered as three independent subsystems of the full system. A normal mode analysis of each of the subsystems then can be performed to find the positive and negative energy modes. These positive

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energy modes interact with the negative energy modes to give rise to instability. The energy for the two-mode coupling thus involved must be conserved – i.e.,

$$|\omega_1| \operatorname{sgn}\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon_i)\right]_{w=w_1} + |\omega_2| \operatorname{sgn}\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon_j)\right]_{w=w_2} = 0, \quad (1)$$

where i and j refer to any one of the subsystems, ω_1 and ω_2 are their normal modes such that ω_1 and ω_2 are the solutions of

$$\varepsilon_i = \frac{q^2c^2}{\omega^2} \cdot \quad (2)$$

The linear dielectric function $\varepsilon_{Li}(\vec{q}, \omega)$ is given by

$$\varepsilon_{Li}(\vec{q}, \omega) = 1 - \sum_n \frac{D_{ni}}{(\omega - sq_zu_i)^2 - n^2\omega_{ce}^2}, \quad (3)$$

where $\vec{q}$ and ω are the wave vector and frequency of the mode; $q_\perp = [q_x^2 + q_y^2]^{1/2}$; $\omega_{pi} = [4\pi n_ie^2/m]^{1/2}$ is the plasma frequency; n_i the particle density; $u_{i=1} = u_1$, $u_{i=2} = u_2$ and $u_{i=0} = 0$; $s = 0$, 1 and -1 respectively for $i = 0$, 1 and 2, corresponding to the stationary plasma, primary and secondary beams; $\sqrt{\alpha_i} = (q_\perp V_{\text{th}\,i})/\omega_{ce} \ll 1$ is the ratio of the wavelength to the Larmor radius ($V_{\text{th}\,i}$ being the thermal velocity of the electrons); and D_{ni} is given for the first few values of n as

$$D_{1i} = \frac{\omega_{pi}^2q_\perp^2}{4q^2}$$

$$D_{2i} = 4\alpha_iD_{1i}$$

$$D_{3i} = \tfrac{3}{16}\alpha_i^2D_{1i}$$

$$D_{4i} = \tfrac{1}{24}\alpha_i^3D_{1i}$$

$$D_{5i} = \frac{5}{3 \times 2^{10}}\alpha_i^4D_{1i}$$

$$D_{6i} = \frac{1}{3 \times 2^{11}}\alpha_i^5D_{1i}.$$

In these equations the ion dynamics has been neglected. The solution of the linear dispersion relation (Krishan *et al.*, 1978) gives the following frequency spectrum with their respective growth rates: for $u_2 = \frac{1}{5}u_1$, we obtain

ω_0/ω_{ce}	$\gamma_0\left[\frac{\omega_{ce}q_\perp^4c^4}{\omega_{p0}^2\omega_{p1}\omega_{p2}}\right]$
$\frac{u_1 - u_2}{u_1 + u_2} = 0.66$	$\omega_0^2/8$
$\frac{2u_1 - u_2}{u_1 + u_2} = 1.5$	$\sqrt{\frac{\alpha_2}{32}}\,\omega_0^2$
$\frac{3u_1 - u_2}{u_1 + u_2} = 2.33$	$\frac{\alpha_2}{32}\,\omega_0^2$

under the assumption that $q_zu_1 > \omega_{ce}$ and $q_zu_2 < \omega_{ce}$, $q_\perp \gg q_z$.

As the wave grows, the electron trajectory is affected by the electromagnetic field associated with the growing wave or, alternatively, the energy of the particle is modified in the presence of the wave field. The technique for finding the change in the particle energy has been described very well by Harris (1969). Following the procedure given there, the modified particle energy $E_{p,n}$ is found to be

$$E_{p,n} = \frac{p_z^2}{2m} + n\hbar\omega_{ce} + \frac{p_\perp^2}{2m}\sum_{q'} \frac{\omega_{Be}^4(\vec{q}\,')}{\Omega_{\mathbf{q}'}^2(\Omega_{\mathbf{q}'} - q_z'(p_z/m))^2}\frac{q_z'^2}{q'^2}J_1^2\left(\frac{q_\perp' p_\perp}{m\omega_{ce}}\right), \tag{4}$$

where $p_\perp$ is the particle momentum perpendicular to the magnetic field; $\omega_{Be}(\vec{q}\,')$, the bounce frequency; $\omega_{Be} = \sqrt{eEq/m}$, where E is the wave field; $\Omega_{\mathbf{q}'} = \omega_0(q') + i\gamma_0(q')$; and $J_1^2(q_\perp' p_\perp/m\omega_{ce})$ is the square of the Bessel function of order unity.

Using this values of the electron energy, after some lengthy calculations, we find the nonlinear dielectric function of the complete plasma system as:

$$\varepsilon_{NL} = 1 - \sum_{n,i} \frac{D_{ni}}{(\omega - sq_z u_i - B_i)^2 - n^2\omega_{ce}^2}, \tag{5}$$

where

$$B_i(\mathbf{q}) = \sum_{\mathbf{q}'} \frac{\omega_{Be}^4(q')\omega_{ce}A_i}{4\Omega_{\mathbf{q}'}^2(\Omega_{\mathbf{q}'} - sq_z'u_i)^2} \tag{6}$$

and

$$A_i = \left[\alpha_i(\mathbf{q})\alpha_i^{1/2}(\mathbf{q}')\frac{q_\perp'}{q_\perp} + \alpha_i^{3/2}(\mathbf{q}')\right]. \tag{7}$$

To solve the nonlinear dispersion relation, let us write

$$\omega = \Omega_0 + \Omega_1, \qquad \Omega_0 = \omega_0 + i\gamma_0, \qquad \text{and} \qquad \Omega_1 = \omega_1 + i\gamma_1.$$

Substituting these expansions, assuming $\omega_1/\omega_0 \ll 1$, and making use of $\sum_{Li} \varepsilon_i(\Omega_0, \mathbf{q}) = q^2c^2/\Omega_0^2$, we find the roots of Equation (5) for the following three cases:

(i) For

$$\frac{\omega_0}{\omega_{ce}} \sim \tfrac{1}{2} \qquad \text{and} \qquad \gamma_0 = \frac{\omega_{p0}^2\omega_{p1}\omega_{p2}}{\omega_{ce}q_\perp^4c^4}\frac{\omega_0^2}{8};$$

the nonlinear damping coefficient γ_1 is given by

$$\gamma_1 = \frac{-5\gamma_0 B_{2R}}{\omega_{ce}},$$

where B_{2R} is the real part of $B_2(\mathbf{q})$. The wave saturates when $|\gamma_1| = |\gamma_0|$, which gives

$$\omega_{Be}^4/\omega_{ce}^4 = \frac{1}{10\alpha_2^{3/2}} \quad \text{for} \quad \frac{\omega_0}{\omega_{ce}} \sim \tfrac{1}{2}. \tag{8}$$

(ii) For

$$\omega_0/\omega_{ce} \sim \tfrac{3}{2} \quad \text{and} \quad \gamma_0 = \sqrt{\frac{\alpha_2}{32}} \frac{\omega_{p0}^2 \omega_{p1} \omega_{p2} \omega_0^2}{\omega_{ce} q_\perp^4 c^4}$$

the nonlinear damping coefficient γ_1 is found to be

$$\gamma_1 = -\frac{7}{2} \frac{\gamma_0}{\omega_0} B_{2R}.$$

The saturation field for this case is given as

$$\frac{\omega_{Be}^4}{\omega_{ce}^4} = \frac{7}{\alpha_2^{3/2}}. \tag{9}$$

(iii) For

$$\frac{\omega_0}{\omega_{ce}} \sim \frac{5}{2} \quad \text{and} \quad \gamma_0 = \frac{\alpha_2}{32} \frac{\omega_{p0}^2 \omega_{p1} \omega_{p2} \omega_0^2}{\omega_{ce} q_\perp^4 c^4}$$

the damping coefficient γ_1 is given as

$$\gamma_1 \sim -\frac{40}{\alpha_2^2} \frac{B_{2R}}{\omega_{ce}} \gamma_0 \frac{\omega_{p0}^4 \omega_{p1}^2}{q_\perp^4 c^4 \omega_{p2}^2}.$$

The mode $\omega_0 \sim \frac{5}{2}\omega_{ce}$ saturates for

$$\frac{\omega_{Be}^4}{\omega_{ce}^4} < \frac{5}{2\alpha^{3/2}}. \tag{10}$$

The last result has been obtained under the assumption that

$$\frac{q_\perp^4 c^4}{\omega_{p0}^4} \ll \frac{16}{\alpha_2^2} \frac{\omega_{p1}^2}{\omega_{p2}^2},$$

which is consistently satisfied for other cases as well. One easily observes from the above treatment that the mode $\omega = \frac{3}{2}\omega_{ce}$ has the maximum amplitude. Therefore, in agreement with the laboratory experimental results of Bernstein *et al.* (1975), one can say with conviction that the interaction of two contra-streaming electron beams is the cause of the observed $\frac{3}{2}\omega_{ce}$ emission at auroral latitudes.

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